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Horizontal inversion of guided wave dispersion data

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Guided waves play an important role both in the investigation of near surface structures and in mining applications; channel waves are commonly used to detect and locate tectonic disturbances of coal seams. On the other hand, guided waves contain information about the structural- and material parameters of the wave-guide model, so — using the methods of geophysical inversion — these characteristics can be determined by means of frequency-dependent phase- and group velocity and absorption coefficient data. Near-surface geological structures often serve as seismic wave-guides. By inverting the frequency dependent dispersion characteristics of surface waves the model parameters of the wave-guide structure can also be determined. For laterally heterogeneous wave-guides the lateral changes of the material or geometrical parameters of the model can also be determined.

In this paper the inversion of in-situ measured surface wave data is presented. The input data of the inversion algorithm are the group traveltimes determined from the seismic traces at various frequencies. The approximate inversion procedure consists of two steps: first the local group velocities are determined at various frequencies by means of tomographic inversion of the group traveltimes, and then the local dispersion characteristics of the Love- or Rayleigh surface waves are inverted in the second step. In our investigations a robust version of the SIRT (Simultaneous Iterative Reconstruction Technique) method is used for tomography and an IRLS (Iteratively Reweighted Least Squares) algorithm using Cauchy weights is applied for the inversion of the group velocity data.

Keywords: wave dispersion, inversion

1. Introduction

In a discussion of the guided-wave seismic inverse problem KREY [1983] introduced the terms: 'vertical- and horizontal inversion'. He used the term 'vertical inversion' for the determination of the thicknesses and the petrophysical characteristics of the (one-dimensional) wave-guide structure by means of the frequency-dependent phase velocities and/or group velocities. A more complicated model with rapid thickness-changes along directions parallel to the coal seam was also discussed by KREY [1983] and the procedure giving the solution of the inverse problem for this case was called 'horizontal inversion'. We shall use

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this latter term in a more general context: changes in both the geometrical (thickness) and petrophysical parameters are allowed.

1.1. The forward problem

The solution of the guided-wave seismic forward problem for horizontally layered, layerwise homogeneous wave-guide structure is well-discussed [SCHWAAB, KNOPOFF 1972, RÄDER et al. 1985, BUCHANAN 1987]. For direct calculation of the absorption-dispersion characteristics of Love-waves in a one dimensional wave-guide we use the algorithm presented by BUCHANAN [1987] with the modification that any dissipative properties of the medium will also be considered. In order to describe inelastic friction the constant Q model is used with the complex shear modulus $\mu = \mu_o [1 + i\varepsilon]$, (μ_0 is the real shear modulus; $\varepsilon = 1/Q$, Q being the quality factor). For a horizontally (weakly) inhomogeneous wave-guide with slowly changing (or constant) thickness the WKB method will be used to determine the absorption-dispersion characteristics [DOBRÓKA 1987, 1988]. A more general dispersion relation of [FANCSIK 1997] can also be applied; this author extended the use of the WKB method to the case of Rayleigh waves, too.

2. Vertical inversion of guided-wave dispersion data

If one solves the complex dispersion relation (for both Love- and Rayleigh waves) the frequency dependent phase velocity (V^c_{ph}), the group velocity (V^c_g), and the absorption coefficient a^c can be computed

$$V^c_{ph} = v_{ph}(\omega, \vec{m}) \qquad V^c_g = v_{gr}(\omega, \vec{m}) \qquad a^c = a(\omega, \vec{m})$$

where $\vec{m}$ is the vector of model parameters:

$$\vec{m} = \{\beta_1, \beta_2, \beta_3, \rho_1, \rho_2, \rho_3, \varepsilon_1, \varepsilon_2, \varepsilon_3\}^T$$

(for a three-layered model), where β denotes the shear velocity and ρ is the density (the superscript c refers to calculation). On forming a joint inversion algorithm one then introduces the combined response function

$$\vec{d}^c = \left\{ V^c_{ph1}, \ldots, V^c_{ph_{N_p}}, V^c_{g_1}, \ldots, V^c_{g_{N_g}}, a^c_1, \ldots, a^c_{N_a} \right\}$$

with the kth coordinate

$$d_k^c = d\{\omega_k, \vec{m}\} \ .$$

Similarly, the combined vector of observations is introduced as

$$\vec{d}^{obs} = \left\{V_{ph_1}^{obs},\ldots,V_{ph_{N_p}}^{obs},V_{g_1}^{obs},\ldots,V_{g_{N_g}}^{obs},a_1^{obs},\ldots,a_{N_a}^{obs}\right\}$$

(the superscript *obs* refers to observation). In the inversion procedure the parameter vector is usually determined by minimizing the weighted norm

$$E = (\vec{e}, W\vec{e})$$

of the

$$\vec{e} = \vec{d}^{obs} - \vec{d}^{c}$$

vector. The W_{kk} weights are usually a priori given.

When the weight matrix is independent of the parameter vector, the linearized procedure of the weighted least squares leads to the normal equations

$$\underline{\underline{G}}^T \underline{\underline{W}}\, \underline{\underline{G}}\, \vec{x} = \underline{\underline{G}}^T \underline{\underline{W}}\, \vec{y}$$

where

$$y_k = \frac{d_k^{obs} - d_k^{(0)}}{d_k^{obs}}, \qquad G_{kj} = \frac{m_j^{(0)}}{d_k^{obs}} \left(\frac{\partial d_k^c}{\partial m_j}\right)_{\vec{m}_0}, \qquad x_j = \frac{\delta m_j}{m_j^{(0)}} .$$

Here $\vec{m}^{(0)}$ is the point in the model space around which the problem is linearized, $d_k^{(0)} = d\left\{\omega_k, \vec{m}^{(0)}\right\}$ and $\delta\vec{m}$ is the parameter correction.

In some cases the weight matrix contains the e_k residuals. For example the Cauchy weights are of the form

$$W_{kk} = \frac{s^2}{s^2 + e_k^2},$$

where s is the scale parameter. It was in order to save the linearity of the normal equations even in such cases that SCALES et al. [1988] introduced the Iteratively Reweighted Least Squares (IRLS) method. The resulting normal equation in the ith iterations of the IRLS procedure is

$$\underline{\underline{G}}^T \underline{\underline{W}}^{(i-1)} \underline{\underline{G}}\, \vec{x}^{(i)} = \underline{\underline{G}}^T \underline{\underline{W}}^{(i-1)} \vec{y} \tag{1}$$

where $W^{(I-1)}$ is the Cauchy weight matrix containing the $e_k^{(I-1)}$ residuals of the previous step of iteration. In the following, this algorithm will be tested using synthetic data.

3. Horizontal inversion algorithms

The geological structure representing the seismic wave-guide often shows lateral inhomogeneities. If the inhomogeneity of the petrophysical parameters is weak and the change in the thickness is slow enough, the WKB method can be used to solve the forward problem. This gives the possibility of rapid calculation of the (local) phase- and group velocities as well as the absorption coefficients. The traveltime data belonging to various guided wave constituents can be calculated by (numeric) integration

$$t(\omega,\vec{m})=\sum_i \frac{\Delta x}{V_{ph}(\omega,\vec{m},x_i)}. \qquad (2)$$

where the $\vec{m}$ parameter vector consists of the variables appearing in the vertical inverse problem and some further ones, depending on the discretization of laterally changing variables.

3.1. Exact inversion

The exact inversion algorithm was introduced by DOBRÓKA [1996]. Assuming a three layered wave-guide structure, the varying (x-dependent) thickness function can be approximated as a power series

$$H(x)=\sum_{j=1}^{P} B_j x^{j-1} \qquad (3)$$

Here B_j are the unknown expansion coefficients. The parameter vector can be written in the form

$$\vec{m}=\{\beta_1,\beta_2,\beta_3,\rho_1,\rho_2,\rho_3,\varepsilon_1,\varepsilon_2,\varepsilon_3\,.B_1,\ldots,B_P\}^T$$

when B_j-s are the coefficients of the (3) series expansion. In the formulation of the linearized inverse problem the $\underline{\underline{G}}$ matrix can be computed as

$$G_{ij} = \frac{m_j}{t_i^{obs}} \left(\frac{\partial t_i}{\partial m_j} \right)_{\vec{m}_o}$$

and using a Cauchy-IRLS algorithm the normal equations can again be written in the form of Eq. (1). The exact horizontal inversion algorithm requires numerical integration in the forward modelling with the solution of the complex dispersion relation at all the x_i points (Eq. (2)). This results in relatively large computation times.

3.2. *Approximate inversion*

In order to reduce the computation time requirements an approximate inversion method was also developed in which the guided-wave seismic horizontal inverse problem is reduced to a (multiply solved) vertical inverse problem. To do this, as a first step in the approximate inversion procedure the local phase- and group velocities are determined at various frequencies by means of the tomographic method. The local absorption–dispersion characteristics given at different frequencies and positions are then inverted in a vertical joint inversion procedure. In our investigations the SIRT method (based on one-dimensional cells along the x-axis) was used for tomography and LSQ (Least Squares) or Cauchy-IRLS algorithms (IRLS method using Cauchy weights) were again applied for joint inversion of the phase- and group velocity as well as absorption coefficient data.

4. Numerical results

In order to test the horizontal inversion algorithms, synthetic data sets were generated. The petrophysical parameters of the model chosen for numerical investigations are the same as in *Table I*, with the thickness function

$$H_2(X) = d\ [1 + \exp(-X^2)]\,, \quad X > 0\,, \tag{4}$$

where $X = \dfrac{x}{20\,d}$ and $d = 1$ (m).

H(m)	β (m/s)	α (m/s)	ρ (kg/dm³)	E
–	2000	3000	2.7	0.02
2	1000	1500	1.3	0.05
–	2500	3800	2.2	0.02

Table I. The model parameters
I. táblázat. A modellparaméterek

The theoretical traveltimes were calculated by means of Eq. (2) in the range of (150–750) Hz at 30 frequency points for all the receivers placed at the points $x_j = j\Delta x$ (j=1,...,10 and Δx=5 m). Random noise of Gaussian distribution was added: either 1% (data set I.) or 5% (data set II.). In order to simulate a data set containing outliers, data set III. is generated by adding 25% extra noise to a randomly selected 20 % portion of the data set I.

The thickness function found in the LSQ exact horizontal inversion is shown in *Fig. 1.* In the discretization of the thickness (Eq. (3)) we used P=5 which gives an acceptable approximation of the thickness function. The relative

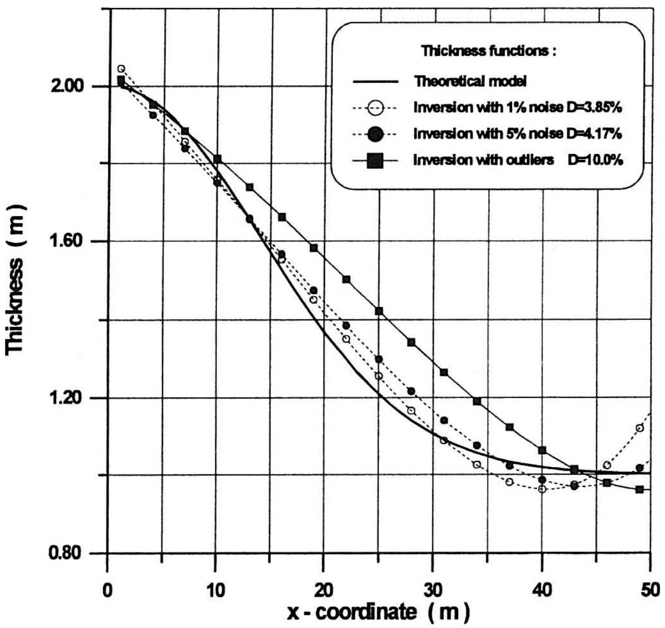

Fig. 1. Thickness function found in horizontal exact inversion
1. ábra. A horizontális egzakt inverzióban kapott vastagságfüggvény

distance between the ($H_0(x)$) exact and the ($H(x)$) estimated thickness is characterized by the scalar

$$D_H = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{H(x_i) - H_0(x_i)}{H_0(x_i)}\right)^2}$$

The approximate inversion result is shown in *Fig. 2*. Both the exact and the approximate inversion of the synthetic dispersion (phase traveltime) data gave acceptable results. The quality of approximate inversion can appreciably be improved by joint inversion of various kinds of dispersion data.

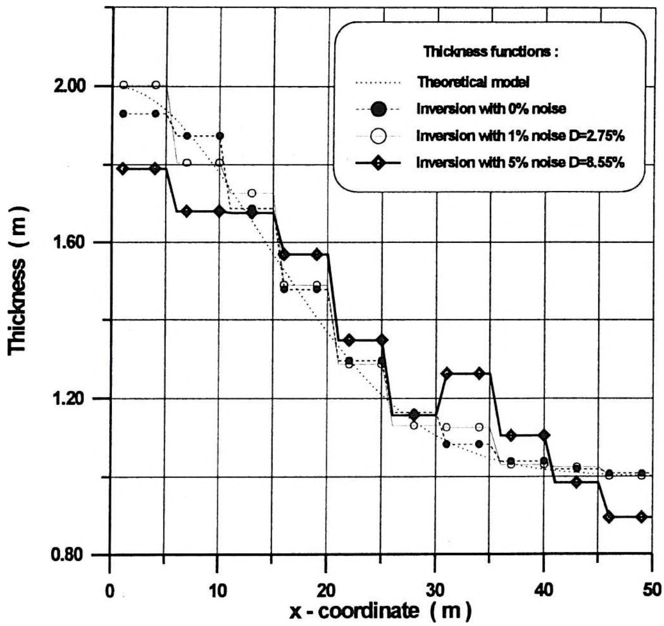

Fig. 2. Thickness function found in phase velocity approximate LSQ inversion
2. ábra. A fázissebesség adatok horizontális közelítő inverziójában kapott vastagságfüggvény

In order to test the approximate horizontal joint inversion algorithm the theoretical data were calculated by means of Eq. (2) and also

$$t_{gr}(\omega, \vec{m}) = \sum_i \frac{\Delta x}{V_{gr}(\omega, x_i)} \quad . \tag{5}$$

$$A(\omega,\vec{m})=\sum_i a(\omega,x_i)\Delta x \quad . \tag{6}$$

in the range of (150–750) Hz at 30 frequency points for all the receivers located in any of the cells with their border defined at points $xj = j\Delta x$ (j=1,...,10 and Δx=5 m is the cell-size). Three data sets (I, II, and III) were again generated by adding noise in the same way as above. The joint inversion of phase- and group traveltimes results in *Fig. 3*, while the result of the phase velocity–absorption coefficient joint inversion is presented in *Fig. 4*. It can be seen that joint inversion gives more accurate parameter estimation.

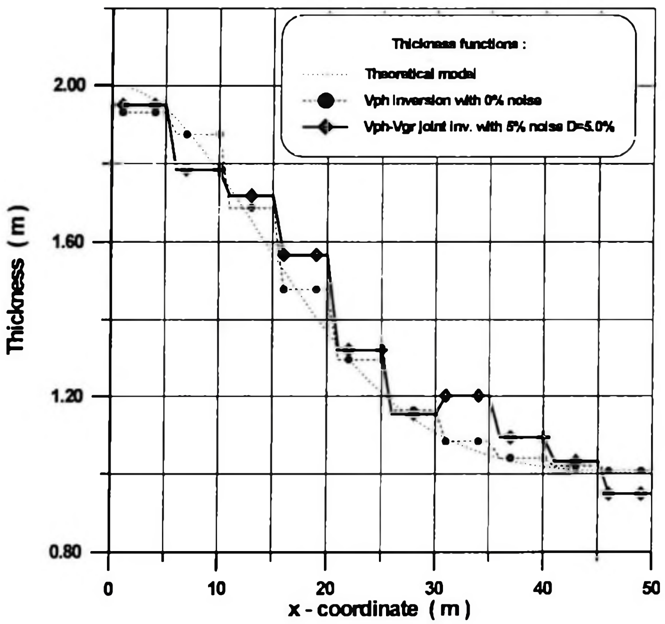

Fig. 3. Thickness function found in phase velocity–group velocity approximate joint inversion

3. ábra. A fázis- és csoportsebesség adatok horizontális közelítő együttes inverziójában kapott vastagságfüggvény

5. Field results

The horizontal inversion methods were also tested by means of in-situ measured surface wave data. The measurements were carried out in Borsod

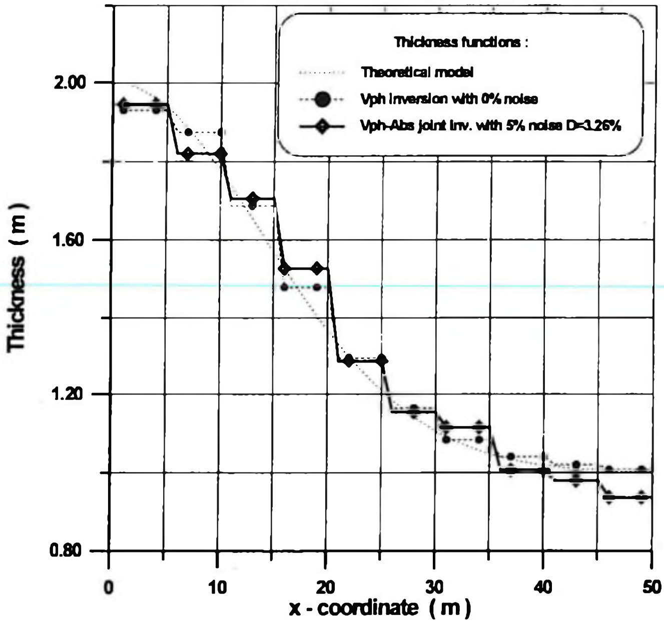

Fig. 4. Thickness function found in phase velocity–absorption coefficient approximate joint inversion

4. ábra. A fázissebesség és abszorpciós tényező adatok horizontális közelítő együttes inverziójában kapott vastagságfüggvény

County. The layout was similar to that assured in the generation of synthetic data. The seismic source and the five receivers were arranged along a straight line; the first receiver was 38 m apart from the source, the distance between receivers was 6 m. Horizontal displacements (perpendicular to the measurement line) were generated. 1 Hz Lennartz geophones were used to measure the displacements of the surface waves of the Love type. The measurement data were collected using the PC-regulated engineering seismic instrument ESS 03–24 of the Eötvös Loránd Geophysical Institute, Budapest.

The seismogram detected in the measurement (and filtered in the frequency range 2–35 Hz) is shown in *Fig. 5*. In order to find group slowness data from the filtered seismic traces the wavelet transform method was used with a Morlet-wavelet, as an analysing function. A typical result of the wavelet analyses — the amplitude as a function of the group traveltime and frequency — is shown in *Fig. 6* (belonging to the first receiver in the frequency range 0–15 Hz). For the sake of simplicity the basic mode group

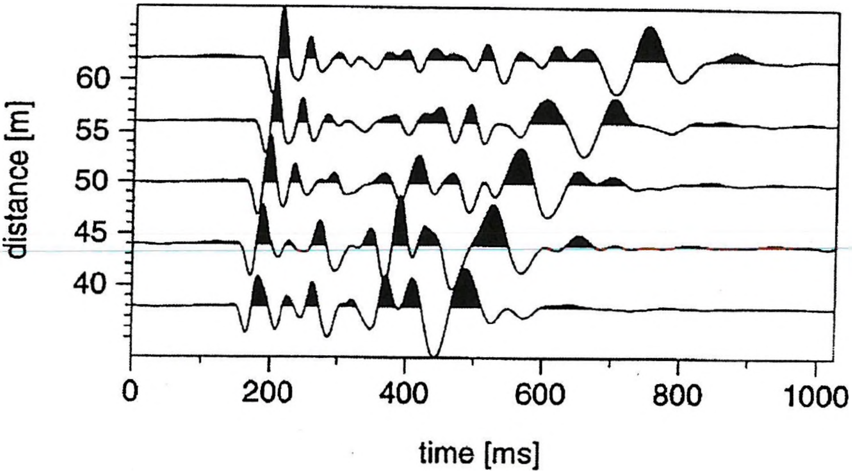

Fig. 5. The six seismic channels of the in-situ measurement

5. ábra. A terepi mérésen regisztrált szeizmikus csatornák

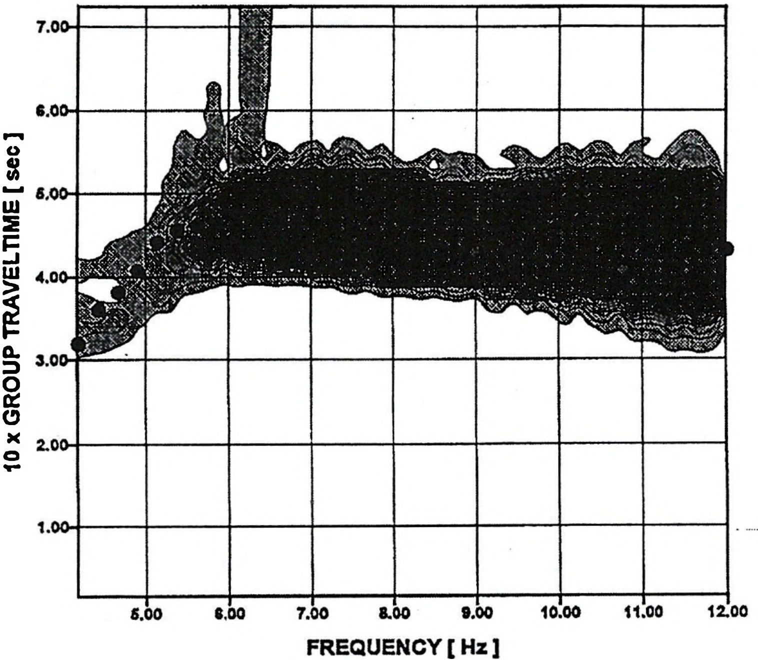

Fig. 6. Dispersion analysis of the first seismic channel

6. ábra. Az 1. szeizmikus csatorna diszperzió analízisének eredménye slowness data belonging to the maximum amplitudes in the 5–12 Hz frequency range were selected for inversion.

The group slowness data derived from the in-situ measured seismogram were used to test the approximate horizontal inversion algorithm. The start model was defined in agreement with the paper by MISIEK et al. [1997]. (In this work geoelectric- and seismic field data were measured at the same location and their joint inversion was discussed assuming a horizontally layered, layerwise homogeneous geological model. The parameters of the three-layered model given by this joint inversion procedure were chosen as the start model for the horizontal inversion.)

The one-dimensional cells required for the method were defined in agreement with the measurement layout: (0,38], (38,44], (44,50], (50,56], (56,62]. The local thickness and shear velocity values (in the first layer) were allowed to change cell by cell. The other model parameters of the wave-guide (H_2, VS_2, VS_3) were considered as common parameters of the inversion procedure. As a result of the approximate horizontal inversion procedure, the thickness- and shear velocity functions of the first layer are shown in *Fig. 7* and *Fig. 8* respectively. By means of the estimated model parameters, theoretical group traveltimes — as functions of the frequency — were also calculated at all geophone positions. As is shown by *Fig. 9*, the group traveltimes (calculated on the estimated model) are in relatively good agreement with those derived from the seismogram.

6. Conclusion

In order to demonstrate the accuracy and stability of approximate horizontal inversion, numerically simulated absorption–dispersion data were used. The inversion method was used to determine the lateral changes of the wave guide model for field measurements carried out in Borsod County. The relatively good fit between the observed and predicted group traveltime data (calculated on the model, given by horizontal inversion) proves that the approximate horizontal inversion method is sufficiently accurate for practical use.

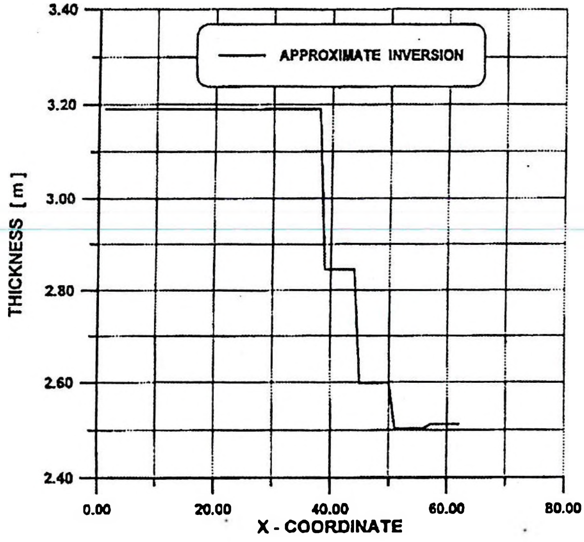

Fig. 7. Thickness function given by horizontal inversion

7. ábra. A terepi adatok horizontális inverziójával kapott vastagságfüggvény

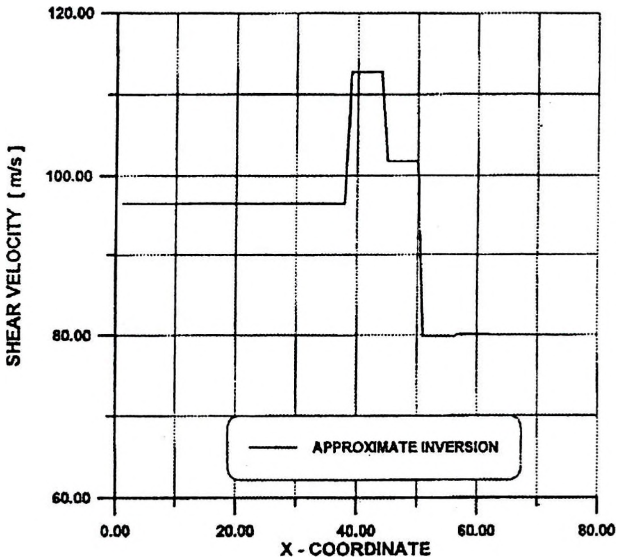

Fig. 8. Shear velocity function given by horizontal inversion

8. ábra. A terepi adatok horizontális inverziójával kapott transzverzális sebességfüggvény

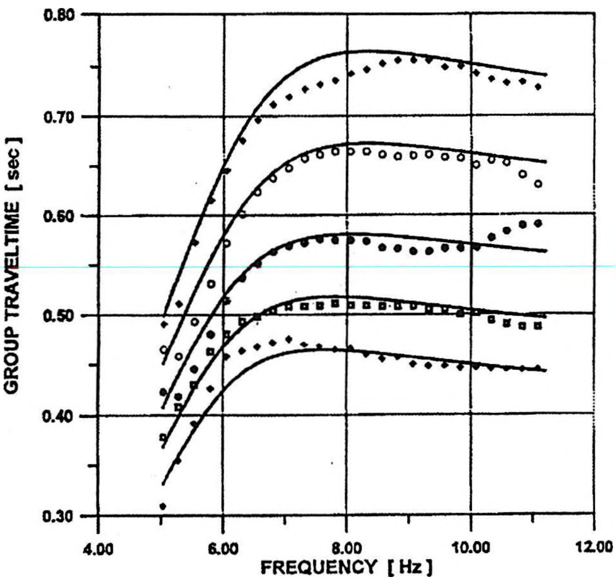

Fig. 9. Fit between the observed and predicted group traveltime data (calculated on the model given by horizontal inversion)

9. ábra. A mért és (a horizontális inverzió eredmény-modelljén) számított csoportsebesség adatok egyezése

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Vezetett hullám diszperziós adatok horizontális inverziója

DOBRÓKA Mihály

A vezetett hullámok fontos szerepet játszanak a felszín közeli szerkezetek kutatásában. Ismeretes, hogy csatornahullámok segítségével a széntelepes összletek tektonikai zavarai kutathatók. Ugyanakkor a vezetett hullámok információt hordoznak a hullámvezető szerkezet geometriai és anyagi jellemzőiről is, ezért a geofizikai inverzió módszereinek alkalmazásával ezek a paraméterek meghatározhatók a frekvenciafüggő fázis- és csoportsebesség ill. abszorpciós tényező adatok alapján.

A felszín közeli szerkezetek gyakran hullámvezetőként viselkednek, így a frekvenciafüggő diszperziós adatok inverziója révén a hullámvezető szerkezet jellemzői vizsgálhatók (HERING et al. 1994]. Ebből adódóan laterálisan változó hullámvezetők esetén mód van a hullámvezető szerkezet geometriai és anyagi jellemzői laterális változásának inverziós meghatározására is.

Ebben a dolgozatban felületi hullám diszperziós adatok inverziójával foglalkozunk. Az alkalmazott horizontális inverziós módszer bemeneti adatai a terepi szeizmikus csatornák diszperzió analízisével (különböző frekvenciákon) előállított csoportsebességek. A közelítő inverziós eljárás két lépésből áll:

1. lokális csoportsebességek előállítása (különböző frekvenciákon) a csoport-futási idő adatok tomográfiai inverziójával;

2. a lokális hullámvezető paraméterek meghatározása a lokálisan különböző frekvenciákon adott csoportsebességek (vertikális) inverziójával.

Vizsgálatainkban az ismert SIRT (Simultaneous Iterative Reconstruction Technique) módszer robusztifikált változatát [DOBRÓKA 1994] alkalmazzuk a tomográfiai rekonstrukcióban ill. az iteratív újrasúlyozás módszerével definiált (Cauchy súlyokat használó) eljárást alkalmazunk a diszperziós adatok vertikális inverziójában.

ABOUT THE AUTHOR

Mihály Dobróka received his M.Sc. in physics (1972) from Kossuth Lajos University (Debrecen). He obtained his university doctor's degree from the Eötvös Loránd University (Budapest) in 1976. The Hungarian Academy of Sciences awarded him his Candidate's degree (C.Sc.) in 1986 and his Doctor of Sciences degree (D.Sc.) in 1996. He has worked continuously at the University of Miskolc since 1972; at present he is professor of geophysics. His main fields of interest are geophysical inversion and tomography, seismic methods, and engineering geophysics.