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Homoeopathic method to increase the accuracy of inversion results

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The basic principle of the surplus-error method is given and its effectiveness is demonstrated on the basis of a microgravimetric example.

Keywords: microgravimetric method, inversion, surplus-error method, relative model-distance

The algorithm presented below supposes that an adequate model is given for inversion.

The basic principle of the method of surplus errors may be explained as follows: *If a surplus error series of considerable number is superposed on the measured data, even though in each case the increase in the error of the model parameters is likely, it may be that the medians of the model parameters (or the median of medians after some kind of reasonable grouping of the parameters) are significantly more accurate than the results of direct inversion based on the original measured data.*

A comprehensive example is given in STEINER [2002] for the application and for the possible effectiveness of the surplus-error method. The 2-D model given below was defined by 6 parameters: two horizontal cylinder-shaped cavities of unknown depth, diameter and horizontal position. The actual values for the two cavities were: depth — 7.5 m and 6.5 m; diameters — 3 m and 3 m; horizontal position — 5 m and 13 m. Along the profile exactly calculated 19 g_i gravimetric values with error of statistical type using S = 4 as parameter of scale were superposed as the semi-intersextile range Q equals 2.2 μGal in this case (see STEINER [1990] page 52 for a=5 which defines the statistical probability distribution), the scatter amounts to 2.8 μGal in full accordance with the well known fact that the er-

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ror of microgravimetric measurements cannot be less than 2–3 μGal. The chosen type was based on DUTTER [1986–87]: the occurrence of such a type is most probable in geostatistics.

Denoted by h_i the random numbers obtained according to the above mentioned way, regarding them as 'natural errors', the 'measured' values can be calculated obviously as $g_i^{measured} = g_i + h_i$ $(I = 0, 1, \dots .\ 18)$.

Inversion needs to determine the six model parameter values on the basis of these 19 measured data. Aptly, the *P*-norm of the deviations (see STEINER (ed) [1997] p. 20.) can be chosen to minimize the deviations because this procedure gives, with sufficient accuracy, the *S* parameter of scale, too, which — in practice — is not a priori known. This *S*-value of the measuring ('natural') error is chosen as the *S*-value of the surplus errors, too.

In STEINER [2002], 21 surplus-error series were superimposed 21 times on the measured microgravimetric values; for all six model parameters the median of the 21 medians was accepted (i.e. 441 inversions were carried out).

Was this amount of calculation really worth while? The answer is given in STEINER [2002]: 'Yes' in all ten investigated cases (see the comprehensive Tables for all six model parameters in the cited article). Here, however, a much shorter way should be used.

The points in *Fig. 1* correspond to the $g_i^{measured}$ values. The thin line demonstrates the direct *P*-fitting results, the thick line corresponds to the six parameter-values which were obtained after the above-mentioned procedure (with 441 inversions). The fitting quality of both curves seems to be (judged on the face of it) quite the same — the chief meaning of these parameter values, however, is not in the fitting of the measured values, but in the final definition of the investigated model as model parameters. The two thin line circles in Fig. 1 correspond to one single inversion; they are fully unacceptable in relation to the true situation (see the data after the formulation of the basic principle: e.g. both true diameters are the same). On the other hand, the model obtained using the surplus-error method (these circles are drawn by thick lines in Fig. 1), is near to the true case.

It seems to be necessary to characterize quantitatively the model distances (instead of using the qualitative expressions 'near' or 'far'). In our case the following formula should be used as the 'relative model distance':

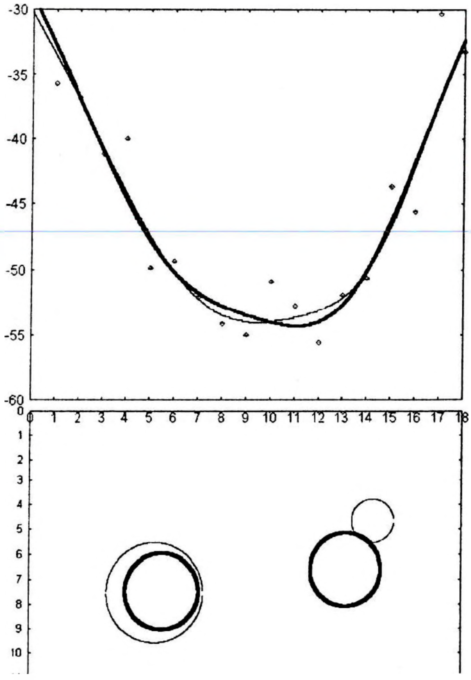

Fig. 1. Two thin line circles show that the results of one single step of inversion based on the measured points may be fully unacceptable whereas both thick line circles — which are the results of the surplus-error method — are acceptably near to the true model (see the true values of the model parameters after the formulation of the basic principle)

1. ábra. A mérési pontok alapján végrehajtott egyetlen inverziós lépés a két, vékony vonallal rajzolt körkeresztmetszetre vezet, amely teljesen elfogadhatatlan eredmény. Ezzel szemben a többlethiba-módszer a vastag vonallal rajzolt köröket szolgáltatja, amelyeknek adatai a (módszerdefiníció után számszerűen megadott) helyes értékekhez nagyon közel állnak

$$\delta = 100 \cdot \sqrt{\frac{1}{6}\sum_{j=1}^{6}\left(\frac{p_j^{calculated} - p_j^{true}}{p_j^{true}}\right)^2}$$

If one single inversion is carried out, δ is denoted by δ_0; on the other hand, if the six $p_j^{calculated}$ model parameters are the results of the surplus-error method, the model-distance is denoted by $\delta_{surplus}$. The cases in Fig. 1 result in the values δ_0=34.4% and $\delta_{surplus}$=5.2%. The latter value as

δ_0 could be achieved only if the error of the microgravimetric values would be 0.4–0.6 μGals; this is, however, in the near (or even in the predictable) future not realizable.

One single example is perhaps not convincing enough (although in STEINER [2002] ten examples were comprehensively investigated) and the question arises, too, whether really many hundreds of inversions are needed to achieve a significant decrease in the relative model distance.

In HAJAGOS and STEINER [2003] one hundred 'natural' error-series of statistical type were generated (otherwise speaking: 100 measured data sets were given) but only 5 surplus-error sets were superimposed on each measured data set and 6 medians of the model parameters determined. The whole procedure was repeated 9 times and finally the median of the 9 medians was accepted for all six model parameters as a result of the surplus-error method (i.e. only 45 inversions were carried out for all 100 cases). Calculating both $\delta_{surplus}$ and δ_0 for all hundred cases, the values of $\delta_{surplus}$ and also those of $\delta_{surplus}$ were ordered, consequently 'empirical distribution functions' could be constructed for both types of model distances. *Figure 2* shows these functions, demonstrating that 45 surplus-error superpositions and inversions are enough to halve the δ_0 model distances which belong to one single inversion.

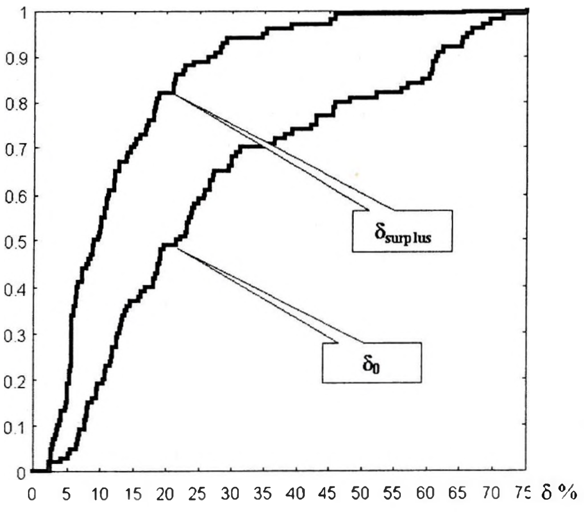

Fig. 2. Empirical distribution functions of model distances from the true situation of two kinds: δ_0 concerns the results of one single inversion step, $\delta_{surplus}$ concerns the results of the surplus-error method using 45 superpositions. The natural error was of statistical type

2. ábra. A valódi hatótól mért kétféle modelltávolság empirikus eloszlásfüggvényei: δ_0 az egyetlen inverziós lépéssel, $\delta_{surplus}$ a 45 szuperpozíciót alkalmazó többlethiba-módszerrel adódó eredményekre vonatkozik. A természetes hiba statisztikus típusú volt mind a száz esetben

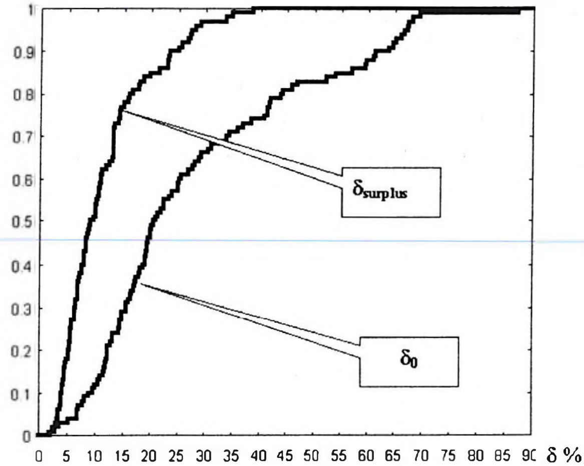

Fig. 3. Empirical distribution functions of model distances from the true situation of two kinds: δ_0 concerns the results of one single inversion step, $\delta_{surplus}$ concerns the results of the surplus-error method using 45 superpositions. The natural error was of Laplace type

3. ábra. A valódi hatótól mért kétféle modelltávolság empirikus eloszlásfüggvényei: δ_0 az egyetlen inverziós lépéssel, $\delta_{surplus}$ a 45 szuperpozíciót alkalmazó többlethiba-módszerrel adódó eredményekre vonatkozik. A természetes hiba Laplace típusú volt mind a száz esetben

Similar halving is shown in *Fig. 3* although the natural errors here are of Laplace type (but are characterized naturally by the same semi-intersextile range of Q=2.2), consequently it seems that the procedure is insensitive to the 'natural' error-types. Even if the natural error is of Cauchy-type and characterized by Q=2.2 (see *Fig. 4*), the $\delta_{surplus}$-curve hardly differs from the $\delta_{surplus}$-curves of Figs. 2 and 3. In contrast, the δ_0-curve of Fig. 4 is more elongated than the δ_0-curves in Figs. 2 and 3.

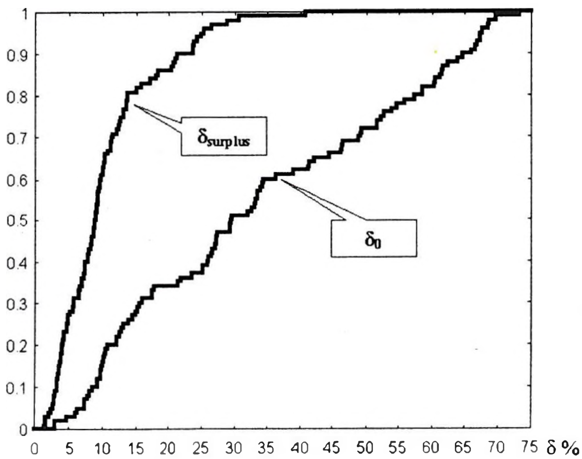

Fig. 4. Empirical distribution functions of model distances from the true situation of two kinds: δ_0 concerns the results of one single inversion step, $\delta_{surplus}$ concerns the results of the surplus-error method using 45 superpositions. The natural error was of Cauchy type

4. ábra. A valódi hatótól mért kétféle modelltávolság empirikus eloszlásfüggvényei: δ_0 az egyetlen inverziós lépéssel, $\delta_{surplus}$ a 45 szuperpozíciót alkalmazó többlethiba-módszerrel adódó eredményekre vonatkozik. A természetes hiba Cauchy típusú volt mind a száz esetben

Even so, it should be recalled that TARANTOLA [1987] mentioned that the Cauchy-type ab ovo contains some amount of outliers and this circumstance can cause this elongation of the δ_0-curve. It seems (astonishingly enough) that the surplus-error procedure is able to eliminate this effect, too: see once more the similarity of the three $\delta_{surplus}$-curves in Figs 2, 3, and 4.

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Homeopatikus módszer az inverzió eredményeinek pontosítására

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A dolgozat megadja a többlethiba-módszer alapelvét, hatékonyságát pedig egy mikrogravitációs példán mutatja be.

ABOUT THE AUTHOR

Ferenc Steiner was born in 1932; he received his diploma in physics in 1954 at the University of Science in Szeged. He started teaching and scientific work in 1954 and has continued this till now at the Geophysics Department of the University of Miskolc. He was qualified as D.Sc. in 1975 (awarded by the Hungarian Academy of Sciences). He is a member of the Editorial Board of Acta Geod. Geophys. Mont. Acad. Sci. Hung., and a member of the Scientific Commission for Geophysics of the Hungarian Academy of Sciences. The author is an Academic Prize winner and his award of the silver medallion of the President of the Hungarian Republic was personally presented by Á. Göncz in 1999. In 2001 he was elected an honorary member of the Association of Hungarian Geophysicists.