P. 4956. Egy csillagászati távcső f fókusztávolságú parabolatükrének tengelye egy adott pillanatban éppen függőleges. A tükör pereme ekkor H-val magasabban van, mint a tükör legmélyebb pontja. Egy m tömegű kis test a tükör peremétől indulva súrlódásmentesen lecsúszik a tükör középpontjáig. Mekkora erővel nyomja ott a tükröt?

(*5 pont*) A *Kvant* nyomán

P. 4957. Egy négyzet alakú drótkeret oldalélei az *ábrán* látható r_1 és r_2 ellenállású huzalokból készültek. A keret az ábra síkjára merőleges, homogén, időben egyenletesen növekvő mágneses indukciójú mezőben van. Mekkora R ellenállású vezetéket kapcsoljunk a négyzet átlójára, hogy az a leggyorsabb ütemben melegedjen?

(*5 pont*) *Izsák Imre Gyula verseny* (Zalaegerszeg) feladata nyomán
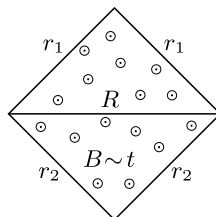
P. 4958. Egy uránércdarabban 200 millió ^{233}U atom található. Az ^{233}U izotóp felezési ideje $1{,}6 \cdot 10^5$ év, és ^{229}Th-ra bomlik, melynek felezési ideje $7{,}8 \cdot 10^3$ év. Ez tovább bomlik ^{225}Ra-ra, melynek felezési ideje 15 nap. Becsüljük meg az uránércdarabban levő ^{225}Ra atommagok számát!
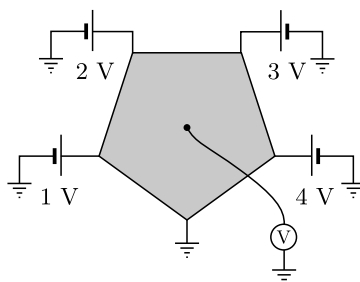
(*5 pont*) *Országos Szilárd Leó Fizikaverseny,* Paks

P. 4959. Egy szabályos ötszög alakú, vékony fémlemez egyik csúcsát lefoldeljük, a többire az ábrán látható módon kis belső ellenállású feszültségforrásokat kapcsolunk. Mekkora feszültséget mutat a lemez középpontjához kapcsolt voltmérő?

(*6 pont*) *Példatári feladat nyomán*

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Problems in Mathematics

New exercises for practice – competition K (see page 353): **K. 547.** Peter thought of a positive integer. He added the number containing the same digits in reverse order. (For example, starting with 26 he added 62 to it, or starting with 530 he

added 35.) He obtained a three-digit number that only contained digits of 6 and/or 9. What may have been Peter's original number? **K. 548.** We have four boxes numbered 1 to 4, and four cards with the numbers 1, 2, 3, 4 on them. We place one card in each box, according to the following rule: every card shows the number of the box that contains the card corresponding to the number of the box containing it. In how many different ways is it possible to place the cards in the boxes? **K. 549.** Three cars are travelling along the same road, in the same direction but at different uniform speeds. In principle, there are six possible orders for the three cars behind each other. Is it possible that all six orders actually occur during their journey? (Proposed by *L. Loránt,* Budapest) **K. 550.** An unusual telegraph company charges for the various words by the letters they contain. Consonants are free, but each vowel costs a certain amount. We do not know these prices, but we do know the charges for a few words we have sent before: TÉGLALAP, PARALELOGRAMMA, NÉGYZET, HÁROMSZÖG, NÉGYSZÖG, ROMBUSZ, TRAPÉZ, DELTOID. (These are all mathematical terms in Hungarian. Y is not a vowel, and vowels with accents on them count as different vowels.) Show a possible method to determine the charge for the word GEOMETRIA. **K. 551.** Find appropriate positive integers $x > y > z$ such that $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{z^2}$ should hold. **K. 552.** What is the largest divisor of 9900 that is divisible by 22, 33 and 55, but not divisible by 44, 50 or 99?

New exercises for practice – competition C (see page 354): **Exercises up to grade 10: C. 1427.** Divide a square into ten acute-angled isosceles triangles. (*Elemente der Mathematik*) **C. 1428.** The product of four consecutive odd numbers ends in a digit of 9. What may be the preceding digit? (*Matlap,* Kolozsvár) **Exercises for everyone: C. 1429.** Ten points are placed in a 5 cm × 8 cm rectangle. Prove that there are two points separated by a distance of at most $\sqrt{10}$ cm. **C. 1430.** Determine all natural numbers x and y such that $\frac{20}{x} + \frac{17}{y} = 1$, and xy is a perfect square. (Proposed by *B. Kovács,* Szatmárnémeti) **C. 1431.** The lengths of the shorter base of a trapezium, then one leg, then the other leg and finally the longer base, in this order, form an arithmetic progression. Given that the length of the shortest side is 3 cm, and one of the angles lying on the longer base is 60 degrees, what is the common difference of the arithmetic progression? **Exercises upwards of grade 11: C. 1432.** Let n be a positive integer. Show that there exists an n-digit number that is divisible by 2^n, and only contains digits of 1 and 2. **C. 1433.** Four $r \times 6r$ rectangles are assembled to form a flexible rhombus, hinged on circles of radius r at the vertices. The circles touch the shorter sides of the rectangles at the midpoints (see the *figure*). The circles can be moved to change the angles of the rhombus, but the rectangles may not overlap. What are the smallest and the largest possible angles?

New exercises – competition B (see page 355): **B. 4885.** Let k and m be two distinct 14-digit positive integers, each containing two of each digit 1, 2, 3, 4, 5, 6 and 7 (like 22133456456717, for example). Prove that $\frac{k}{m}$ cannot be an integer. (*4 points*) (*M&IQ*) **B. 4886.** How many different convex polyhedra are determined by the vertices of a cube? (Two polyhedra are considered different if they are not congruent.) (*3 points*) **B. 4887.** Prove that there are infinitely many number pairs (a, b), such that $a + \frac{1}{b} = b + \frac{1}{a}$, where $a \neq b$. Find the possible values of ab. (*3 points*) (Proposed by *J. Szoldatics,* Budapest) **B. 4888.** From his third birthday onwards, Sebastian always gets a birthday cake shaped like a triangular prism, with one candle in each of the three upper vertices, and as many further candles on the top as needed to make the total equal to his age. No three candles are collinear. Sebastian wants to cut the cake into triangular pieces with vertices at the positions of the candles, without other candles in the interior of the triangles. How many pieces can he form on his kth birthday? (*4 points*) **B. 4889.** The trapezium $ABCD$ has

an inscribed circle. The circle touches base AB at point T, and the parallel base CD at point U. Let M denote the intersection of the lines of legs AD and BC, and let V be the intersection of side AB with line MU. Show that $AT = VB$. (*4 points*) **B. 4890.** Solve the following equation on the set of positive integers: $x - y - \dfrac{x}{y} - \dfrac{x^3}{y^3} + \dfrac{x^4}{y^4} = 2017$. (*5 points*) (Proposed by *B. Kovács,* Szatmárnémeti) **B. 4891.** The circles S_1, S_2, S_3 pairwise touch each other on the outside. Let A, B and C denote the common points of the circles S_1 and S_2, S_1 and S_3, S_2 and S_3, respectively. Line AB intersects the circles S_2 and S_3 again at points D and E, respectively. Line DC intersects circle S_3 again at F. Prove that triangle DEF is right-angled. (*5 points*) (*Kvant*) **B. 4892.** Two players, First and Second, play the following game: they place 2017 pebbles on the table. First starts by removing 1 pebble. Then Second may choose to remove either 1 or 2. Then First may remove 1, 2, 3 or 4. Then Second may remove any number from 1 to 8. And so on, the player in the ith step needs to remove at least 1 and at most 2^{i-1} pebbles. The player removing the last pebble from the table wins the game. Who has a winning strategy? (*5 points*) **B. 4893.** In a triangle ABC, $AB \neq BC$. The angle bisector drawn from point B intersects side AC at point D, and intersects the circumscribed circle again at point E. The circle of diameter DE intersects the circumscribed circle again at a point F, different from E. Prove that the reflection of line BF about the line BD results in a median of triangle ABC. (*6 points*)

New problems – competition A (see page 356): **A. 701.** An airline operates flights between any two capital cities in the European Union. Each flight has a fixed price which is the same in both directions. Furthermore, the flight prices from any given city are pairwise distinct. Anna and Bella wish to visit each city exactly once, not necessarily starting from the same city. While Anna always takes the cheapest flight from her current city to some city she hasn't visited yet, Bella always continues her tour with the most expensive flight available. Is it true that Bella's tour will surely cost at least as much as Anna's tour? (*Based on a Soviet problem*) **A. 702.** Fix a triangle ABC. We say that triangle XYZ is elegant if X lies on segment BC, Y lies on segment CA, Z lies on segment AB, and XYZ is similar to ABC (i.e., $\angle A = \angle X$, $\angle B = \angle Y$, $\angle C = \angle Z$). Of all the elegant triangles, which one has the smallest perimeter? **A. 703.** Let $n \geqslant 2$ be an integer. We call an ordered n-tuple of integers primitive if the greatest common divisor of its components is 1. Prove that for every finite set H of primitive n-tuples, there exists a non-constant homogenous polynomial $f(x_1, x_2, \ldots, x_n)$ with integer coefficients whose value is 1 at every n-tuple in H. (Based on the sixth problem of the *58th IMO,* Brazil)

Problems in Physics
(see page 378)

M. 370. Measure the tilt angle of at least three different types of grainy food (e.g. rice, poppy seed, flour, granulated sugar or powdered sugar).

G. 605. Two trains are travelling along two parallel tracks. The speed of one of them is 80 km/h. The distance between them is 4.8 km, and after a quarter of an hour the distance between them is the same. What is the speed of the other train if both trains have the same length of 200 m? **G. 606.** The heat capacity of a calorimeter is to be measured, therefore 65 g water at a temperature of 45 °C is poured into the calorimeter, which has already contained for a long time 150 g water at a temperature of 17 °C. The final temperature of the mixture is 25 °C. What is the heat capacity of the calorimeter? **G. 607.** Calculate the equivalent resistance of the circuit shown in the *figure* across points A and B, and across the points C and D. **G. 608.** The magnetic effect of a relatively weak electric current flowing in a piece of vertical wire is to be shown by means of a compass.