A NEM-SZIMETRIKUS FOGALAKÚ HENGERES FERDE FOGÚ FOGASKERÉKPÁROK KAPCSOLÓSZÁM SAJÁTOSSÁGAI

CHARACTERISTICS OF CONTACT RATIO IN CASE OF CYLINDRIC HELICAL GEAR PAIRS WITH NON-SYMMETRICAL TOOTH SHAPE

Drágár Zsuzsa, Dr. Kamondi László, PhD
Miskolci Egyetem, Gép- és Terméktervezési Intézet

ABSTRACT
A cikk a nem-szimmetrikus alapprofilú hengeres ferde fogú fogaskerékpárok geometriai meghatározásán belül a kapcsolószám meghatározásával foglalkozik. Az irodalomból ismert a kapcsolószám meghatározásának elmélete, de nem tartalmazza azt az általánosítást, mely akkor is alkalmazható, ha a kapcsolómezőt meghatározó fejtetőhenger a szokásostól eltérő. Jelen estben csak szabályos négyszögű kapcsolómezőre mutatunk be általánosítást, mely a későbbiekben kiterjesztésre kerül.

1. INTRODUCTION

The application of gears in power train has not decreased as earlier predicted, but rather increased. Contact solutions of specific element pairs help to understand this. This study deals with the application conditions of non-symmetrical gear pairs, including some features of contact ratio. It is because of that, it determines and influences substantially the load carrying capacity and meshing excitation, as source of vibration and noise.

2. REASONS OF NON-SYMMETRICAL GEAR PAIRS PRESENCE

The possibilities of the gear manufacturing technology have resulted many new solutions in the past decade [1]. Roughing and finishing process of tools with regular and standard edge-geometry gave several constrains for designers, because the gear pair could not have been suited to perform its required function. It appeared especially at the design of drive systems in the automotive industry and energy converters.

The technological development enabled to produce gears on other principles. For example, injection moulding, non-metallic and then metallic 3D printing. The base of these technologies is the exact geometrical mapping of the teeth, including one tooth complying with the meshing principles. The exact mapping consists of top land, face and flank, fillet and bottom land surface geometry.

3. SPECIFIC CHARACTERISTICS OF MESHING

The non-symmetrical base profile is non standardized, so developers of toothed element pairs have more possibilities to design contacts with more favourable properties [2]. It can be appeared in one hand at forming of the load carrying capacity, on the other hand of tribological attributes. The base profile can be altered in two way:

- by base profile angles (α_d, α_c), which are different from each other (Fig. 1. a.),
- by alternative forming of tool top land (Fig. 1. a.: rounded, Fig. 1. b.: a general curve, Fig. 1. c.: without correction).

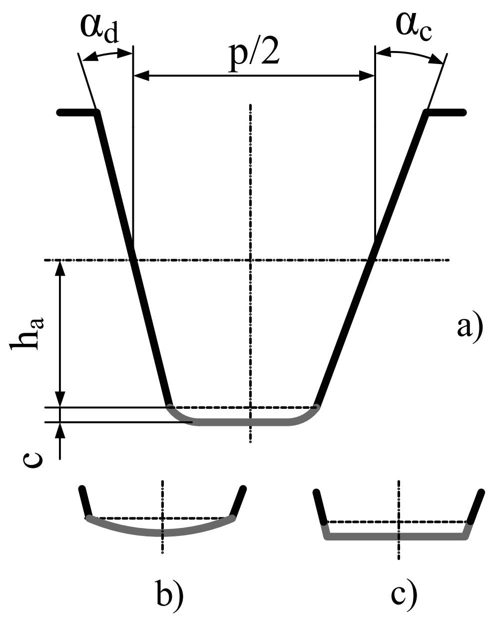

Figure 1. Non-symmetrical base profile tool top land with variations

The unusual shaping of the tool top land defines basically the surface geometry of fillet and bottom land [3], which influences the determination of nominal and maximum tooth root stress. It also largely depends on the determinability of form factor [4].

4. CONTACT FIELD DEPENDS ON DIRECTION OF ROTATION

Different base profile angles (in indexes **c**- coast, **d**- drive side) cause changing of the actual pressure angle depending on the direction of rotation. Not considering the tooting systems and analysing elementary meshing only, the pressure angle always equals with value of base profile angle in transverse plane ($\alpha \leftrightarrow \alpha_t$). The pressure angle is $\boldsymbol{\alpha_{t,d}}$, when drive sides mesh (with smaller basic profile angle, in general), the pressure angle is $\boldsymbol{\alpha_{t,c}}$ when coast sides mesh (reverse direction of rotation) (Fig. 2.). In case of given β tooth directional angle on pitch cylinder

$$\alpha_{t,i}=\mathrm{arctg}\left(\frac{\mathrm{tg}\,\alpha_i}{\cos\beta}\right) \qquad (i=d,c)\,. \qquad (1)$$

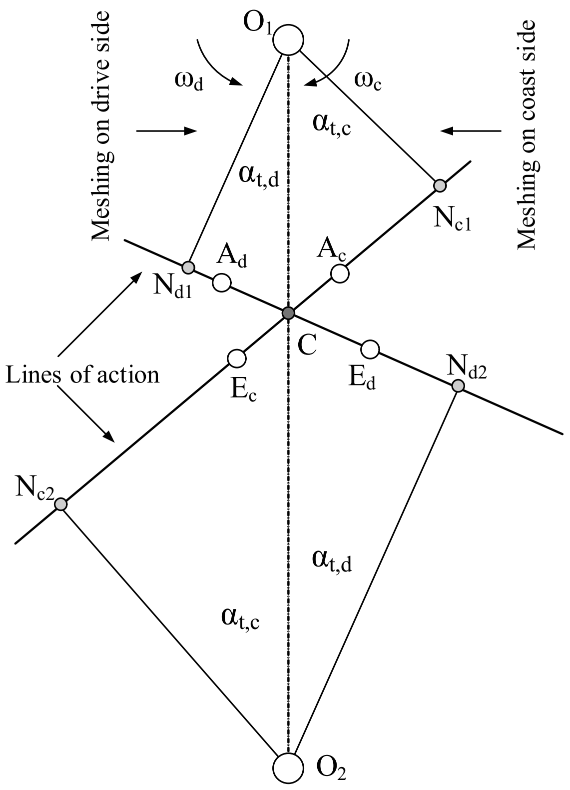

Figure 2. Meshing characteristics depends on direction of rotation

Letters $\mathbf{A_i}$ sign the actual beginning of mesh, letters $\mathbf{E_i}$ sign the apparent end of mesh, which are resected by the head cylinders of gears. The apparency is given by the characteristics of contact field, in which the common face width (b) and tooth directional angle ($\boldsymbol{\beta_{b,i}}$) in contacting plane have important role, where

$$\beta_{b,i}=\arcsin(\cos\alpha_i \cdot \sin\beta) \qquad (i=d,c)\,. \qquad (2)$$

Changing of direction rotation of the driver gear results that meshing areas/fields are changed (Fig. 3. and 4.) Values of tooth direction angles in base plane ($\boldsymbol{\beta_{b,c}}$, $\boldsymbol{\beta_{b,d}}$), normal section values of base pitch in transverse plane ($\mathbf{p_{bt,d}}$, $\mathbf{p_{bt,c}}$) and its axial directed components ($\mathbf{p_{x,d}}$, $\mathbf{p_{x,c}}$) changes.

$$p_{bn,i} = p_n \cdot \cos\alpha_i \qquad (i=d,c)\,, \qquad (3)$$

$$p_{bt,i} = \frac{p_{bn,i}}{\cos\beta_{b,i}}\,, \qquad (4)$$

$$p_{x,t} = \frac{p_{bn,i}}{\sin\beta_{b,i}}\,. \qquad (5)$$

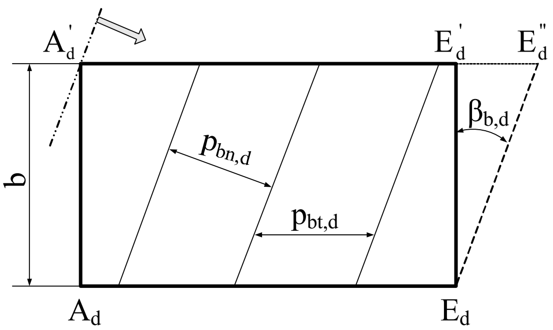

Figure 3. Contacting field on the drive side

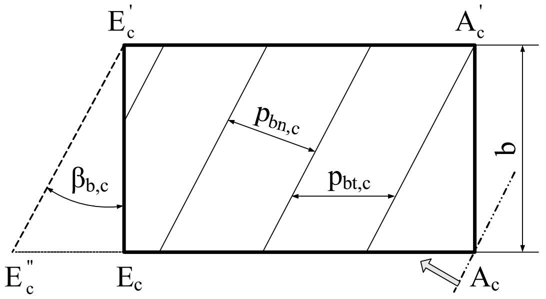

Figure 4. Contacting field on the coast side

Changing of direction rotation changes $\overline{A_dE_d}$ and $\overline{A_cE_c}$ field lengths in the contacting / meshing plane, with relation this, the $\overline{E'_dE''_d}$ and $\overline{E_cE''_c}$, apparent contact lengths will change, as increment of field lengths.

5. GENERALIZED DETERMINATION OF CONTACT RATIO

One of the meshing characteristics of the helical cylindric gear pairs, the contact ratio is determined by the geometry of the geometry generated contacting field. The basic geometry data (module, tooth numbers, base profile geometry, addendum modification coefficients) determine the geometric dimensions, which generate the contacting field. These basic geometrical data are as follows: axle distance, pitch- or rolling circle diameters, addendum cylinder diameters, tooth directional angles (helix angle) on pitch cylinder and derived pressure angles. The shape of the contacting field is defined by its dimensions and addendum cylinder surface of gears. In that case, when a line parallel to a shaft – generates the addendum cylinder, the connecting field is a regular tetragon. When the generating curve is not straight, but an arbitrary curve, then the connecting field loses its regular tetragonal shape. Now let us consider only the regular tetragon field. Fig. 5. shows base determination of contacting field in case of non-symmetrical tooth shape contacts. Results can be seen in Fig. 3. and 4.

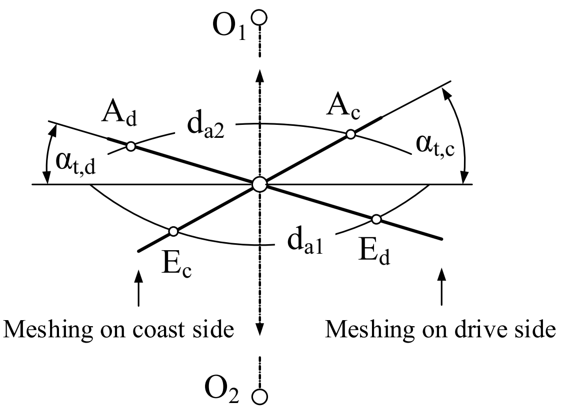

Figure 5. Generation of connecting field, geometrical bases

For the generalization of the contact ratio determination in the meshing field, field proportions perpendicular to the contacting components have to be chosen. (Fig. 6.). The size of field is influenced by the common face width and the dimensions of addendum cylinder and tooth directional angle (helix angle) defined in datum define field dimensions as follows

$$L_i = f(\overline{A_i E_i'}, \beta_{b,i}), \qquad (6)$$

with geometrical dimensions

$$L_i = \frac{\overline{A_i E_i'} + b \cdot \mathrm{tg}\,\beta_{b,i}}{\cos\beta_{b,i}}. \qquad (7)$$

Contact ratio is the quotient of typical field dimensions (L_i) and base pitch in normal section ($p_{bn,i}$)

$$\varepsilon_i = \frac{L_i}{p_{bn,i}}, \qquad (8)$$

with exposing this, a more simply and well-known form is created

$$\varepsilon_i = \frac{\overline{A_i E_i'} + b \cdot \mathrm{tg}\,\beta_{b,i}}{\cos\beta_{b,i} \cdot p_{bn,i}}. \qquad (9)$$

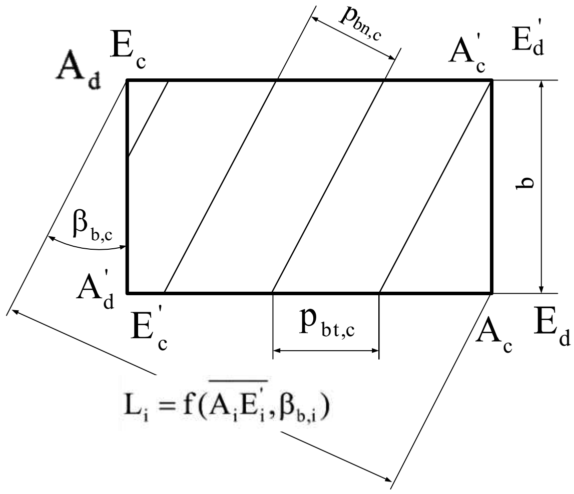

Figure 6. Generalization of contact ratio

This relation let us conclude to the interpretation that contact ratio can be shared, as the gear literature does as well [5]. The contact ratio can be divided into two parts

- interpreted in transverse plane and
- interpreted in axial plane.

It is important, that this sharing is possible only in case of regular tetragonal field, but no corrected top land cylinder. Typical field dimensions in transverse plane comes from the following relationship

$$\overline{A_i E_i'} = \overline{N_{1i} E_i} + \overline{N_{2i} A_i} - \overline{N_{1i} N_{2i}}, \qquad (10)$$

in which addendum circles, base circles, axle-base and actual pressure angle take part,

$$\overline{A_i E_i'} = \sqrt{\left(\frac{d_{a1}}{2}\right)^2 - \left(\frac{d_{b1i}}{2}\right)^2} + \sqrt{\left(\frac{d_{a2}}{2}\right)^2 - \left(\frac{d_{b2i}}{2}\right)^2} - a \cdot \sin\alpha_{t,i}. \qquad (11)$$

According to the above representation, the transverse contact ratio

$$\varepsilon_{\alpha,i} = \frac{\overline{A_i E_i'}}{p_{bt,i}}, \qquad (12)$$

the axial contact ratio

$$\varepsilon_{\beta,i} = \frac{b}{p_{x,i}}, \qquad (13)$$

and the totted contact ratio can be determined as the sum of transverse and axial contact ratio

$$\varepsilon_i = \varepsilon_{\alpha,i} + \varepsilon_{\beta,i}\,. \qquad (14)$$

6. THE CHARACTERISTICS OF CONTACT RATIO CHANGING

The presented method, which is aimed to provide the generalized determination of contact ratio, apart from top land correction, is able to draw conclusions with the help of the attached example. Table 1. contains parameters, which construct gear pair geometry. In this example the base profile angle on the drive side is fixed (close to applications of automotive industry), the base profile angle on the coast side is changed in the given scope, but everything else stays the same.

Table 1. Gear pair geometry

Parameter	Values					
m (mm)	3	3	3	3	3	3
z1	25	25	25	25	25	25
z2	50	50	50	50	50	50
α_d	17°	17°	17°	17°	17°	17°
α_c	20°	21°	22°	23°	24°	25
β	10°	10°	10°	10°	10°	10°
b (mm)	20	20	20	20	20	20

The results of calculations are summed up in the Table 2.

Table 2. Calculations

$\varepsilon_{\alpha d}$	1,769	1,769	1,769	1,769	1,769	1,769
$\varepsilon_{\alpha c}$	1,605	1,558	1,515	1,475	1,438	1,404
ε_{axd}	0,368	0,368	0,368	0,368	0,368	0,368
ε_{axc}	0,368	0,368	0,368	0,368	0,368	0,368
ε_d	2,137	2,137	2,137	2,137	2,137	2,137
ε_c	1,974	1,927	1,884	1,844	1,806	1,772

Analysing the results, the followings are statable, which can be considered as new results of this research:

- Changing of tooth shape (non-symmetrical) in case of leaving unvaried the geometry of gear body (addendum cylinder, common face width) doesn't change the axial component of the summed contact ratio. It keeps at fixed value.
- In application of non-symmetrical tooth shape, direction of rotation changes the extent of summed contact ratio, which is caused by changing of contact ratio in the transverse plane.
- Enlargement of base profile angle decreases the contact ratio, and vice versa.

7. SUMMARY

The goal of this study is to be able to take into account of the specifics from difference to the adequate calculations in case of gear contacts with non-symmetrical tooth shape. The contact ratio is important parameter for opinion of right operation and determination of the conditions of loading capacity. Definition of contact ratio is presented supposing regular tetragonal contacting field.

ACKNOWLEDGEMENT

„The described article was carried out as part of the EFOP-3.6.1-16-2016-00011 "Younger and Renewing University – Innovative Knowledge City – institutional development of the University of Miskolc aiming at intelligent specialisation" project implemented in the framework of the Széchenyi 2020 program. The realization of this project is supported by the European Union, co-financed by the European Social Fund."

LITERATURE

[1] Kapelevich, A.: Geometry and design of involute spur gears with asymmetric teeth, Mechanism and Machine Theory, Pergamon, 35 (2000) pp. 117-130.

[2] Kamondi, L.: A rezgésgerjesztés csökkentésének egy lehetősége hengeres fogaskerékpárok kapcsolódásában. OGÉT 2003. XI. Nemzetközi Gépész Találkozó. Kolozsvár. 2003. május 8-11. p: 129-132.

[3] Drágár, Zs., - Kamondi, L.: Tooth Root Stress Calculation for Non-symmetric Tooth Shape. Géptervezők és Termékfejlesztők XXIX. Szemináriuma, Miskolc, 2013. november 7-8., GÉP, ISSN 0016-8572, LXIV. évf., 2013/6., pp. 25-28.

[4] Drágár, Zs., - Kamondi, L.: Nem-szabványos alapprofilú fogaskerekek tervezésének kérdései. Géptervezők és Termékfejlesztők XXVII. Szemináriuma, Miskolc, 2011. november 10-11., GÉP, ISSN 0016-8572, LXII. évf., 2011/7-8., I. kötet, p. 35-38.

[5] Ernei, Gy.: Fogaskerekek, Műszaki könyvkiadó, Budapest, 1983