

of ways it is possible to select i points out of A_1, A_2, A_3, A_4, A_5 such that the convex hull of the selected points should contain P . Show that $k_3 = k_4$. (5 points) **B. 4916.** Let $P(a, b, c)$ be a fixed point in the 3D Cartesian coordinate system. Let $a, b, c > 0$ and let O denote the origin. Let S be a plane passing through P that intersects the positive coordinate axes at the points X, Y and Z . Show that the volume of tetrahedron $OXYZ$ is a minimum if and only if P is the centroid of triangle XYZ . (4 points) **B. 4917.** Determine all functions $f : (\mathbb{R} \setminus \{0, 1\}) \rightarrow \mathbb{R}$ for which $f\left(\frac{x-1}{x}\right) + f\left(\frac{1}{1-x}\right) = 4 - \frac{2}{x}$. (5 points) (Proposed by *B. Kovács, Szatmárnémeti*) **B. 4918.** Given M unit vectors in the space ($M \geq 2$), prove that it is possible to select $M - 1$ unit vectors such that the length of their sum is at least unity. (5 points) **B. 4919.** Let I be the centre of the inscribed circle of triangle ABC . The inscribed circle touches the sides BC, CA and AB at the points A_1, B_1 and C_1 , respectively. The lines AB and A_1B_1 intersect at point K . Show that the circle centred at K and passing through C_1 , and the inscribed circles of the kites AC_1IB_1 and BA_1IC_1 have a tangent in common, and that tangent passes through the point C . (6 points) **B. 4920.** A frame of width 2 units is drawn around an 8×8 chessboard. In how many different ways is it possible to cover that frame without gaps or overlaps with 1×2 dominoes? (The two arrangements shown in the figure are considered different.) (6 points) (Proposed by *S. Róka, Nyíregyháza*)

New problems – competition A (see page 553): **A. 710.** For which n can we partition a regular n -gon into finitely many triangles such that no two triangles share a side? (Based on a problem of the *2017 Miklós Schweitzer competition*) **A. 711.** For which pairs (m, n) does there exist an injective function $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ under which the image of every regular m -gon is a regular n -gon. (Note that $m, n \geq 3$, and that by a regular N -gon we mean the union of the boundary segments, not the closed polygonal region.) (Proposed by: *Sutanay Bhattacharya, Bishnupur, India*) **A. 712.** We say that a strictly increasing positive real sequence a_1, a_2, \dots is an *elf sequence* if for any $c > 0$ we can find an N such that $a_n < cn$ for $n = N, N + 1, \dots$. Furthermore, we say that a_n is a *hat* if $a_{n-i} + a_{n+i} < 2a_n$ for $1 \leq i \leq n - 1$. Is it true that every elf sequence has infinitely many hats?

Problems in Physics

(see page 570)

M. 373. Measure the heat capacity of a great piece of pebble (or a small sized cobble).

G. 617. A lorry travels at a speed of 70 km/h along a horizontal, circular path of radius 120 m. What is the minimum value of the coefficient of static friction if the vehicle does not slide? **G. 618.** At most what amount of water can be pumped up in a quarter of an hour from a depth of 50 m by means of a submersible pump of power rating 2 kW? **G. 619.** Point C can be connected to either point X or to point Y by means of a piece of flexible wire as shown in the figure of the circuit. *a)* What is the value of current I in the main branch in each case? *b)* What is the value of this current if the flexible wire is disconnected from point C ? **G. 620.** When there is a thunderstorm, and a lightning strikes a lake, then although it has negligibly small probability that a fish is just struck by the lightning, there are bodies of dead fish floating on the surface of the water. What is the reason?

P. 4980. Same as exercise **G. 620.** **P. 4981.** An object of mass m and of speed v is shot through the suspended object of mass M , as shown in the figure, and leaves it at a speed of $v/2$. The hung object of mass M is first suspended on a negligible mass rigid rod of length ℓ , and second it is suspended on a thread of length ℓ . After the object

of mass M was shot through, it moves along the circular path of radius ℓ in both cases. Determine the values of the necessary speed of v in both cases. What is the ratio of these speeds? **P. 4982.** A solid cylinder of radius 5 cm can be rotated about its own horizontal symmetry axis. A long piece of thin thread is wrapped around the lateral surface of the cylinder, such that an object, which has the same mass as the cylinder, is attached to the free end of the thread. *a)* The system begins to move from rest. How many revolutions does the cylinder turn in 1.2 seconds? *b)* What is the speed of the suspended object after N complete turns of the cylinder? (Neglect air resistance.) **P. 4983.** A cylinder, whose walls are thermally insulated, is attached to the inside part of the wall of a fixed closed cylinder as shown in the *figure*. The wall of the container is made of some heat conducting material. The thermally insulated piston of cross sectional area of 1 dm^2 in the cylinder can move frictionlessly. Initially both in the container and in the cylinder there is a sample of air of volume 2.6 dm^3 at a pressure of 10^5 Pa , and at a temperature of 27°C . The air in the cylinder is then heated by means of an electric heater, while the temperature of the air in the container remains constant. *a)* How much distance does the piston move if the temperature of the air in the cylinder increases to 77°C ? *b)* Sketch the change of states of the air inside the cylinder on the p - V diagram. *c)* Estimate how much heat is absorbed by the air inside the cylinder. **P. 4984.** Express, in terms of the ratio of the specific heat capacities, $\kappa = c_p/c_v$, what portion of the absorbed heat by a sample of gas, expanding at constant pressure, is the work done by the expanding gas. **P. 4985.** The focal length of the objective lens of a camera is 3 cm. A photo is taken about a distant object and then the image is enlarged by a scale factor of three. What is observed to be greater, the object or its image on the picture? By what factor is the greater one larger than the other? **P. 4986.** At each vertex of an equilateral triangle a point-like charge Q is fixed. A point-like charge of mass m and of charge q is oscillating in the middle of the triangle along one of the medians of the triangle. The amplitude of the oscillation is much smaller than the diameter D of the circumscribed circle of the triangle. What is the angular frequency of the oscillation? (Consider only the electric forces.) **P. 4987.** The flux linkage of a circular copper loop of radius R and of cross sectional area A , is changing in time as $\Phi(t) = \Phi_0 - kt$. The loop is in uniform magnetic field and the magnetic induction is perpendicular to the plane of the loop. *a)* What is the tension in the loop at the moment of t_0 ? *b)* What is the value of the current in the loop at the moment when the magnetic induction is zero? *Data:* $R = 10 \text{ cm}$, $A = 0.5 \text{ mm}^2$, $\Phi_0 = 0.04 \text{ Vs}$, $k = 5 \text{ mV}$, $t_0 = 2 \text{ s}$. **P. 4988.** The Sun has a radius of 1.4 million km and is making one complete rotation in approximately 25.4 days about its axis, which is approximately perpendicular to the plane of the orbit of the Earth. Consequently one part of the Sun is moving away from us whilst the other is approaching us, which leads to the broadening of the spectral lines due to the Doppler effect. What is the value of this broadening in nm for a spectral line of wavelength approximately 550 nm? **P. 4989.** The half-life of the alpha decaying Uranium isotope of ^{235}U is 704 years. Besides the alpha decays spontaneous fissions may occur as well (resulting in daughter nuclides of greater mass). As an average 0.0056 fissions occur in 1 kg Uranium 235 in one second. *a)* What percent of the ^{235}U nuclides undergo spontaneous fission? *b)* What would the half-life of ^{235}U be if only spontaneous fissions occurred? **P. 4990.** The cross section of the wider part of a fixed vertical tube is A_1 , whilst that of its narrower part is A_2 . In the tube, between two pistons, there is some liquid of density ρ . The pistons are connected by means of a rigid rod of length ℓ . The masses of the rod and the pistons are negligible. The ambient air pressure is p_0 . What is the magnitude and the direction of the force exerted in the rod if *a)* the narrower, *b)* the wider part of the tube is on the horizontal tabletop? What strange thing happens if ℓ is “relatively large”?