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New geoelectric–seismic joint inversion method to determine 2-D structures for different layer thicknesses and boundaries

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The basics of a new joint inversion method are presented. This new method is able to determine 2-D structures employing geophysical measurements based on various kinds of physical principles or arrays together, even for different layer boundaries for the different geophysical methods. The method was used on geoelectric and seismic refraction data. Applications are presented for synthetic computer-generated data and for a set of field data. It will be shown that the so-called function inversion method — by means of the appropriate conditional functions — also gives a good estimation for non-identical boundaries. This method also supports the view that joint inversion is a powerful means to investigate complicated geophysical and geological structures.

Keywords: geoelectrics, seismic methods, joint-inversion, thickness

1. Introduction

If one carries out only a single inversion of geophysical data it can often lead to model parameter estimation which is unreliable; this then appears in the value of the correlation coefficient near ± 1 and in the overall estimation error [INMAN, 1975, SALÁT et al. 1982]. This problem is particularly well-known in geoelectric measurements (VES), where quotient *h*® for *S*-type equivalence and $h \cdot \rho$ *K*-type equivalence are determined but the parameters separately are not [KOEFOED 1979].

Ambiguity of inversion estimation can be diminished by using one of the most important tools in data processing: joint inversion [VOZOFF, JUPP 1975, DOBRÓKA et al., 1991, GYULAI, ORMOS 1999].

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Joint inversion can be carried out with data of geophysical methods based on similar or different physical parameters. The former one is known as simultaneous inversion, the latter being the joint inversion method.

The hitherto known joint inversion methods are efficient (and can be called joint inversion) only if the different geological–(geo)physical models have at least one common parameter: the common parameter(s) is (are) usually the layer-thickness(es). The mutual correlation of the boundaries cannot always be carried out because they are not always identical to any other method. HERING et al. [1995] and MISIEK et al. [1997] developed a new joint inversion method which allows the boundary differences if there are other identical boundaries. The difference of the boundaries may already be considerably large when the application of the joint inversion method is already prohibited. The present joint inversion method developed by us enables joint inversion to be used for a real field model, when there are neither mutual geometrical nor (geo)physical parameters between the two geophysical models.

With this new method the joint inversion method can be used more widely.

2. Function inversion as a joint inversion method

The main idea of function inversion methods is to describe the layer-thicknesses (indirectly the boundaries) and (geo)physical parameters along the profile with functions expanded in series. At first, we determine the function coefficient by inversion and then we compute the local parameters of the model and use them point by point along the profiles.

Inversion for the function coefficients means joint- or simultaneous inversion as the coefficients refer to the whole length of the profile, and we use all the data observed in the geophysical measurement stations for their determination.

The idea of describing model parameters by functions and their application in a seismic inversion was first proposed in applied geophysics by DOBRÓKA [1996, 1997] and DOBRÓKA et al. [1995].

In function inversion, Fourier expansion is very often used to describe the changes of the model parameters [GYULAI, ORMOS 1999, GYULAI 2000, ORMOS et al. 1998] as follows

$$\rho_n(s)=\frac{1}{2}d_{n_0}+\sum_{k=1}^{K}d_{n_k}\cos k\frac{2\pi s}{S}+\sum_{k=1}^{K}d^{\bullet}_{n_k}\sin k\frac{2\pi s}{S},\quad \text{where } n=1,\ \ldots,\ N$$

$$h_n(s)=\frac{1}{2}c_{n_0}+\sum_{l=1}^{L}c_{n_l}\cos l\frac{2\pi s}{S}+\sum_{l=1}^{L}c^{\bullet}_{n_l}\sin l\frac{2\pi s}{S},\quad \text{where } n=1,\ \ldots,\ N-1$$

where function h_n is the thickness function of the nth layer-thickness, ρ_n is the resistivity function of the nth layer and $d_{n_k}, d^{\bullet}_{n_k}, c_{n_l}, c^{\bullet}_{n_l}$ denote the function coefficients. N is the number of layers, and s is the distance of the stations along the profile with length S.

To describe 'slow' change of model parameters the power functions can also be used successfully

$$\rho_n(s)=\sum_{p=1}^{P}a_{n_p}s^{(p-1)},\quad \text{where } n=1,\ \ldots,\ N$$

$$h_n(s)=\sum_{q=1}^{Q}b_{n_q}s^{(q-1)},\quad \text{where } n=1,\ \ldots,\ N-1.$$

The great advantage of using power functions is that we can successfully carry out the joint inversion even for 2 or 3 stations simultaneously.

2.1. Inversion of geoelectric measured data

We have developed a new inversion procedure, which we call the 1.5-D inversion method, for interpreting conventional VES measurements. This involves all the data of the VES stations measured in the direction of the structural strike of the section being linked in one joint inversion procedure [GYULAI, ORMOS 1997, 1999]. We used a 1-D solution of the direct problem at each VES station (i.e. local models) in the section. This procedure increases the extent of reliability of the parameter estimation compared to single inversion as will be shown by synthetic and field examples.

For investigating 2-D structures with parameters changing laterally 'quickly', new methods have been introduced which are different from the conventional VES method. The important element of these procedures is that the measurements are done with an electrode configuration which is a combination of horizontal profiling and vertical electric sounding in the dip direction. The measured data are plotted in pseudo sections and are inverted by 2-D an inversion method [LOKE, BARKER 1996]. The reliability of the inversion results of pseudo sections can be increased by using a function inversion algorithm. The 1.5-D function inversion is one of these methods [GYULAI, ORMOS 1997, 1999] which can increase the reliability of the estimation in such a way that it can compensate the errors caused by 1-D forward modelling [ORMOS et al. 1999]. Another version of geoelectrical function inversion is the combined function inversion method. Here we realize a two-step inversion. At the beginning of the inversion we apply in the inversion algorithm the faster but less accurate 1-D forward modelling (1.5-D inversion), and then the slower but more precise 2-D forward modelling (e.g. finite difference method) [GYULAI 2000].

2.2. Inversion of time data of refracted waves

If the geometrical and (geo)physical model parameters change laterally slowly and continuously in the given 2-D structure, furthermore, conditions for using the refraction method are accomplished, the function inversion method can be applied for the kinematic inversion of refraction time data. BERNABINI et al. [1988] developed a method in which the layer boundaries are described by power functions and it is assumed that the layer velocities are horizontally non-variable. However, in practice it is not unusual for the layer velocities to change horizontally and that is why we developed a function inversion method for the refraction data that allows lateral changes of the physical parameters [ORMOS 2002].

A further advantage of this new method is that we can use several types of functions (e.g. Fourier expansion) in the same inversion procedure. Thus the method is suitable for estimating the parameters of real (complicated) field structures in a joint inversion process.

2.3. Joint inversion of geoelectric apparent resistivity and arrival time data of refracted waves

As we have seen in GYULAI and ORMOS's works [e.g. 1999] on synthetic and field geoelectric data, 1.5-D inversion (function inversion) can considerably reduce uncertainty in estimating the local layer parameters in the 2-D section. To increase the vertical and horizontal resolution of the inversion the information about the model from a single geophysical method for enhancement might not always be enough: we need the joint inversion of data of several geophysical methods. If we do it with function inversion, it means — in practice — a double joint inversion procedure. In engineering and environmental geophysics one of the possibilities of this kind of inversion is the joint inversion of geoelectrical and refraction data for 2-D structures.

ORMOS et al. [1998] and KIS [1998] developed a joint inversion method to evaluate refraction and VES data located in the drift of the 2-D structure. DOBRÓKA et al. [1999] have developed a new, so-called hybrid joint inversion method to evaluate the joint inversion of dip refraction and drift VES data. The algorithm allows the change of either the geometrical or the physical parameters. Furthermore, these joint inversion methods assume that the geoelectric and seismic layer boundaries are the same.

3. Relative efficiency of the joint function inversion method

To characterize the uncertainty of the model parameters estimated by inversion we use the covariance matrix [SALÁT et al. 1982, DOBRÓKA et al. 1991] to compute the parameter error and the correlation matrix. One of the indices of the efficiency of function inversion is the error of the parameter estimation. We define it as:

$$\sigma_{p_s} = \left[\frac{1}{M} \cdot \frac{1}{2N-1} \sum_{m=1}^{M} \sum_{n=1}^{2N-1} \sigma_{p_{mn}} \right]^{1/2},$$

where M — number of measuring stations (local models),
N — number of layers
$\sigma_{p_{mn}}$ — error of nth parameters in the mth local model.

The first step in 1.5-D function inversion is to determine the function coefficients and their errors. From these we can compute the error of the nth parameter in the mth model based on error propagation law:

$$\sigma_{p_{mn}}(\%) = 100 \cdot \frac{\sum_{i=1}^{J(n)} \sum_{j=1}^{J(n)} F_{mni} F_{mnj} (\Delta\sigma_{c_{ij}})^{1/2}}{p_{mn}},$$

where $J(n)$ is the number of function coefficients for the nth layer ($1<n<N$).

In the case of 1.5-D function inversion, the joint inversion method is the most successful if we consider the geometrical parameters of the model (e.g. layer-thickness) or the physical (geophysical) parameters to be non-variable along the profile. It means that the model parameters are the zeroth elements of the function series ($j=1$). Let us define this special case of j as j^*. Real geophysical models often require that we allow horizontal changes of both types of parameters (thickness and physical parameters, too). In this case, we have to take it into consideration that the efficiency of joint inversion may be reduced. If the number of coefficients (for both parameters) reaches the number of measurement stations (local models) ($j=M$) along the profile, the joint inversion turns into sequences of single inversions [GYULAI, ORMOS 1999].

To extend the reliability of the parameter estimation let us define the relative efficiency of the function inversion. The relative efficiency of the function inversion is 100% if the geometrical or the (geo)physical parameters are non-variable along the profile so the number of function coefficients is $j=1$. The relative efficiency of joint inversion is 0 when at each parameter $j=M$ is fulfilled. The relative efficiency is given by:

$$\eta(\%) = \frac{\sigma_{p_s}(\max) - \sigma_{p_s}(estimated)}{\sigma_{p_s}(\max) - \sigma_{p_s}(\min)} \cdot 100.$$

This definition may cause a distortion in the computation of efficiency at high σ_{p_s} values. With high σ_{p_s} values the efficiency improves; in other words, the greater the value of σ_{p_s}, the lower the physical value (it loses its error values) and it relates only to the large degree of uncertainty. This is because the deviation of parameters close to the estimation point are defined with the help of linearization (with the zeroth and first element of Taylor's expansion) so that the characterization of the extent of the confidence region is very inaccurate [SALÁT et al. 1982].

That is why we define the efficiency in another way, deriving it from the model distance

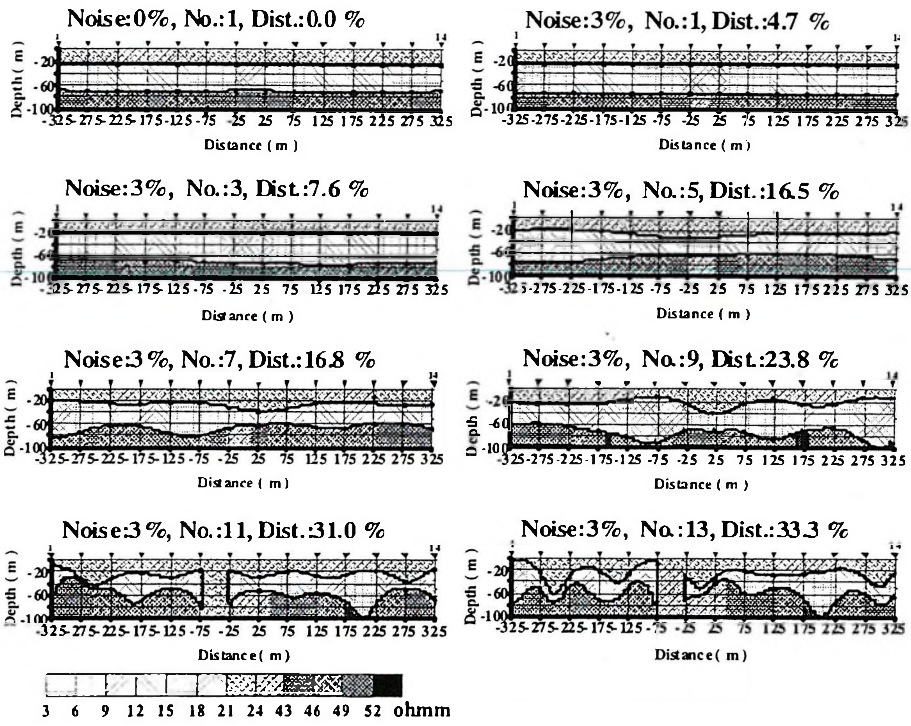
$$\eta^*(\%) = \frac{d(\max) - d(estimated)}{d(\max) - d(\min)} \cdot 100,$$

where d corresponds to the relative model distance [GYULAI, ORMOS 1999], such as the distance of the estimated model from the exact model. One of the problems of this definition is that it can only be used for known models (for investigating synthetic data) and we have to take into consideration that it also shows deviation because it is a probability parameter. We can expect that the relative efficiency determined in two ways is the same and gives similar values if σ_{p_s} is not too high ($\sigma_{p_s} < 50\%$) and the expected value of the model distances is estimated by a sequence of computer runs.

In *Figs. 1* and *2* we demonstrate the changing of the efficiency of joint inversion due to the number of unknown function coefficients. For these investigations we computed synthetic input data for lateral non-variable layer-thicknesses in the first case (Fig. 1), and in the second case for lateral non-variable resistivities (Fig. 2). In both cases we added errors to the synthetic data. During the inversion we allowed the change of the non-variable model parameters along the profile in the first case for the layer-thicknesses, in the second for the resistivities because of the error added to the data. During the test we used more than one coefficient to describe the model parameter mentioned above. (The first, i.e. zeroth, coefficient describes lateral non-variable model parameters.) With increasing number of these coefficients the degree of freedom of the inversion increases. We call them free coefficients. We made a series of inversions with a different number of free coefficients in both cases.

Figures 1 and 2 show the results of 1.5-D function inversions of apparent resistivity with added 3% Gaussian noise for 14 VES stations. VES data were calculated on a 2-D structure in the strike direction for logarithmically equidistant, AB/2=5–800 distances. In the joint inversion there were 14×23 data.

It can be seen from the figures that the increasing number of free coefficients leads to a decrease in the efficiency of inversion (the degree of efficiency), i.e. due to the data errors the estimated model parameters get farther from the theoretical values. In the figures summarizing the results we can also see that the relative model distances (differences from the theoretical models) are very close to the norm of the estimation errors in the case of a low number of free coefficients. With a high number of free coefficients — when the joint inversion turns into a single one — the model



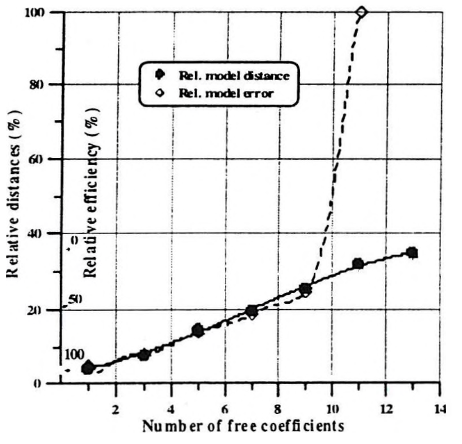
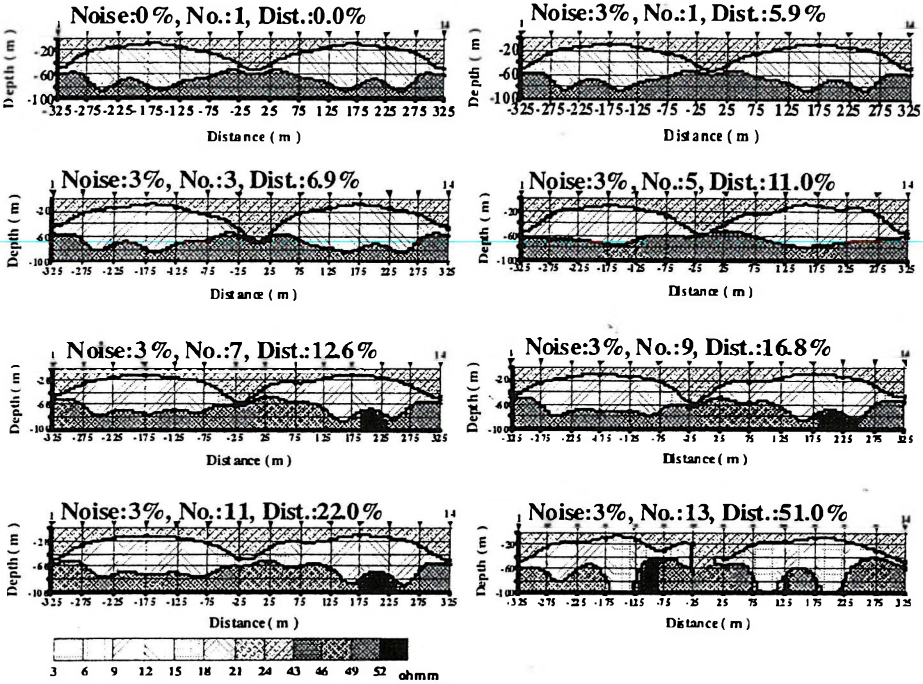


Fig. 1. Efficiency of function inversion for constant layer thickness and variable resistivity (No: number of free coefficients, Dist: distinctness between estimated model parameters and theoretical values)

1. ábra. A függvényinverzió hatékonysága konstans rétegvastagság, változó fajlagos ellenállás esetén (No: a szabad együtthatók száma, Dist.: a becsült és elméleti modell paraméterek közti eltérés)

parameter errors suddenly increase. The ‘relative model error’ curve separates from the model distance curve.

We have already referred to the fact that above a certain level the parameter estimation errors lose their physical values. In our example an



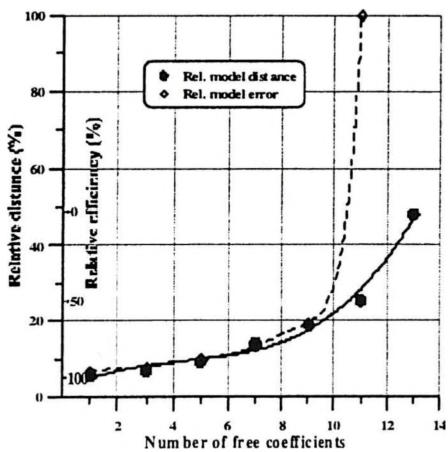


Fig. 2. Efficiency of function inversion for constant resistivity and variable layer thickness (No: the number of free coefficients, Dist: distinctness between estimated model parameters and theoretical values)

2. ábra. A függvényinverzió hatékonysága konstans fajlagos ellenállás, változó rétegvastagság esetén (No: a szabad együtthatók száma, Dist.: a becsült és elméleti modell paraméterek közti eltérés)

increase occurs in the estimation errors in the second layer which, in turn, causes a remarkable increase in the mean value of the whole model.

4. Joint inversion with different layer boundaries

Based on Figs. 1 and 2 we can make an important deduction: the changing of the relative efficiency based on model distance shows that the efficiency of the joint inversion is about 50% with half of the allowable coefficients. From this we can conclude that the joint inversion of different geophysical methods can also be carried out with different layer boundaries (with physical boundaries) by function inversion.

For joint inversion we do not need to demand the agreement of coefficients (mutual correlation of the boundaries) so that we can have free coefficients at different geophysical procedures. This may reduce the efficiency of joint inversion but the different geophysical layer boundaries are allowed to differ from each other.

To solve this problem we chose one of the simplest function inversion methods to describe the parameters by power functions

$$h_{e_j} = a_0 + a_1 s_1 + a_2 s^2 + a_3 s^3 + \ldots + a_n s^n,$$

where h_{e_j} is the thickness of the *j*th layer of the geoelectric model.

$$h_{s_j} = b_0 + b_1 s_1 + b_2 s^2 + b_3 s^3 + \ldots + a_n s^n$$

where h_{s_j} is the thickness of the *j*th layer of the seismic model, apart from the fact that the seismic direct problem is given for boundary depth. If the stations are located very close and

if $a_i = b_i$, i=1,...,n the two boundaries are the same

if $a_0 \neq b_0$ and any other $a_i = b_i$ the parallel translation of the two layer boundary surfaces is allowed

if $a_1 \neq b_1$ and any other $a_i = b_i$ dip changes of the boundary are allowed

etc.

Describing the layer-thicknesses by Fourier expansion we can give the conditions for the similarities of the boundaries from the low or high frequency elements of amplitude spectrum besides the parallel translation of the boundaries. We composed the adjoint function for boundary conditions for linear inversion and we developed the inversion algorithm with it [GYULAI et al. 2000]. It is mentioned that there is a possibility to weight the condition function for the coefficients. We made use of our a priori knowledge for the freedom of coefficients and the weighting of the adjoint auxiliary conditional equation.

4.1. 1.5-D joint inversion of synthetic geoelectric data

We generated synthetic VES data for a given geoelectric model (broken line in *Fig. 3a, b*). In order to simulate a moderately noisy measuring data system we added Gaussian noise of 3% to the calculated apparent resistivity values. In *Fig. 4* the dots denote this sounding data. We carried out the geoelectric joint inversion with 5×21 sounding data. We described the changes of the model parameters by power functions using 5,5,5 coefficients to describe the thickness and 5,3,3,3 coefficients to describe apparent resistivity in the inversion. The estimated model parameters are shown in Fig. 3 (continuous line). The data distance was 2.9% which is equal to 3% Gaussian error. The fitted data (between the theoretical curve and the data denoted by dots) can be seen in *Fig. 6*. The value of the model distance is 16.2%, which derives from the estimation error of the second layer thickness and the apparent resistivity of the second and the fourth layer. The mean of the model error is $\sigma_{p_s} = 31.2\%$.

3.2. Function inversion of synthetic seismic refraction data

The geometrical parameters of the seismic model used by us differed from the geoelectric model so that we could investigate the joint inversion, (described later) for different layer boundaries.

The local parameter changes of the seismic model can be seen in *Fig. 5a, b* plotted by broken lines. We calculated seismic refraction arrival times for the estimated model for 80 geophones. The shot points were positioned equidistantly every 25 m with the first one on the zeroth point of the profile and the last one on its 200 m mark. To simulate field data we added errors to the calculated data. The errors were generated from source synchronizing deviations and high frequency model noise and the sum of Gaussian noise. The generated data system is shown in *Fig. 6* denoted by dots.

In 2-D seismic function inversion [ORMOS 1999] we described the layer-thicknesses by power functions with 4–4 degrees and the velocity by power functions with 6–3–2 degrees. The result of the inversion is shown

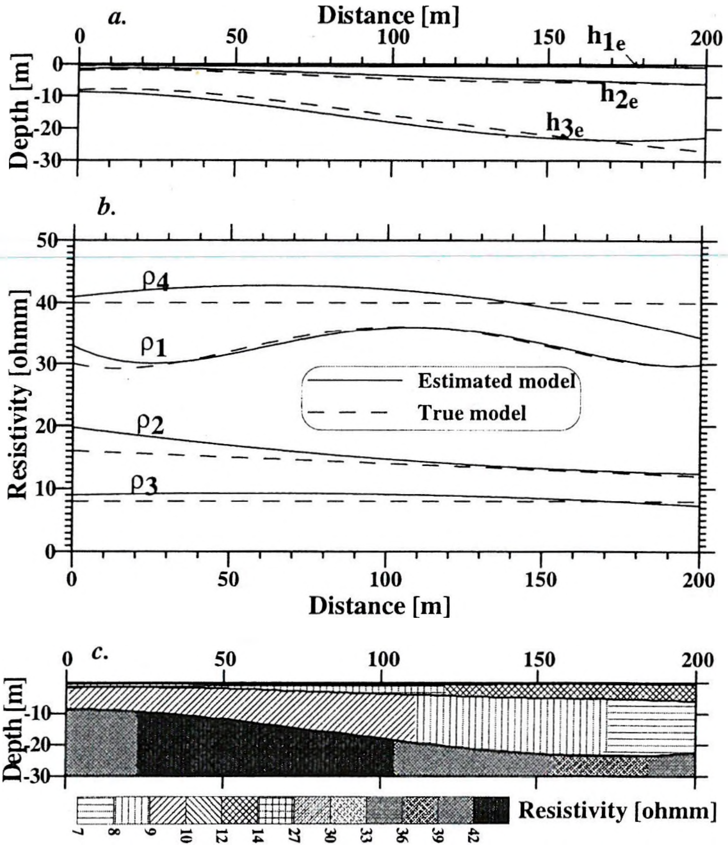


Fig. 3. Result of 1.5-D geoelectric inversion and the theoretical model parameters (plotted with broken line)

3. ábra. Az 1,5-D geoelektromos inverzió eredménye és az elméleti modell paraméterek (szaggatott vonallal rajzolt)

in *Fig. 5a, b* (continuous lines) and *Fig. 5c*. The data fitting error D=6.8%, the model error σ_{p_s} = 45.8%, and the model distance d=17.5% were obtained. In calculating the model distance and the value of the mean error (here and also in the joint inversion) we neglected the parameter data of the two ultimate points because at these points parameter estimation is very unstable due to the lack of refracted arrivals.

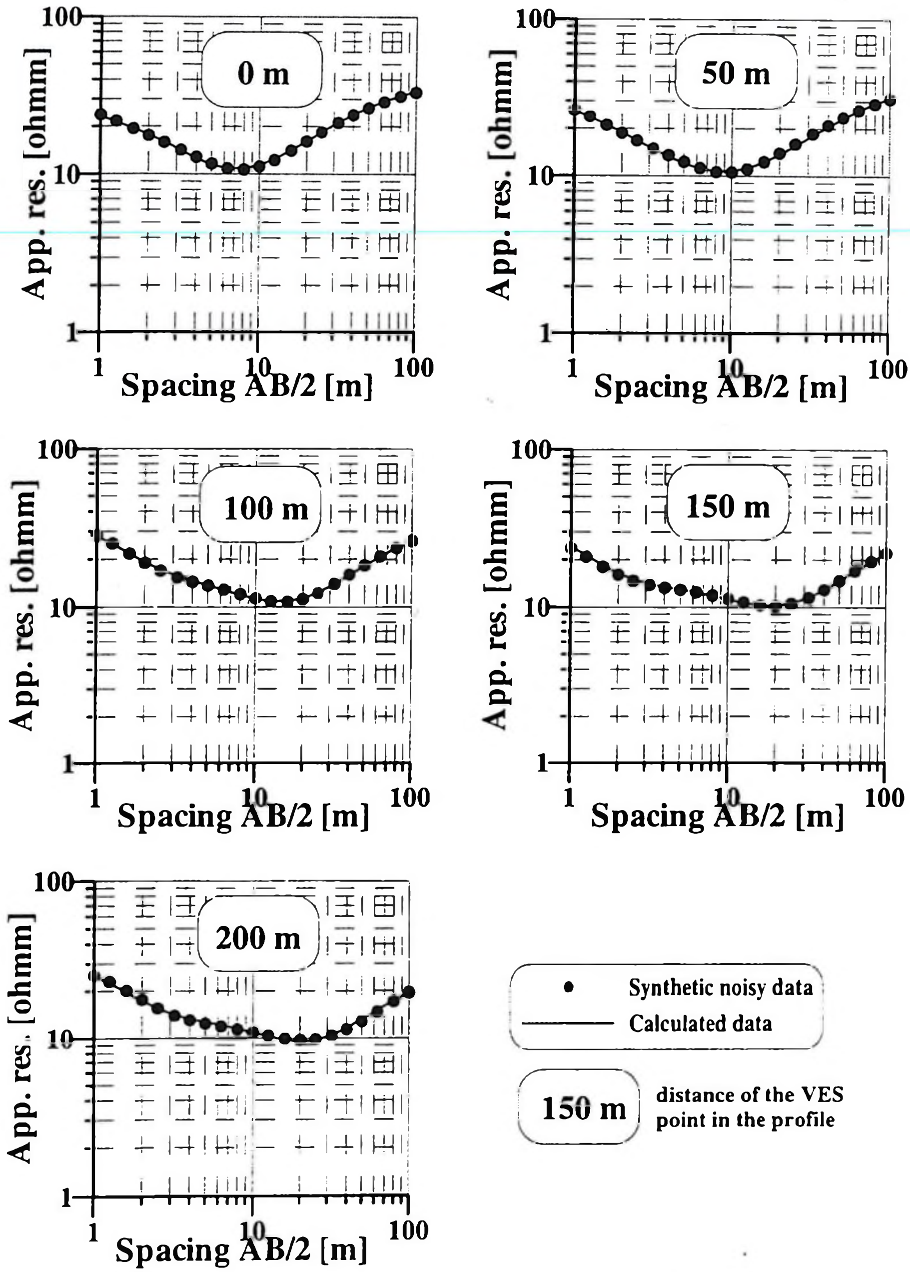


Fig. 4. VES sounding data with added Gaussian noise of 3% and the calculated theoretical curves for the model estimated by inversion

4. ábra. 3%-os Gauss hibával terhelt VESZ szondázási adatok, és a becsült modellből számított elméleti adatok

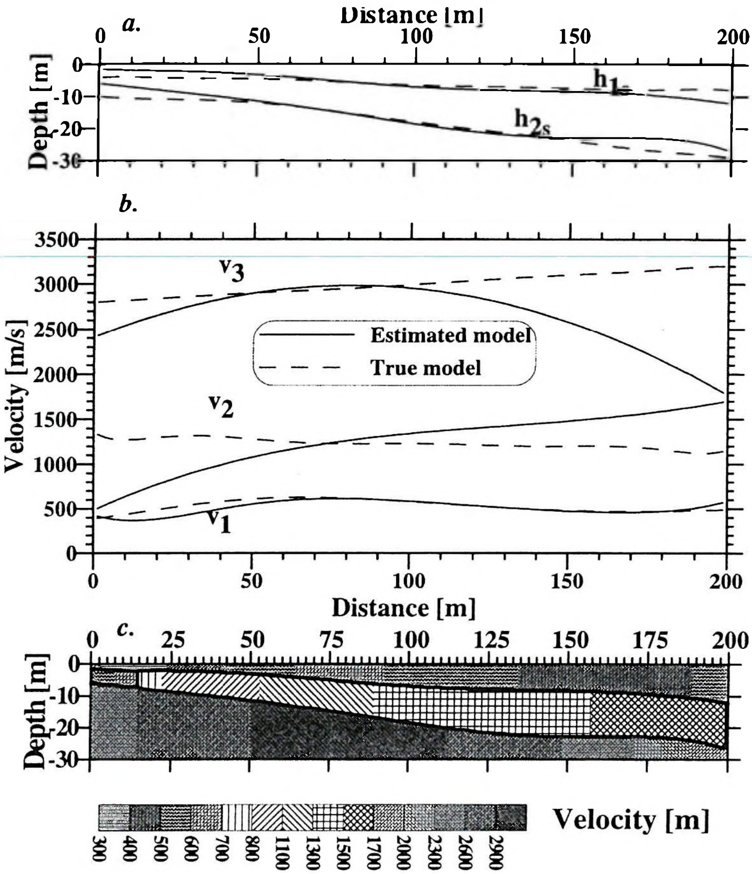


Fig. 5. Result of 2-D seismic inversion and the theoretical model parameters (plotted with broken line)

5. ábra. A 2-D szeizmikus inverzió eredménye és az elméleti modell paraméterek (szaggatott vonallal rajzolt)

4.3. Investigation of the joint inversion of synthetic geoelectric apparent resistivity and synthetic seismic refraction time data

As we have seen in Figs. 3a, b and 5a, b (dotted lines) the thicknesses of theoretical models in the joint inversion differ from each other and the geoelectric model is three-layered whereas the seismic model is two layered:

$$h_{e_1} + h_{e_2} = h_{s_1} - c,$$

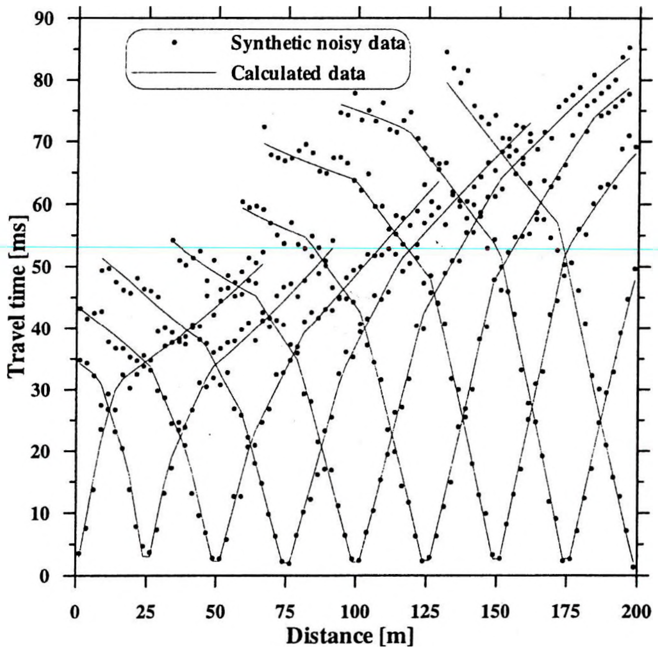


Fig. 6. Synthetic seismic refraction time data (with added noise) and the theoretical curves for the model estimated by inversion

6. ábra. Zajjal terhelt szintetikus refrakciós időadatok, és a becsült modellből számított elméleti görbék

$$h_{e_3} = h_{s_2}$$

where h is given in meters and c=2 m.

We carried out the joint inversion with the same thickness function inversion as described in geoelectric and seismic inversion. We would mention that in the joint inversion algorithm developed by us, the equality of the number of coefficients is not necessary as is shown in the latter field case.

The result of the joint inversion is shown in *Table I*, which contains the changes of the quality parameters as a function of c. It is obvious that during the inversion the theoretical value of $c \approx 2$ m changes due to the data errors. The average model distance minimum was obtained between c=2.2 and 2.4. The well-described model distance minimum shows that the unknown c value can be estimated by an appropriate inversion algorithm.

Geometric const. (c)[m]	Inversion of geoelectric data			Inversion of seismic data			Mean values
	data distance (D) [%]	model error σ_{p_e} [%]	model distance (d) [%]	data distance (D) [%]	model error σ_{p_s} [%]	model distance (d) [%]	model distance (d) [%]
2	2.9	25.3	12.6	7.0	11.9	9.9	11.25
2.2	2.9	25.5	12.4	7.0	11.7	9.2	10.80
2.4	2.9	24.8	12.1	7.1	11.7	9.3	10.70
2.6	2.9	23.9	13.3	7.1	11.3	11.0	12.15

Table I. Quality parameters of geoelectric–seismic joint inversion results, depending on geometrical parameter 'c'
I. táblázat. Geoelektromos–szeizmikus együttes inverzió eredményeinek minősítő paraméterei a „c" geometriai paraméter függvényében

	Inversion of geoelectric data			Inversion of seismic data		
	data distance (D) [%]	model error σ_{p_e} [%]	model distance (d) [%]	data distance (D) [%]	model error σ_{p_s} [%]	model distance (d) [%]
Individual	2.9	31.2	16.2	6.8	45.8	17.5
Joint	2.9	24.8	12.1	7.1	11.7	9.3

Table II. Comparison of quality parameters of geoelectric–seismic single and joint inversions (c=2.4 m)
II. táblázat. A minősítő paraméterek összehasonlítása geoelektromos–szeizmikus egyedi és együttes inverzió esetében (c=2,4 m)

Tables I and *II* show that the distance of the geoelectric model for 34% and the distance of the seismic model for 88% were reduced in the inversion, which results in an average of 61% reduction in model distance. It demonstrates that joint inversion increases the reliability of the parameter estimation for different layer boundaries as well. The result of the estimation by joint inversion is demonstrated along the profile (*Figs. 7, 8*). The relative distances during the iteration are shown in *Fig. 9* for joint

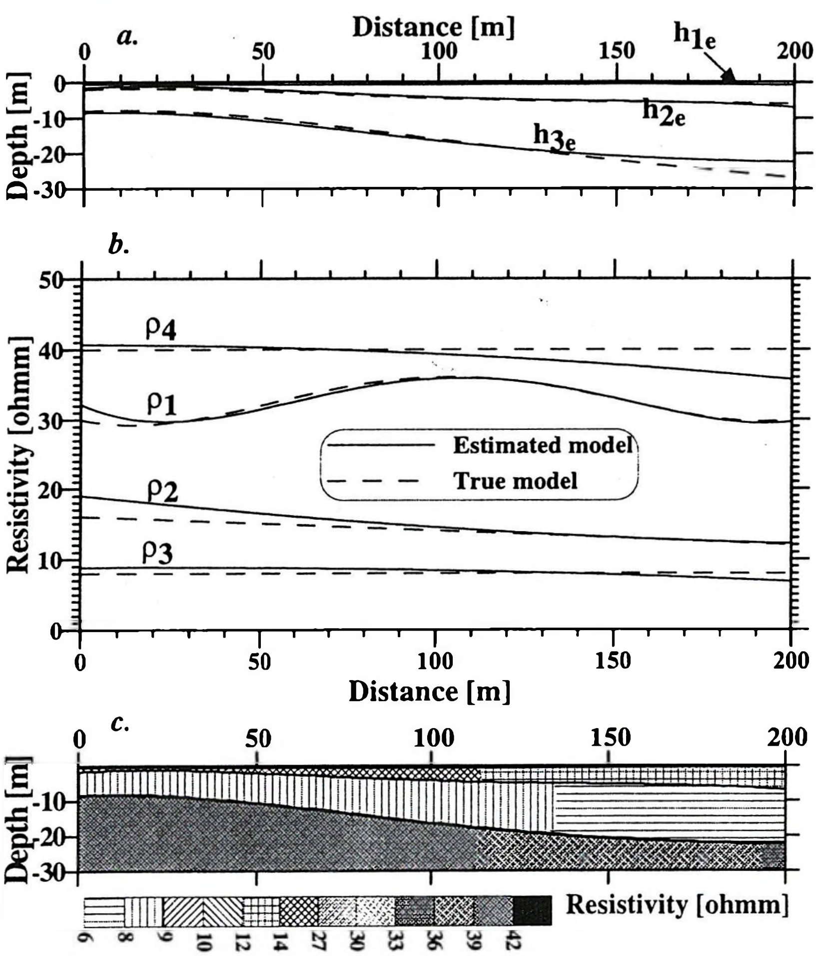


Fig. 7. Geoelectric model estimated by geoelectric and seismic joint inversion and the theoretical values (plotted with broken line)

7. ábra. Geoelektromos–szeizmikus együttes inverzióval becsült geoelektromos modell és az elméleti modell (szaggatott vonallal rajzolt)

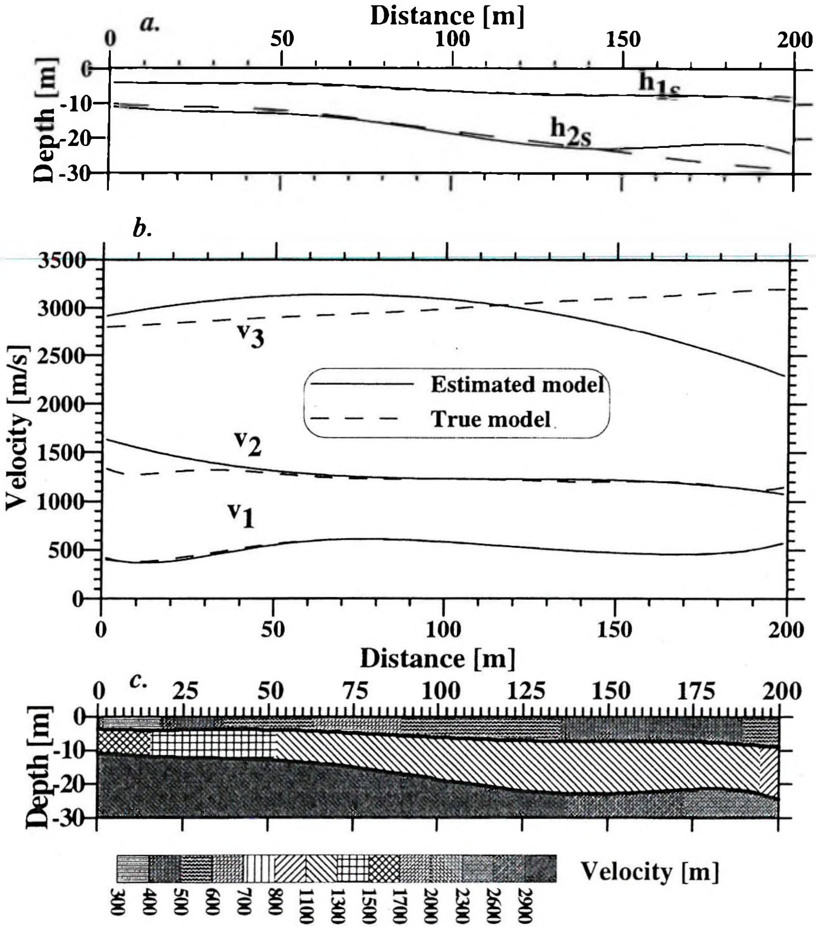


Fig. 8. Seismic model estimated by geoelectric and seismic joint inversion and the theoretical values (plotted with broken line)

8. ábra. Geoelektromos–szeizmikus együttes inverzióval becsült szeizmikus modell és az elméleti modell (szaggatott vonallal rajzolt)

inversion. It is noteworthy that besides the variability of the coefficient correction the model correction changed little during the iteration.

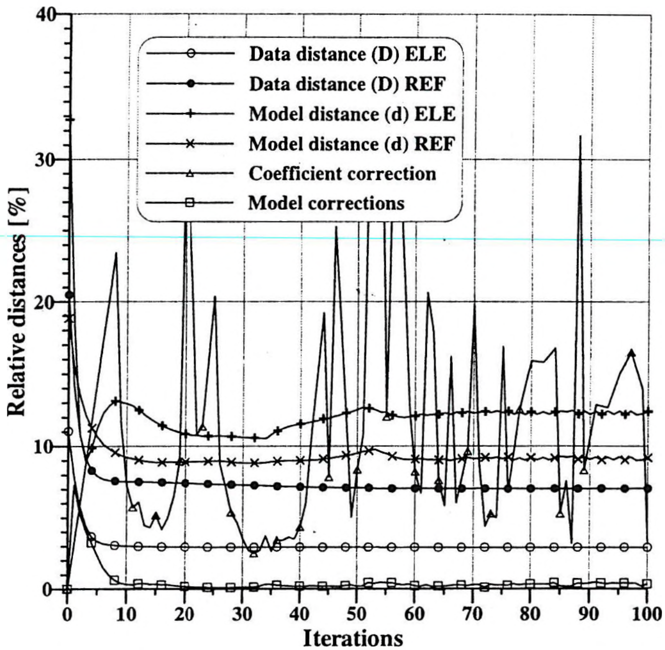


Fig. 9. Iterations of geoelectric and seismic joint inversion for synthetic data
9. ábra. A geoelektromos–szeizmikus együttes inverzió iterációs folyamata szintetikus adatokra

5. Inversion with different layer boundaries, a field study

The geophysical field observations were carried out in the village of Abaújszántó with the primary aim of investigating the positioning of cavities representing inhomogeneity, and not to explore rock stratification. The seismic investigation was planned for the exploration of surface waves and we got the relatively sparse refraction time data as its by-product.

5.1. 1.5-D inversion of geoelectric data

The geoelectric measurements were carried out along a 200 m profile. The 45–95 m long section for the inversion was extracted from this profile. We carried out the measurement using equidistant electrodes with electrode spacing of 2 m. We gained data in two configurations: axial dipole and pole-pole. In the former case the penetration depths were 2–3–4–5–6 m. We described the measured data in pseudosections (see *Fig. 10*). It can be seen clearly from the profile that the geological structure is layered and

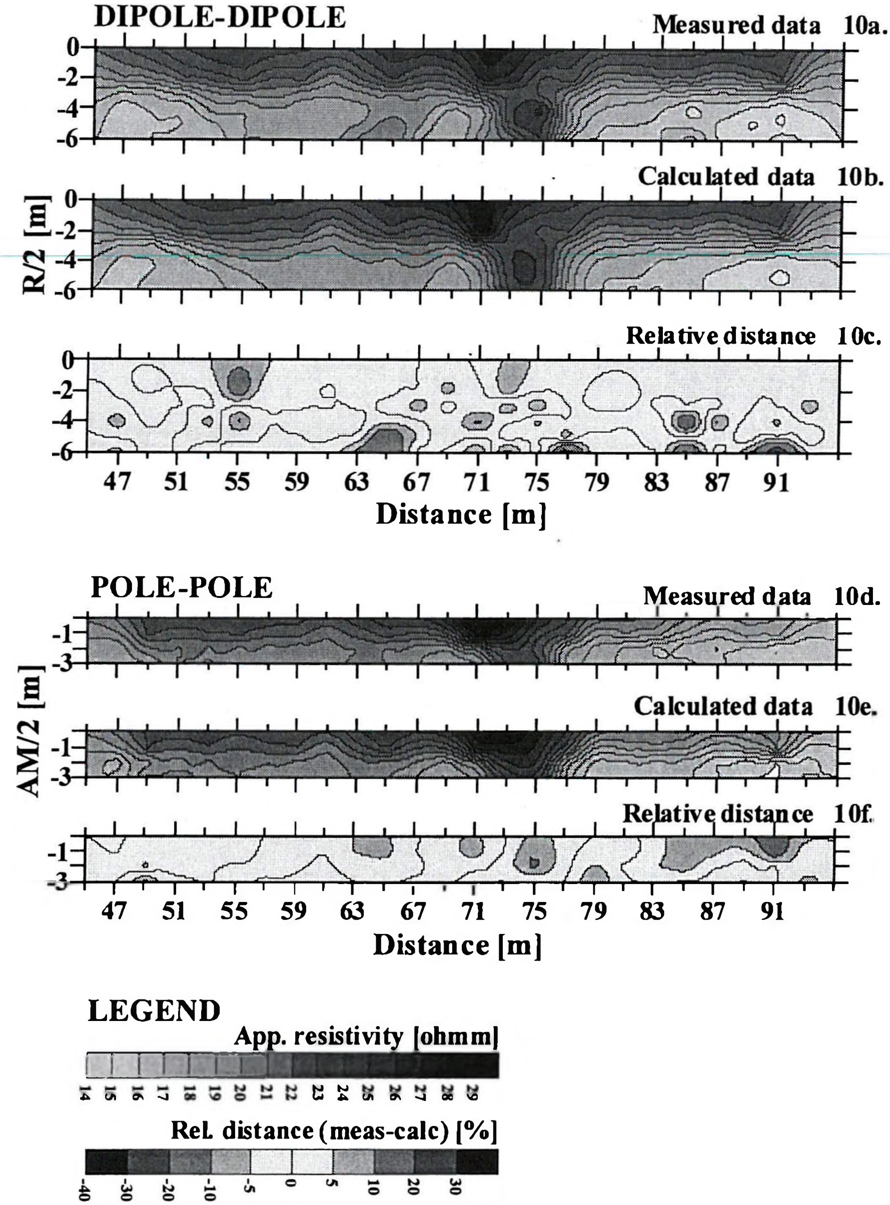


Fig. 10. Dipole-dipole and pole-pole pseudosections plotted with the results of field measurement, the theoretical values of the model estimated by inversion, and the relative distance between the data sets

10. ábra. Terepi dipól-dipól és pól-pól mérésekből, az inverzióval becsült modellből számított elméleti, valamint a kettő relatív eltéréséből számított látszólagos fajlagos ellenállás szelvények at 75 m there is inhomogeneity related to cavity. In Fig. 10 we can also see the data of apparent resistivity in the form of a pseudosection for the model estimated by inversion and the difference of calculated and measured data.

Significant data errors were confined to individual points. In the inversion there were 216 apparent resistivity data, which realized 135 effective data by the MFV-algorithm used here [DOBRÓKA et al. 1991]. In the inversion the data distance was 2.8%, the mean model error $\sigma_{p_s} = 25.0\%$. The result section of the inversion is shown in *Fig. 11*. The 25% mean model error refers to an average model uncertainty which is due to the non-appropriate density of the data.

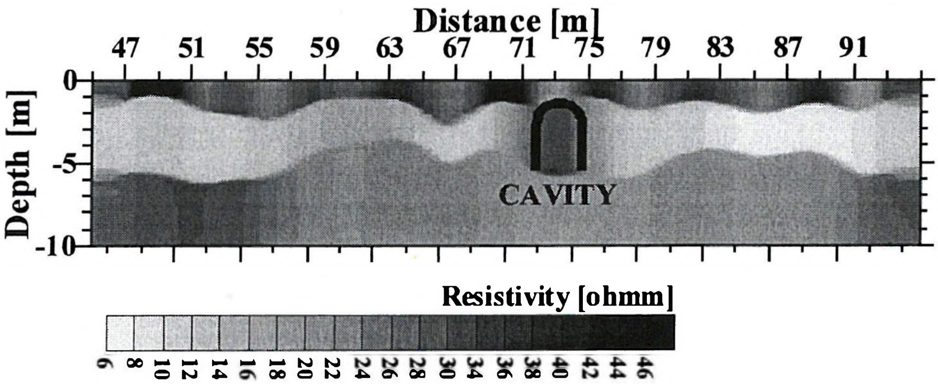


Fig. 11. Result of 1.5-D inversion from field data (interior part of Abaújszántó)
11. ábra. 1,5-D inverzió eredménye Abaújszántó belterületéről származó mérési adatokból

5.2. 2-D inversion of refraction data

Our refraction measurements were also carried out along a 200 m profile, but with 1 m geophone distances, where time data were obtained from 3 shot points. These data and the theoretical curves for the inversion model are shown in *Fig. 12*. We used the MFV-algorithm during the seismic inversion, too. The data distance is D=6.8%, the $\sigma_p = 23.2\%$ value of the average model error refers to a 'moderately low' model estimation. This is due to the incomplete data sets as was already mentioned in the first part of section 4.2. The local values of estimated seismic model parameters are shown in *Fig. 13*.

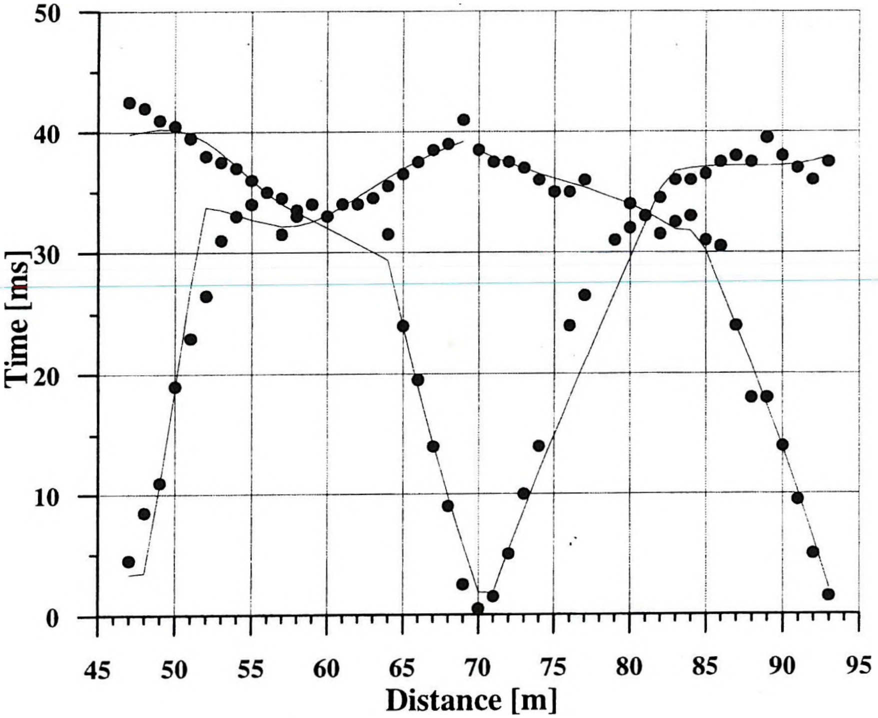


Fig. 12. Seismic refraction time data and the calculated theoretical curves estimated by inversion (Abaújszántó)

12. ábra. Szeizmikus refrakciós mért és az inverzióval becsült modellből számított elméleti időadatok (Abaújszántó)

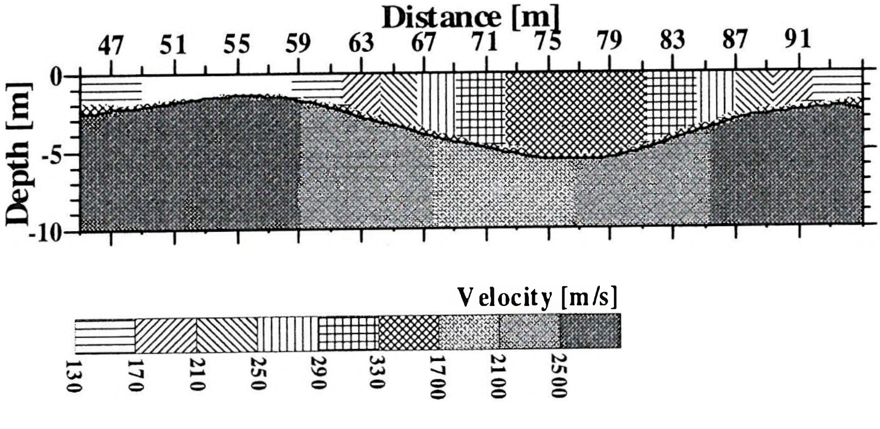


Fig. 13. Seismic field model estimated by 2-D seismic inversion (Abaújszántó)

13. ábra. A 2-D inverzióval becsült szeizmikus terepi modell (Abaújszántó)

5.3. Geoelectric and seismic joint inversion with different layer boundaries

As we can see from the geoelectric and seismic inversion results for the two methods, the boundaries are not identical. The geoelectric one is a three layer and the seismic one is a two layer model and they have not got the same boundary.

For joint inversion as well as in single inversion we used Fourier expansion in the function inversion. In the geoelectric inversion for the layer-thickness we used the expansion up to 9 and 4 harmonics and in seismic inversion up to 3 harmonics. In the joint inversion we assumed that the sum of the first and second geoelectric layer-thicknesses up to the third upper harmonic is the same as the seismic layer-thickness (here the first layer is created with a constant parallel shift). The relative freedom of layer boundaries is allowed by freedom of the further upper harmonics. We would mention that joint inversion failed (was not convergent) for identical geoelectric and seismic layer boundaries. The geoelectric and seismic model estimated by joint inversion is shown in *Fig. 14*. In the inversion the average estimation error of the geoelectric model was reduced from 25.0% to 11.1%; in the seismic model the mean estimation error was improved from 23.2% to 15.7% — which is rather efficient. The values of the S correlation norm — characterizing the correlations between the coefficients [GYULAI, ORMOS 1999] — is S=0.296 in geoelectric inversion, S=0.406 in seismic inversion, and S=0.221 in the joint inversion. The reduction of S in the joint inversion represents a better model parameter reliability than in the single inversion. The changing of the relative distances in the iteration can be seen in *Fig. 15*. The data distance values in joint inversion are practically equal to the values in single inversions.

5. Conclusions

Non-identical model boundaries are often one of the problems in the joint inversion of different geophysical methods. If the difference is significant for certain boundaries, we cannot efficiently use the known inversion methods to reduce the estimation errors. In contradistinction to what has just been said, the new joint inversion method developed by us makes it possible — as has been demonstrated in the synthetic and field examples. With this new procedure 1-D, 2-D, 3-D models can be used in

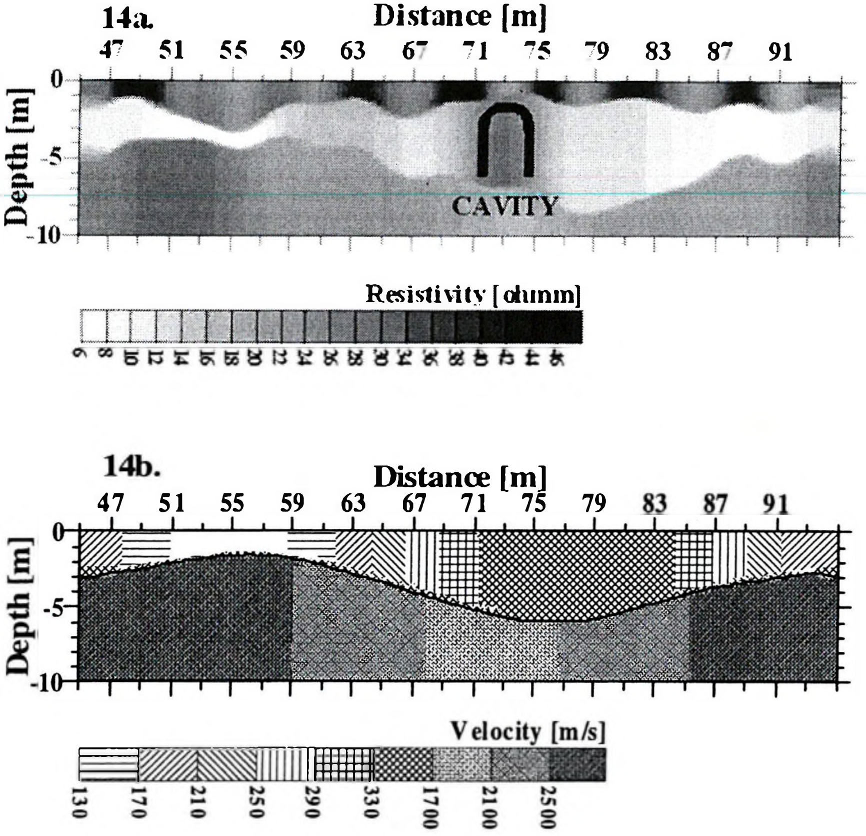


Fig. 14. Geoelectric and seismic models estimated by seismic and geoelectric joint inversion (Abaújszántó)

14. ábra. Geoelektromos és szeizmikus együttes iverzióval becsült geoelektromos és szeizmikus modellek (Abaújszántó)

joint inversion and with the use of the most appropriate geophysical method for a given type of model we can estimate the parameters of complex models more accurately. The new joint inversion method promises to be an efficient tool for interpreting geological, hydrogeological, and environmental investigations.

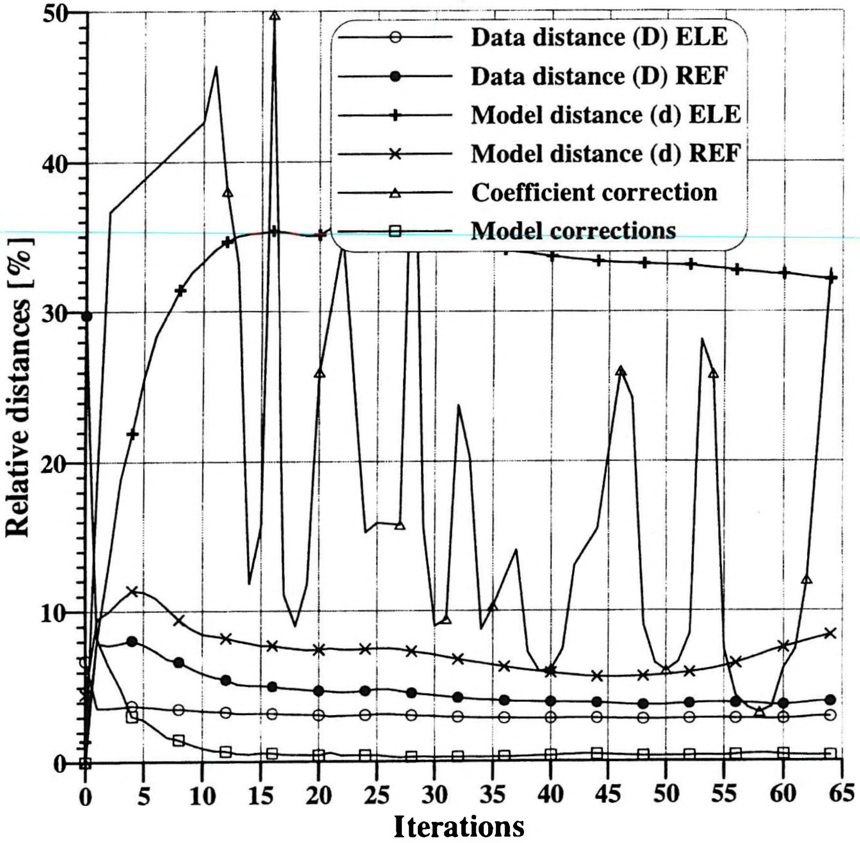


Fig. 15. Iterations of seismic and geoelectric joint inversion for field data (Abaújszántó)
15. ábra. A geoelektromos–szeizmikus együttes inverzió iterációs folyamata terepi adatokra (Abaújszántó)

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Új geoelektromos–szeizmikus együttes inverziós módszer 2-D struktúrák meghatározására eltérő rétegvastagságok, illetve határfelületek esetére

GYULAI Ákos és ORMOS Tamás

A dolgozatban új együttes inverziós módszer alapjait mutatjuk be. Az inverziós módszer 2-D struktúrák meghatározására alkalmas különböző fizikai elven, vagy különböző mérési geometriájú geofizikai módszerek együttes alkalmazásával módszerenként eltérő réteghatárok esetében is. A módszert geoelektromos és szeizmikus refrakciós adatokra alkalmaztuk. Mind ismert modellekre számítógéppel generált adatrendszereken, mind terepi mérési adatokon mutatjuk be a módszer alkalmazását. Bemutatjuk, hogy az együttes inverziós módszerben alkalmazott úgynevezett függvényinverziós eljárás — az együtthatókra vonatkozó megfelelő feltételi egyenletek segítségével — nem identikus határfelületek esetén is jó becslést ad. Ez a módszer az együttes inverzió alkalmazásának az eddigieknél szélesebb lehetőségét teremti meg a bonyolult geológiai–geofizikai struktúrák kutatása területén.

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