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2-D simultaneous inversion method to determine dipping geological structures

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A parameter estimation for a three-layer dipping structure through a field example by using the 2-D simultaneous inversion method is presented. In the inversion we solved the direct problem with an analytical procedure. In the simultaneous inversion the data in dip and in strike direction are presented together. Besides the local values of the layer thickness and the apparent resistivity we also determine the dipping of the layers from VES curves by using the L_2 inversion. The reliability of the estimated parameters is characterized by correlation and variance values. We compare the 2-D inversion results of the measured data in the dip-direction with the 1.5-D inversion results of measured data in the strike direction.

Keywords: inversion, dipping layers, VES

1. Introduction

In the investigation of simple dipping structures the local 1-D approach is often used. This approach results in only small errors up to a dip angle of 20 degrees, in the inversion of the Schlumberger VES data measured in the strike direction. In this method we do not use the characteristic feature that the structural changes are the most pronounced when the measurement is carried out in the dip direction [GYULAI 1995]. That is why, in practice, dip measurements are preferred as a means of determining the horizontal structure changes. It is done in spite of the fact that the interpretation of the data can be carried out only by using 2-D inversion. In these methods the a priori geological knowledge cannot be taken into consideration and the reliability of estimation is unsatisfactory even for simple structures.

For two-layer models with dipping layers CHASTANET de GERY, KUNETZ [1956] published analytical formulae to compute the potential in

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the case of a plane surface. For multi-layer models — starting from only one surface cross line — HMELEVSZKOJ and BONDARENKO [1989] published a solution to calculate the potential. For parallel dipping boundaries BERNABINI, CARDARELLI [1991] plotted apparent resistivity curves which were computed with the help of the so called Alfano integral equations. GYULAI [1995], making use of the potential equations of CHASTANET de GERY, KUNETZ [1956] and HMELEVSZKOJ, BONDARENKO [1989], determined the apparent resistivity equations for different arrays. To compute the resistivity, an algorithm was developed which made it possible to carry out simultaneous inversion on the basis of analytical forward modelling [GYULAI, ORMOS 1996].

Inversion can be realized with data of geophysical methods based on similar or different physical parameters. The former one is called simultaneous inversion, the latter joint inversion.

2. Solution of the direct problem

By simplifying the equations of GYULAI [1995], the potential of the first layer at the surface point M is:

$$U_M = \frac{I\rho_1}{2\pi}\left\{\frac{1}{R}+\frac{2}{\pi}\int_0^\infty f_1(t)\,\mathrm{d}t\int_0^\infty \frac{\cos ts}{\left[z^2+r^2+r_0^2+2rr_0\cosh s\right]^{1/2}}\,\mathrm{d}s\right\} \tag{1}$$

where

r_0 is the distance of the source from the cross line of the surface and of the dip boundary,

r is the distance of the measuring point (M) from the cross line of the surface and of the dip boundary,

R is the distance of M from the source, and $R^2 = r^2 + z^2$,

α_1 is the angle of the dipping layer to the surface.

The function containing $f_1(t)$ layer parameters is:

$$f_1(t) = \frac{\rho_1 T(t)}{Y(t)} \tag{2}$$

$$T(t) = -\sinh(\alpha_n - \pi)t + k_{21}\sinh(\alpha_n + \pi - 2\alpha_1)t + \\ + k_{32}\sinh(\alpha_n + \pi - 2\alpha_2)t - k_{21}k_{32}\sinh(\alpha_n - \pi + 2\alpha_1 - 2\alpha_2)t \quad (3)$$

$$Y(t) = -\sinh\alpha_n t - k_{21}\sinh(\alpha_n - 2\alpha_1)t - \\ - k_{32}\sinh(\alpha_n - 2\alpha_2)t + k_{21}k_{32}\sinh(\alpha_n + 2\alpha_1 - 2\alpha_2)t \quad (4)$$

where

$$k_{i,i-1} = (\rho_i - \rho_{i-1})/(\rho_i + \rho_{i-1})$$

and I=1, ..., 3. α_1, α_2 are the dip angles from the cross section of the layers computed from the horizontal; α_n=π if the surface is flat (*Fig. 1*); I is the inflow current to the rocks; $\rho_1 ... \rho_i$ is the resistivity of the layers.

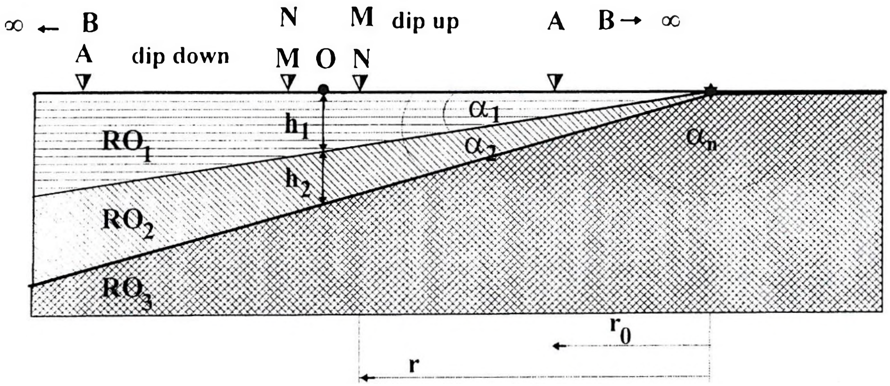

Fig. 1. Measuring array and model
1. ábra. Mérési elrendezés és a modell

Related to the horizontal layered case in the direct problem, the α_1, α_2 dip angles are new elements including the α_n cross and the point of the layer outcrop expressed by r_0 and the dip azimuth value expressed by r and z. These parameters of the dipping structure can be determined from the measured data, provided they have enough parameter sensitivity. For this investigation GYULAI [1995] introduced a new parameter sensitivity function for dipping structures.

Using Eqs. (1)–(4) the apparent resistivity function can be calculated for arbitrary measuring arrays, viz.

$$\rho_a = k \cdot \frac{\Delta U}{I} \tag{5}$$

where

$$\Delta U = \left(U_A^M - U_A^N\right) - \left(U_B^M - U_B^N\right) \tag{6}$$

I is the inflow current to the rock
k is the geometrical constant known in geoelectric practice.

With the help of Eqs. (1)–(6) the sounding theoretical data are generated:

$$\rho_a = \rho_a\left(r, \beta, \vec{p}_e\right) \tag{7}$$

where r is the measuring distance, β is the azimuth of the measuring direction (direction of the sounding compared to the N direction), $\vec{p}_e$ is the vector of electrical model parameters.

The vector of model parameters for a two layer structure is given by

$$\vec{p}_e = \left(h_1, \alpha_1, AZI, \rho_1, \rho_2\right)^T \tag{8}$$

where h_1 is the local depth of the dipping layer at the reference point of soundings, AZI is the azimuth of dipping (clockwise direction from N), ρ_1 and ρ_2 are the apparent resistivities of the layers. The reference point for calculating local parameters in the case of pole-pole array represents the M electrode; in the case of three-electrode Schlumberger- and radial dipole arrays it represent the mid-point of M and the potential electrodes. With this kind of notation of reference points we differ from the conventional reference point used in apparent resistivity pseudosection studies. The reason for the difference is to ensure separation between dip down and dip up data at asymmetrical arrays.

The vector of the model parameter in the multi-layer case is:

$$\vec{p}_e = \left(RKIB, \alpha_1 \ldots \alpha_i, AZI, \rho_1 \ldots \rho_i\right)^T \tag{9}$$

where $RKIB$ is the distance of the common outcrop of dipping layers from the reference point of the sounding where

$\alpha_1 \ldots \alpha_i$ is the measure of dipping
AZI is the azimuth of the dipping

$\rho_1 \ldots \rho_i$ is the resistivity of the layers.

It is mentioned that it is useful to use different algorithms for two- and for multi-layer models. The solution of the direct problem is much simpler, faster and more precise in the case of two-layer models, as is demonstrated by GYULAI [1995].

3. Simultaneous inversion

In the case of dipping layers the number of unknown model parameters is larger than for the horizontally layered (1-D) case. Due to this, the single inversion of the unknown parameters does not give a reliable solution. At the same time the parameter sensitivity investigations [GYULAI 1995] show that the sensitivity for the same array is different in different measuring directions.

A similar phenomenon also occurs for different arrays. In view of the above, not only is it advisable but it is also necessary to apply such a simultaneous inversion method for the parameter estimation that uses the different directions and/or azimuth geoelectric sounding together.

If we use only geoelectric sounding the common parameter vectors of the soundings contain the same elements as each of the soundings would contain. If $\vec{X}$ is the simultaneous inversion parameter vector

$$\vec{X} = \vec{p}_e^{\,single} \tag{10}$$

In simultaneous inversion we estimate the same parameters as in single inversion but the data number is increased and we get more information about the model parameters. In the field case shown later we carried out the simultaneous inversion of a three-electrode (so-called half-Schlumberger) sounding measured in two azimuths. Let these measured data be ρ_{aik}^{AZI1}, ρ_{aik}^{AZI2}. According to the notations used in DOBRÓKA et al. [1991] the theoretical apparent resistivity values are:

$$Y_{ik}^{cal} = Y\left(\vec{X}, \beta_k, r_{ik}\right) \tag{11}$$

where β_k is the azimuth of the kth measurement and r_{ik} is the ith sounding distance belonging to the kth azimuth.

Respectively relating the values of $Y_{ik}^{cal}\left(\vec{X},\beta_k,r_{ik}\right)$ to ρ_{aik}^{AZI1} values and to ρ_{aik}^{AZI2} values and solving the equation

$$\underline{\underline{G}}^T \cdot \underline{\underline{G}} \cdot x = \underline{\underline{G}}^T \cdot y \tag{12}$$

[DOBRÓKA et al. 1991], we can estimate the model parameters of dipping layered models.

To qualify the results we use the relative data distance D, the relative model distance d^{*} (which shows the difference from the start model) defined by HERING et al. [1995], the matrix *cov* and the parameter reliability $\vec{\sigma}_p$ and the correlation matrix *corr* [SALÁT et al. 1982]. In order to get a reliable value for the whole estimation we also computed the

$$\vec{\sigma}_p = \left[\frac{1}{M}\sum_{j=1}^{M}\sigma_{pj}\right]^{1/2}$$

mean value, where M is the number of estimated model parameters.

4. Field case and parameter estimation with simultaneous inversion

The field surveys were carried out at the village of Korlát, Hungary. According to our former knowledge the geological structure can be approximated by a dipping layer geoelectric geophysical model, since on the deepening andesite and andesite tuff there is a gradually thickening clay layer as a cover. The synthetic inversion model studies show that the investigation of such structures can be carried out the most successfully by using asymmetric geoelectric arrays, e. g. three-electrode (so-called half-Schlumberger) surveys and the measurements should be made in at least two directions in order to obtain a reliable parameter estimation.

These two directions are, in practice, the dip down and the up ones, provided we know the azimuth of the dipping. The model and the array are shown in Fig. 1. It can be seen that the dip angles α start from the surface. The surface intersection of the layer boundaries has a specific role because the r and r_0 coordinates of electrodes start from here. Referring back to the fact that in Eq. (9), i.e. in the parameter vectors, it is the distance *RKIB* (the distance between the outcrop line and the point 0) that is represented and not the depths h_1 and h_2 which play a role. In view of this, we can calculate the local depths h_1 and h_2 only indirectly from *RKIB* and dip angle α. Furthermore, the estimation errors can be calculated by error propagating law.

The three-electrode VES data can be seen in *Fig. 2*. At the starting (100 m) and ending (200 m) points of the profile we could only make measurements in the dip down or dip up directions due to some field constraints. At these sites, as the second measuring direction the strike direction (shown in Fig. 4) was used in the simultaneous inversion. At the inner points of the profile it can be seen that the VES data in the two directions

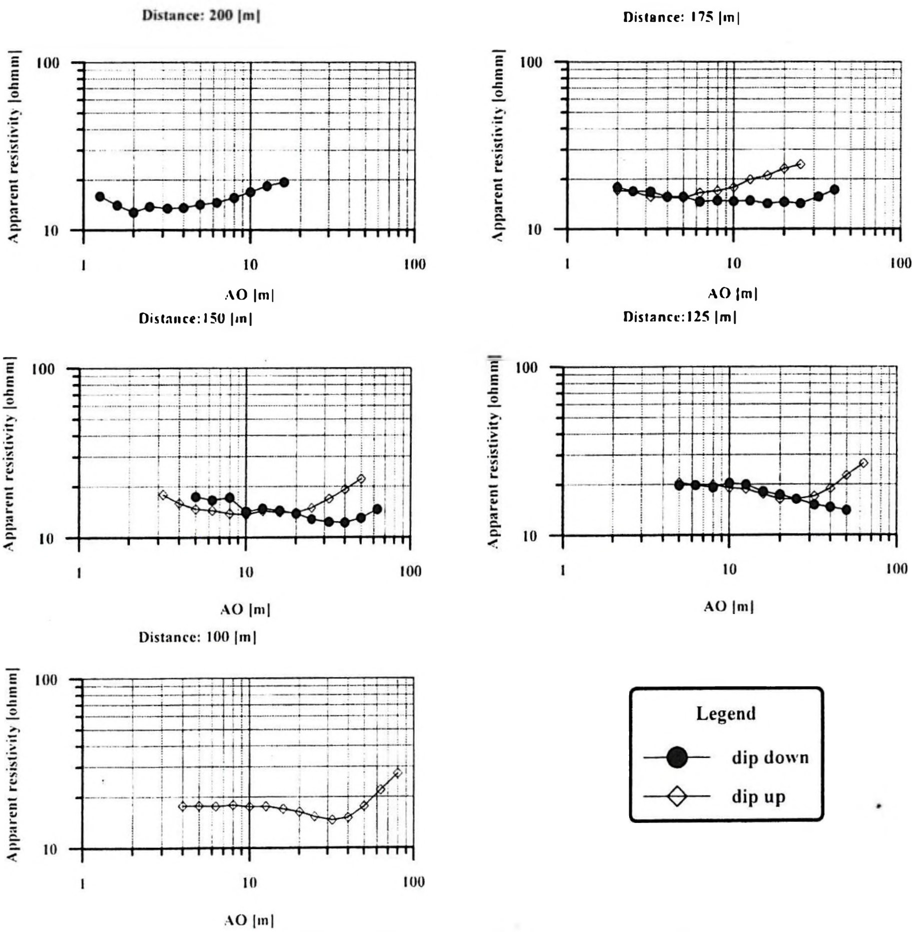

Fig. 2. Three-electrode (half-Schlumberger) VES sounding curves in dip directions
2. ábra. Háromelektródos (fél-Schlumberger) VESZ mérések dőlésirányban

differ significantly, and this fact forms the basis of the simultaneous inversion.

The 2-D simultaneous inversion result for VES stations is shown in *Fig. 3.* The ● denotes the local thickness and the local depth values of the layers. The shorter thick lines denote the estimated layer dipping at the VES stations. Connecting these elementary layers enabled us to determine the general slope of the dipping layers. At 150 m and 175 m a fault in the andesite is assumed with a faulting height of about 5 m. For better identification of the fault, we ought to have more data. The 35 ohm apparent resistivity of the andesite was considered as a fixed value along the whole length of the profile because the other VES surveys had been earlier carried out in the field. The results of 2-D inversion together with the reliability parameters are shown in *Table 1.*

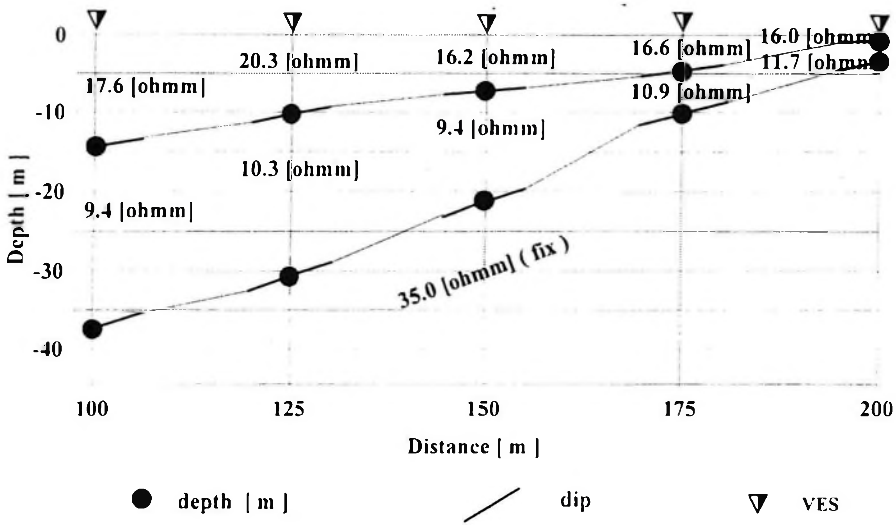

Fig. 3. Result of dip layered 2-D inversion for VES investigation

3. ábra. VESZ mérések dőlt réteges 2-D inverziójának eredménye

X/m	100	125	150	175	200
ρ_1 (ohmm)	17.6 (1%)	20.3 (1%)	16.2 (2%)	16.6 (2%)	16 (fix)
ρ_2 (ohmm)	9.3(63%)	10.3 (22%)	9.4 (37%)	10.9 (107%)	10.9 (22%)
ρ_3 (ohmm)	35 (fix)	35 (fix)	35 (fix)	35 (fix)	35 (fix)
α_1 (degree)	9.0 (35%)	8.1 (13%)	5.0 (36%)	7.6 (87%)	3.8 (44%)
α_2 (degree)	21.7 (47%)	23.3 (27%)	14.4 (36%)	17.0 (57%)	15.5 (13%)
h_1 (m)	14.4 (43%)	10.1 (21%)	7.3 (45%)	4.3 (92%)	0.6 (71%)
h_2(m)	22.0 (74%)	20.5 (46%)	13.8 (56%)	5.5 (47%)	1.8 (39%)
h_1+h_2 (m)	36.4	30.6	21.1	9.8	2.4
azimuth (degree)	180 (fix)	180 (fix)	180 (fix)	180 (fix)	180 (fix)
D (%)	3.3	1.9	5.8	4.8	2.2
$\vec{\sigma}_p$ (%)	50	26	39	74	43

Table I. Dipping flat layered 2-D simultaneous inversion result for VES stations
I. táblázat. Dőlt síkréteges, VESZ állomásonkénti együttes 2-D inverzió eredménye

The inversion procedure for the VES station of 125 m is shown in *Fig. 4*. We give the model distances represented by ▲ related to the start model, which data are shown in detail in *Table II*. The relatively low 19 % model distance and low iteration number result from our successful parameter estimation of the start model, by using 1-D inversion. The data distance of 1.8 % represents a good data fitting.

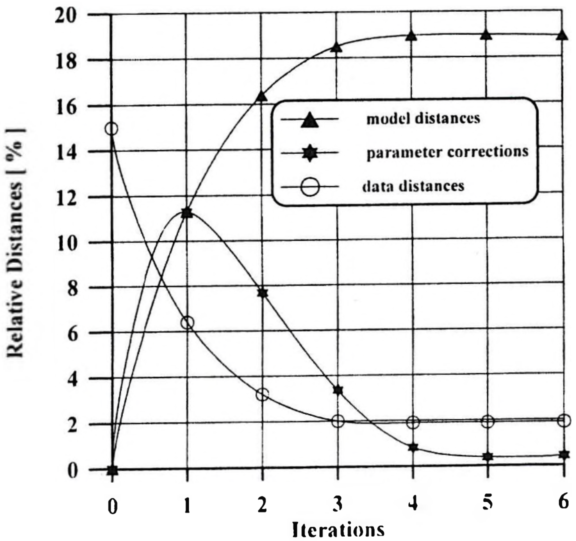

Fig. 4. The iterations, relative distances
4. ábra. Az iteráció menete, relatív távolságok

Start model		Exact model
ρ_1 (ohmm)	18	20.3
ρ_2 (ohmm)	15	10.3
ρ_3 (ohmm)	35	35
α_1 (degree)	10	8.1
α_2 (degree)	20	23.3
h_1 (m)	12	10.1
h_2(m)	30	20.5
h_1+h_2 (m)	42	30.6

Table II. 2-D dip flat layered inversion at 125 m VES point
II. táblázat. Dőlt síkréteges 2-D inverzió a 125 m-es VESZ pontban

In order to be sure about the reliability of the model parameter estimation along the profile, we carried out independent Schlumberger VES surveys in the strike-direction, too. The measured data are shown in *Fig. 5.* We inverted these data both by single 1-D inversion and by 1.5-D joint inversion. The results are shown in *Tables III.* and *IV.*

X	100	125	150	175	200
ρ_1 (ohmm)	17.3 (1%)	18.2 (1%)	17.2 (2%)	15.1 (1%)	15.1 (70%)
ρ_2 (ohmm)	7.9 (179%)	7.8 (105%)	9.8 (60%)	1.5 (125%)	11.9 (7%)
ρ_3 (ohmm)	35 (fix)	35 (fix)	35 (fix)	35 (fix)	35 (fix)
h_1 (m)	17.1 (59%)	12.7 (40%)	8.6 (43%)	5.6 (147%)	0.5 (231%)
h_2(m)	14.5 (235%)	13.0 (137%)	9.7 (93%)	4.8 (289%)	4.2 (40%)
h_1+h_2 (m)	31.6	25.7	18.3	10.4	4.7
D (%)	2.8	3.0	2.4	2.0	3.9
$\vec{\sigma}_p$ (%)	150	89	60	173	122

Table III. Result of 1-D single inversion
III. táblázat. Egyedi 1-D inverzió eredménye

Comparing Tables I, III, and IV, we can see that the reliability of the 2-D inversion for dipping layers (e.g. by comparison of the $\vec{\sigma}_p$ values) is better than that of the 1-D single inversion, in spite of the greater number of the unknowns. This is because of the simultaneous inversion. In *Fig. 6*, the result of the 1.5-D joint inversion and those of the 2-D joint inversion can be seen together at all VES points. The results of this latter inversion are also shown in *Table V.* With the two inversions we got more or less the same estimation. According to Table V the reliability values are higher in the case of 1.5-D inversion. In spite of this we think that the great advantage of 2-D inversion for dipping layers is the fact that we can make a direct estimation about the dipping of the layers. Based upon the layer dipping the tectonic elements can also be determined, but for this purpose we need the exact estimation of local dip angles.

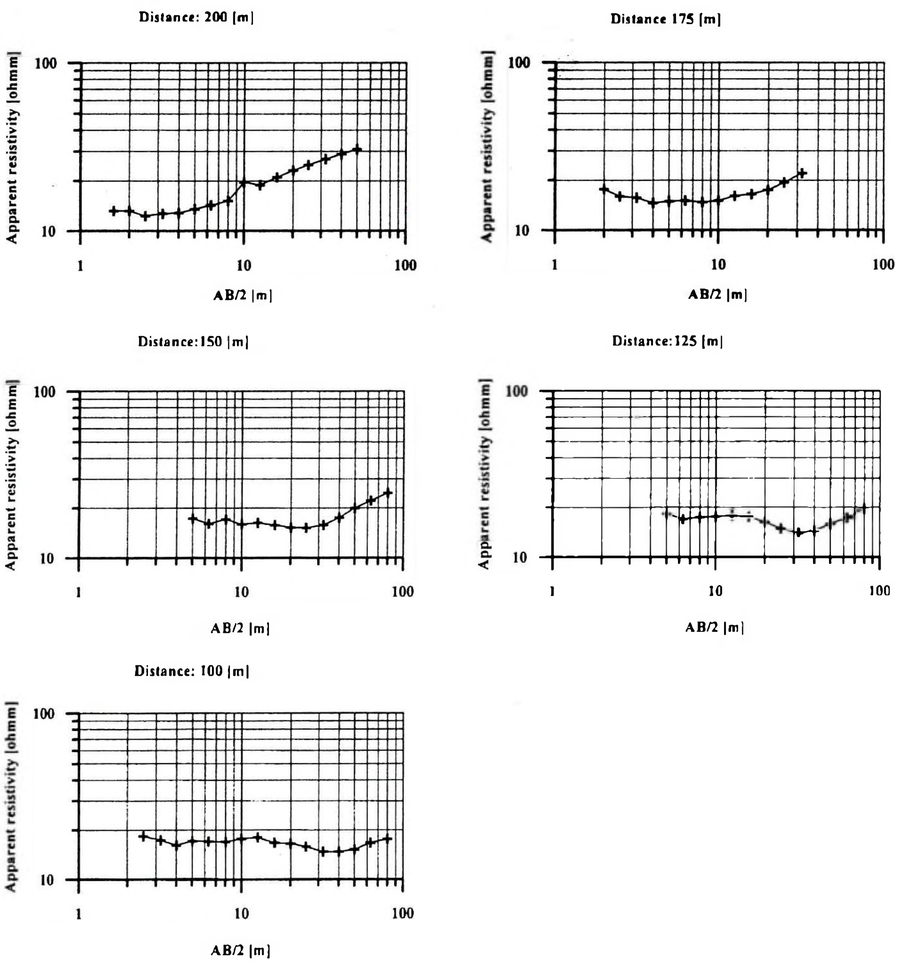

Fig. 5. Schlumberger VES investigations in strike direction
5. ábra. Schlumberger VESZ mérések csapásirányban

	α_1	α_2	ρ_1	ρ_2	*RKB*
α_1	1	–0.61	–0.72	–0.83	0.21
α_2	–0.61	1	0.43	0.94	–0.93
ρ_1	–0.72	0.43	1	0.52	–0.31
ρ_2	–0.83	0.94	0.52	1	–0.73
RKB	0.21	–0.93	–0.31	–0.73	1

	h_1	h_2	ρ_1	ρ_2
h_1	1	–0.99	–0.61	–0.99
h_2	–0.99	1	0.54	1.00
ρ_1	–0.61	0.54	1	0.54
ρ_2	–0.99	1	0.54	1

Table IV. Comparison of correlation matrices for the VES station at 125 m
IV. táblázat. Korrelációs mátrixok összehasonlítása a 125 m-es VESZ állomáson

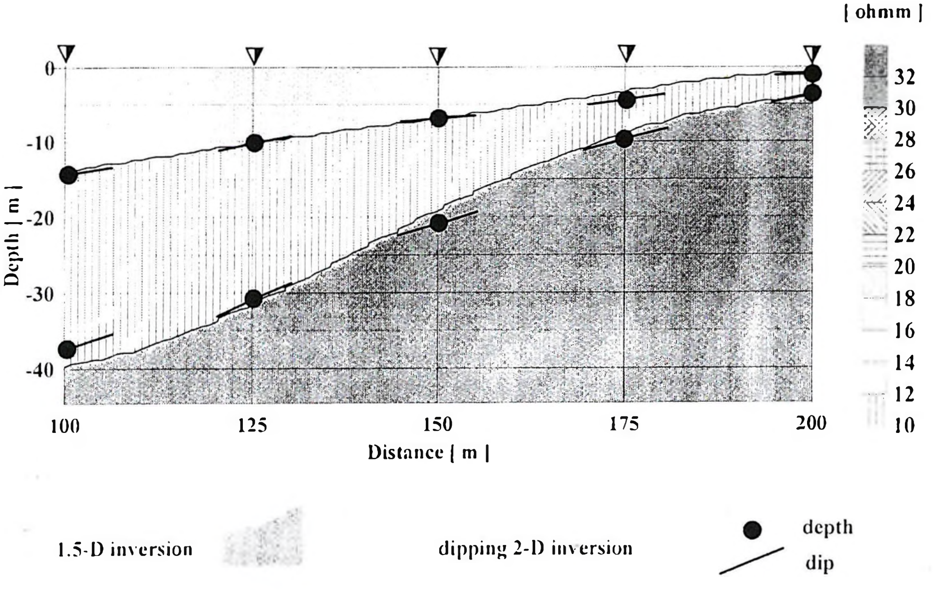

Fig. 6. Inversion result of VES investigations (comparison of dips 2-D and 1.5-D)
6. ábra. VESZ mérések inverziós eredménye (dőlt 2-D és 1,5-D összehasonlítás)

X (m)	100	125	150	175	200
ρ_1 (ohmm)	17.3 (24%)	18.1 (2%)	17.5 (2%)	16.3 (7%)	17.9 (29%)
ρ_2 (ohmm)	11.4 (13%)	11.4 (4%)	11.4 (4%)	11.4 (4%)	11.4 (4%)
ρ_3 (ohmm)	33.6 (7%)	33.6 (3%)	33.6 (3%)	33.6 (3%)	33.6 (3%)
h_1 (m)	13.8 (13%)	10.1 (11%)	6.9 (10%)	3.3 (24%)	0.5 (75%)
h_2 (m)	25.0 (3%)	21.7 (11%)	12.4 (10%)	5.7 (13%)	3.8 (34%)
h_1+h_2 (m)	38.8	31.8	19.3	9.0	4.2
D (%)	3	3.7	2.9	3.8	3.7
$\vec{\sigma}_n$ (%)	7	4	4	7	21

Table V. Result of 1.5-D simultaneous inversion

V. táblázat. 1,5-D együttes inverzió eredménye

Thanks to the former model investigations, the more reliable estimation of the layer dippings can be given by using simultaneous inversion of VES data measured in different directions and by using several array geometries.

5. Summary

The equations of the potential to solve the direct problem of dipping multi-layer models are known. Based upon the solutions, we set up an algorithm and a computer program in order to calculate the apparent resistivity [GYULAI 1995]. Based upon the analytical solution of the direct problem we elaborated a 2-D simultaneous inversion method for VES data. We carried out investigations on synthetic data in order to determine the parameter estimation reliability of dipping layers [GYULAI, ORMOS 1996]. Up till that time, no field testing had been carried out. Subsequently, however, field investigations proved successful so we now consider this method appropriate for such use.

Despite the fact that the cross line of the layer boundaries and the common outcrops can be considered as very strict requirements in field practice, our view is that such an approximation is allowable.

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Dőlt réteges földtani szerkezet meghatározása együttes 2-D inverziós módszerrel

GYULAI Ákos

A dolgozatban dőlt, háromréteges földtani szerkezet paramétereinek becslését mutatjuk be egy terepi példán 2-D együttes inverziós módszerrel. Az inverzióban a direkt feladatot analítikus módszerrel oldjuk meg. Az együttes inverzióban közösen szerepelnek a dőlés- és csapásirányú mérések adatai. A VESZ görbékből L_2 inverzióval a rétegvastagságok és fajlagos ellenállások lokális értékei mellett a rétegek dőlését is meghatározzuk. A becsült rétegparaméterek

megbízhatóságát a korreláció és a variancia értékekkel jellemezzük. Összehasonlítjuk a dőlésirányú mérési adatoknak 2-D inverziós módszerrel és az előbbiektől független csapásirányú geoelektromos mérési adatoknak 1,5-D inverziós módszerrel történő becslési eredményét.

ABOUT THE AUTHOR

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