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Analysis of inversion of secondary EM field obtained by 'casing-surface electrodes' source

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An electromagnetic field's secondary components are highly informative in the geological mapping of lateral changes. They are especially useful when certain components are missing from the primary field of the source and the anomalous — secondary — part of the field can be measured directly. Consequently it is also worth while — besides the possibility of indicating the lateral changes — to be concerned with the inversion of the secondary components, which means determining the subsurface charge-, or dipole-distribution linked with the inhomogeneities.

On the one hand the paper examines the peculiar influences that the charges and dipoles have in creating the secondary components; on the other hand, it tests by simulated data D. Patella's procedure elaborated originally for the inversion of SP data.

Keywords: electrical sounding, borehole casing electrode, electromagnetic field, secondary components, charge distribution, inversion

1. Introduction

Secondary electromagnetic fields caused by geological inhomogeneities are capable of perceptibly modifying the more stronger primary field of the source only at a rather large separation. Therefore considerable spacings are needed for deep prospecting. The situation is quite different when some components of the primary field are missing. In such a case the weak secondary field becomes measurable and will indicate deep inhomogeneities even at small separations [TAKÁCS, HARSTON 2000]. Consequently it is worth while to be concerned with the inversion of the secondary components.

This paper intends to outline a train of thought using examples gained by an excitation mode, whose primary field is peculiar. On the one hand we examine the roles of the charges and dipoles in creating the secondary

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components, on the other hand we test a possible inversion procedure by simulated data.

The selected method uses for current injection a steel borehole casing and surface electrodes circularly grounded around it. The principles of the geometric and frequency sounding with this kind of source were elaborated in the framework of OTKA (Hungarian Scientific Research Fund) Project 2383 beginning from 1990 [TAKÁCS et al. 1995]. Later the Domestic Research Division of the Hungarian Oil and Gas Company (MOL Rt) provided support to carry out experimental measurements and numerical simulation [TAKÁCS et al. 2001].

It is mentioned that instead of the casing a central surface electrode can also be used, as in the case of the vertical electric current transient sounding, which is based on the same principles [MOGILATOV, BALASKOV 1996].

The 'casing-surface electrodes' source can be regarded as a sequence of vertical electric dipoles (VED) connected in series. The primary field of such a source has at the surface only one component, namely the radial electric one [TAKÁCS 1995]. Along a radial profile the tangential electric EFI (perpendicular to the profile) and all of the magnetic components (radial HR, tangential HFI, vertical HZ) are caused solely by subsurface inhomogeneities. The primary currents are concentrated in much smaller volume compared with the traditional sources. Therefore this method is especially suitable for investigating local, 3-D structures. Because the casing injects current into the deeper horizons, too, the depth of investigation at a given separation is larger than in the case of traditional current-sources. At the same time the layering appears in the sounding curves unusually owing to the VED character of the 'casing-surface electrodes' configuration.

2. Origin and features of secondary components

The model presented in *Fig. 1* will be investigated several times. The lateral inhomogeneities are prisms elongated towards N–S. The lower one is a horst of the basement — with a height of 300 m, a width of 1000 m, a length of 1400 m — and its top is located at a depth of 1450 m. The upper prism is created by a local downward thickening of a layer of 300–350 m and covers the lower one. The casing penetrates the prisms asymmetrically.

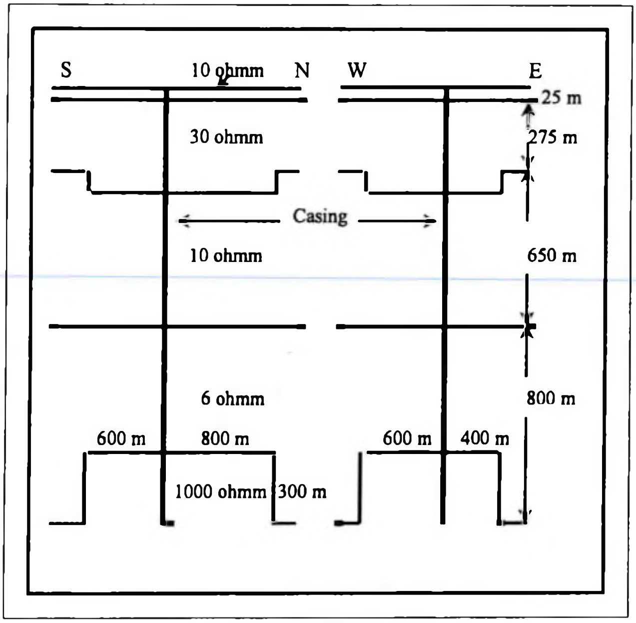

Fig. 1. Side view of 3-D inhomogeneity model penetrated by the casing
1. ábra. A béléscsővel harántolt laterális inhomogenitások modelljének oldalnézeti vetületei

Let us examine first the case when the horst of the basement is the only inhomogeneity. The current emitted from the casing accumulates electrical charge on the surface of the prism, as is seen in *Fig. 2*. Charges on the top and side faces are shown from above. This charge distribution can be substituted by point-sources distribution, which produces the secondary electric field components at the earth's surface. The maxima of the charge distribution can be found on the side faces where they are closest to the casing and on the top and bottom face around the casing. On the top and bottom faces the signs of the charges are opposite, they act against each other thereby reducing the effects of the horizontal planes.

Extreme values of the *secondary radial electric field* isolines reflect the horst contour rather well (*Fig. 3*). However, this component itself cannot be measured because it is superposed on the primary field of the source.

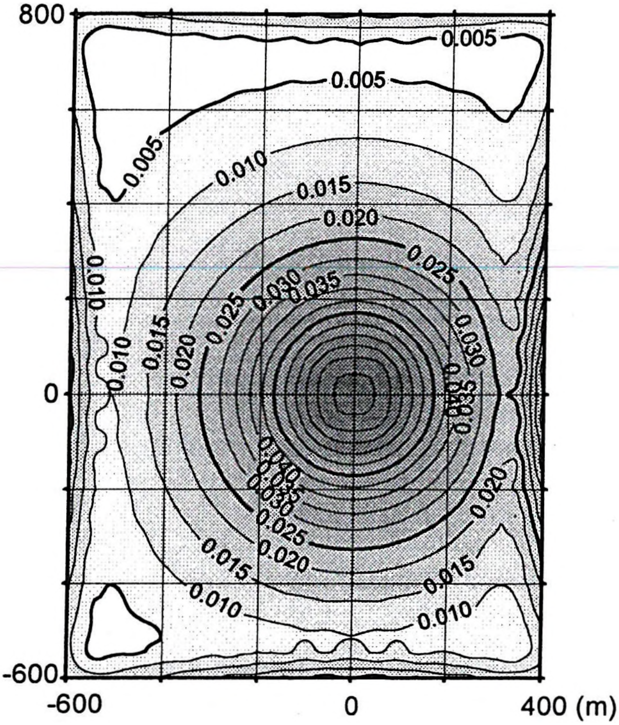

Fig. 2. 'Current intensity' distribution on the upper and side faces of the lower inhomogeneity in Fig. 1.

2. ábra. „Áramerősség" eloszlás az 1. ábra alsó inhomogenitásának felső és oldalsó lapjain

Fig. 3. Contour map of anomalous radial electric component caused by the lower inhomogeneity

3. ábra. Az alsó inhomogenitástól származó anomális radiális elektromos összetevő felszíni izovonalai

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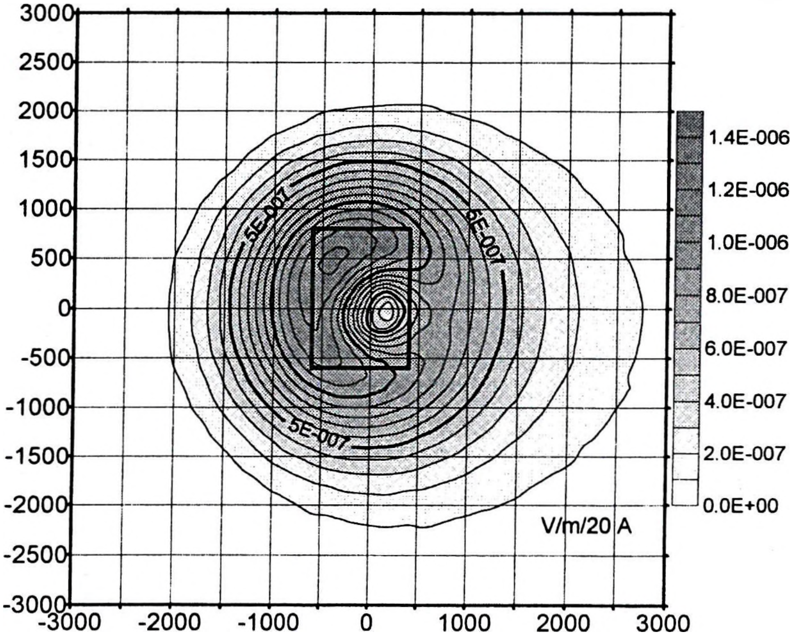


Apparent resistivities calculated from the total radial electric field — the sum of the primary and secondary fields — already reflect less clearly the shape of the inhomogeneity (*Fig. 4*). It should be noted that the resistive basement — in consequence of the VED behaviour of the source — will diminish the apparent resistivity until separation occurs. The nearly circular strip of the somewhat reduced apparent resistivity values at about 1600 m separation — ROA< 9.7 ohmm — appears outwards from the contour of the horst. Clear conclusions on the asymmetrical position and on the orientation of the horst can be drawn on the deformation of the isolines only at rather large separation.

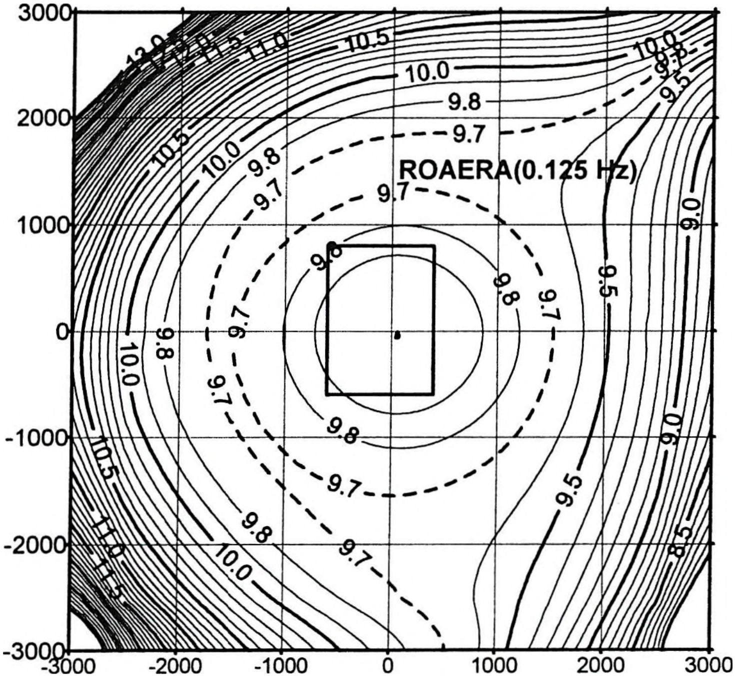


Fig. 4. Effect of lower inhomogeneity on the apparent resistivity contour map calculated from radial electric amplitude

4. ábra. Az alsó inhomogenitás hatása a radiális elektromos összetevőből számított látszólagos fajlagos ellenállás izovonalaira

The tangential electric component will appear only when the charge distribution has an asymmetry relating to the radial profile of the observation. Therefore on the contour map of its isolines — in *Fig. 5* — there is a sector with very small values which is in the east nearly perpen-

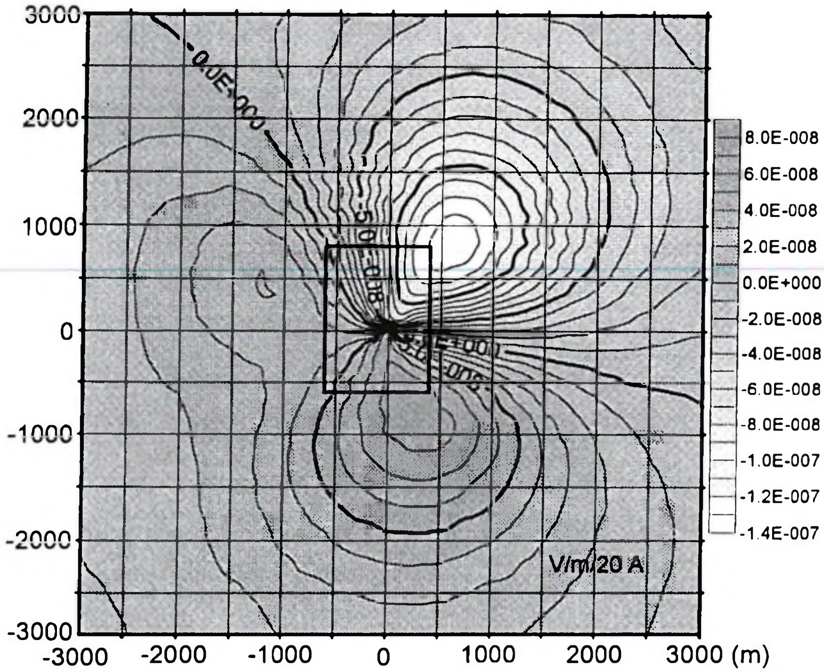


Fig. 5. Contour map of tangential electric amplitude caused by the lower inhomogeneity
5. ábra. Az alsó inhomogenitástól származó tangenciális elektromos összetevő felszíni izovonalai

dicular to the side face, where the charge values have their maximum. Extreme values are located very near to the eastern boundary of the horst. The sharp zone with minimum values — from where, looking towards the casing, there is a symmetry in the charge distribution — turns out to be a reliable indicator of a lineamentum near to the borehole. As a result of the asymmetrical position of the horst the minimum zone passes over it in a curved manner. The pattern of isolines looks like one over a horizontal dipole with a bent axis. It should be noted that the tangential electric component could not exist without lateral inhomogeneities and its source is exclusively the charge accumulation on the surface of the horst. Owing to that, if the current density at the depth of the inhomogeneity is strong enough, its presence can be detected at small separation, even in the vicinity of the source.

Secondary magnetic components are caused by currents flowing inside the inhomogeneity to maintain the charge distribution on its surface. These currents, in addition to the horizontal one, will have a vertical component, too. However, the magnetic fields caused by the vertical currents vanish at the earth's surface [KAUFMAN 1992] consequently the secondary magnetic components observed at the earth's surface are caused only by the horizontal currents; in other words, by the multitude of horizontal electric dipoles distributed inside the inhomogeneity. *Figure 6* shows the momentum distribution of the horizontal electric dipoles inside the horst. The length of the arrows is proportional to the resultant momentum in the volume-element of 100 m x 100 m x 300 m. Because the 'casing-surface electrodes' source has no primary magnetic field component at the surface, the observed magnetic field components will be connected — based on their own individual behaviour — to the resultant current filament flowing along the minimum zone of the tangential electric field (Fig. 5). In *Fig. 7* the vertical magnetic field is given as an example.

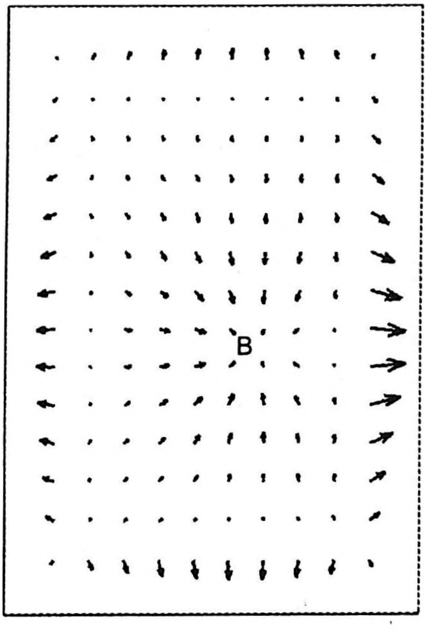


Fig. 6. Momentum distribution of horizontal electric dipoles inside the volume-elements of the lower inhomogeneity

6. ábra. A térfogatelemenkénti vízszintes elektromos dipólusok momentumainak eloszlása az alsó inhomogenitáson belül

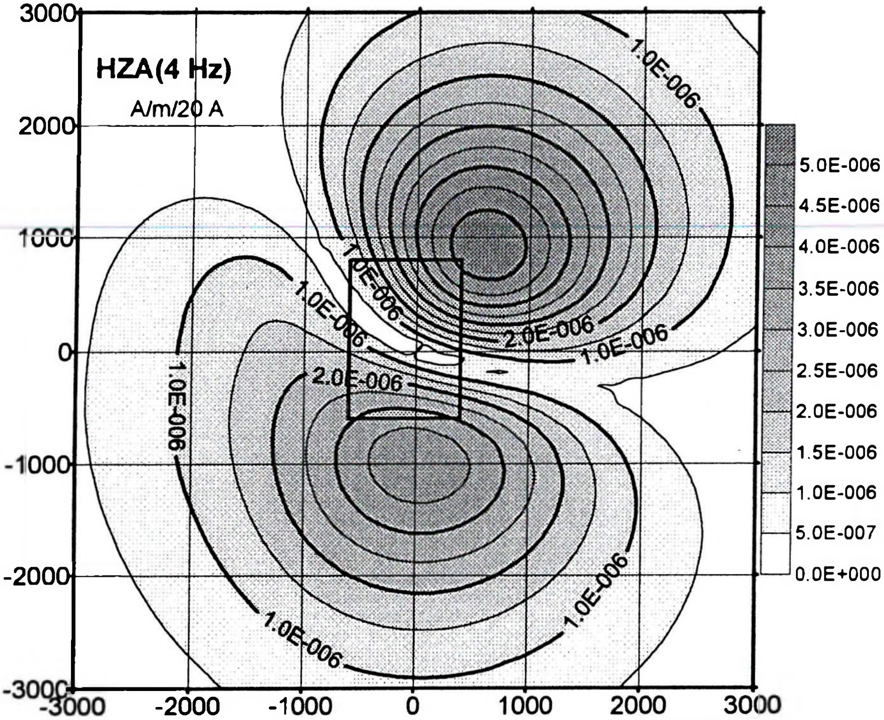


Fig. 7. Contour map of vertical magnetic amplitude caused by the lower inhomogeneity
7. ábra. Az alsó inhomogenitástól származó vertikális mágneses összetevő felszíni izovonalai

3. Role of subsurface horizontal electric dipoles in the inversion of the secondary components

Analysis of the secondary components has shown that

—in the inversion of the secondary electric components we have to determine the location of surface charges, or horizontal and vertical electric dipoles inside the inhomogeneities,

—in the inversion of the secondary magnetic components we have to determine the locations of horizontal electric dipoles inside the inhomogeneity.

We can conclude that the dominant effects on the secondary components are caused — in the case of prismatic inhomogeneities — by the extreme values of the charge, or dipole distribution on the nearest part of the side walls to the casing (Figs. 2, 6). With the increase of the elongation this dominance will be more and more pronounced and the patterns due to a single horizontal electric dipole will appear. This means, however, that by inversion the depth can be deduced more reliably than the contour.

Taking into account the leading role of the horizontal electric dipoles in the creation of the secondary components, let us now consider the behaviour of their field components at the earth's surface. Tangential electric, tangential and vertical magnetic amplitude curves caused by a subsurface horizontal electric dipole — normalized to their own maximum — are shown in *Fig. 8* as a function of the 'separation/depth' quotient along a profile perpendicularly oriented to the dipole axis. Characteristic points can be selected — e.g. maximum, null-crossing, half value, etc. — for the depth estimation, which in my experience is quite reliable for a single inhomogeneity.

Inversion needs the solution of the direct problem. Simple expressions exist for calculating the stationary field components caused by the subsurface charges and dipoles in the uniform halfspace. After the men-

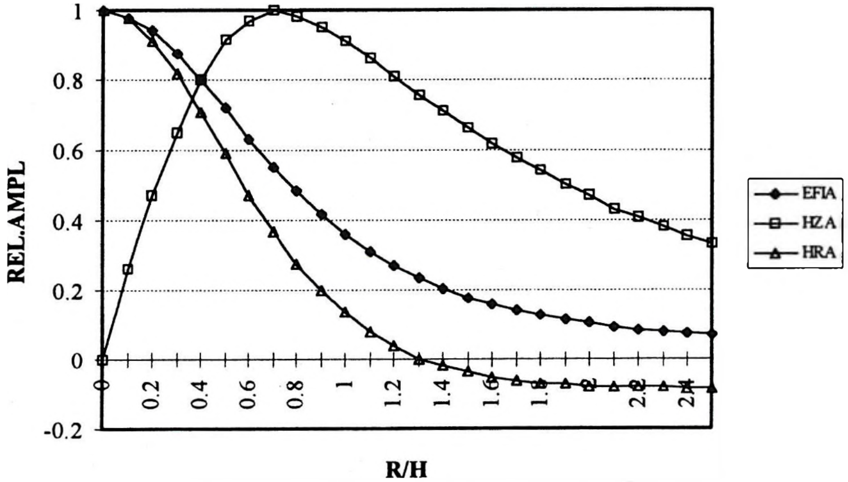


Fig. 8. Normalized to their maximum amplitude curves of the tangential electric (EFI), radial (HR) and vertical magnetic (HZ) components due to a buried electric dipole as a function of 'separation/depth' quotient

8. ábra. A mélybeli horizontális elektromos dipólus tangenciális elektromos (EFI), radiális (HR), és vertikális mágneses (HZ) térerősségének maximumukra normált értékei a felszíni, a dipólus irányára merőleges szelvényen a távolság és mélység hányadosának függvényében tioned normalization, they contain only geometrical data. Equations of the electric field above layered earth are already rather complicated. At the same time, formulae of the stationary magnetic fields are independent of layering. Moreover, they remain valid up to a frequency limit. To get some information in this respect the position of the vertical magnetic field amplitude maximum was studied as a function of the 'conductivity × frequency' product for dipoles of different depth (*Fig'. 9*). We can see that at 2.5 Hz and at a dipole-depth of 1000 m the shift of the maximum is within 10 %. Theoretically, a correction is possible, too.

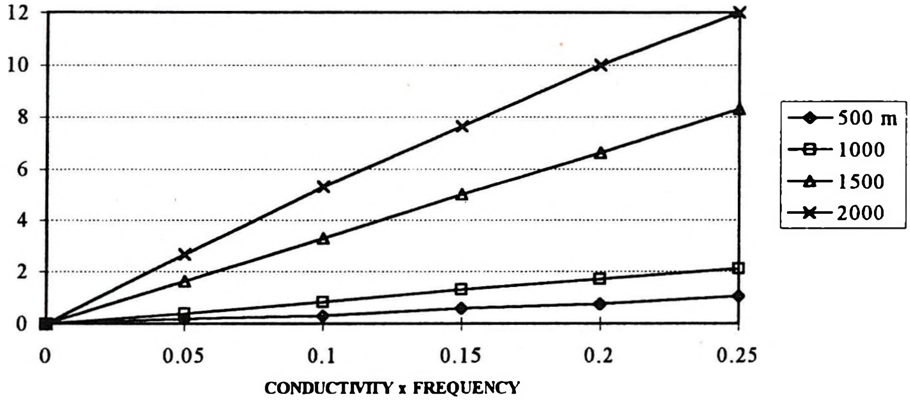


Fig. 9. Relative shift — caused by the frequency increase — of the vertical magnetic maxima due to a buried, horizontal direct current dipole and low frequency electric dipoles at different depths as a function of the conductivity × frequency product

9. ábra. A különböző mélységű felszin alatti, egyenáramú és kis frekvenciás, vízszintes elektromos dipólusok vertikális mágneses térerőssége maximumainak relatív eltolódása a frekvencia növekedése miatt a fajlagos vezetőképesség és a frekvencia szorzatának függvényében

4. Inversion attempt with Patella's method

It was mentioned above that the aim of the inversion of the secondary components is some kind of reconstruction of the subsurface electric charge-, or dipole-distribution. Inversion of the observed SP data has the same task. However, analogies could be found with the inversion procedures for gravity and magnetism, too.

For my first attempt I selected Patella's method [MAURIELLO et al. 1998, PATELLA 1997] elaborated initially for the inversion of SP data and later for DC vertical electric sounding. For simplicity my first attempt was limited to a 2-D cross-cut inversion used for the synthetic data of the numerical simulation treated above. The selected SW–NE profile passes through the extreme values in the presence of both the upper and the lower inhomogeneity.

Patella's method consists of scanning the vertical section through the profile along the regular grid by a unit-strength charge, or dipole. For each point of the grid field-strength components are calculated to the observation point of the profile. These calculated field data are cross-correlated with the observed values. Then, at each point of the grid charge or dipole occurrence, probability values are calculated from the cross-correlation data. Expressions for the steps of the inversion are shown in *Fig. 10.* The meanings of the symbols are as follows:

x — observation point coordinate along the profile,

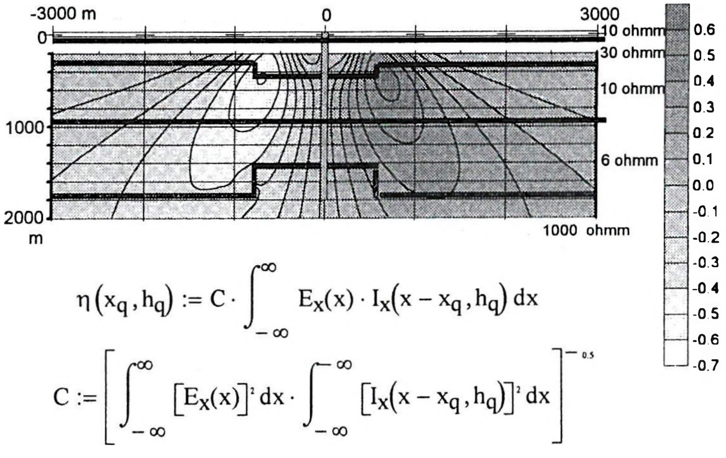
x_q, h_q — coordinates of the continuously displaced unit source,

$E_x(x)$ — secondary radial *electric* field, or instead of it any observed field component,

$I_x(x_q, h_q)$ — scanning function depending on the type of source and the measured component, it contains only x_q, h_q coordinates,

C — normalization factor, which includes normalization by the total power of the field component recorded at the earth's surface,

$\eta(x_q, h_q)$ — charge , or dipole occurrence probability.



$$\eta(x_q, h_q) := C \cdot \int_{-\infty}^{\infty} E_x(x) \cdot I_x(x - x_q, h_q)\, dx$$

$$C := \left[\int_{-\infty}^{\infty} [E_x(x)]^2 dx \cdot \int_{-\infty}^{\infty} [I_x(x - x_q, h_q)]^2 dx \right]^{-0.5}$$

Fig. 10. Charge occurrence probability section calculated from anomalous radial electric amplitude along the SW–NE profile

10. ábra. Az anomális radiális elektromos térerősségből számított töltés-előfordulási valószinűség metszete a DNY–ÉK szelvényre

The complete set of calculated grid values of $\eta(x_q, h_q)$ is used to draw a contoured section in order to single out the zones of highest probability of concentration.

Figure 10 shows the result of the secondary radial electric field inversion. The location of the side walls of the upper prism are marked out quite well. The presence of the lower prism can only be guessed from the elongation of isolines downwards. The reason for this is that in the electric component — especially at small separation — the signal from the lower inhomogeneity is comparatively insignificant. However, the main problem with this component is rather the elimination of the primary field from the observed total field. Furthermore, in principle a scanning function valid for the layered section should have been used.

The situation is more favourable for magnetic components. In this case

—the primary field is missing at the earth's surface,

—stationary magnetic components do not depend on layering: this means scanning functions remain even in very simple expressions,

—the effect of inhomogeneities near to the surface is less dominant than in the electric field.

For example, the vertical magnetic component reflects the horst of the basement, as *Fig. 11* shows. The image is rather smoothed owing to the

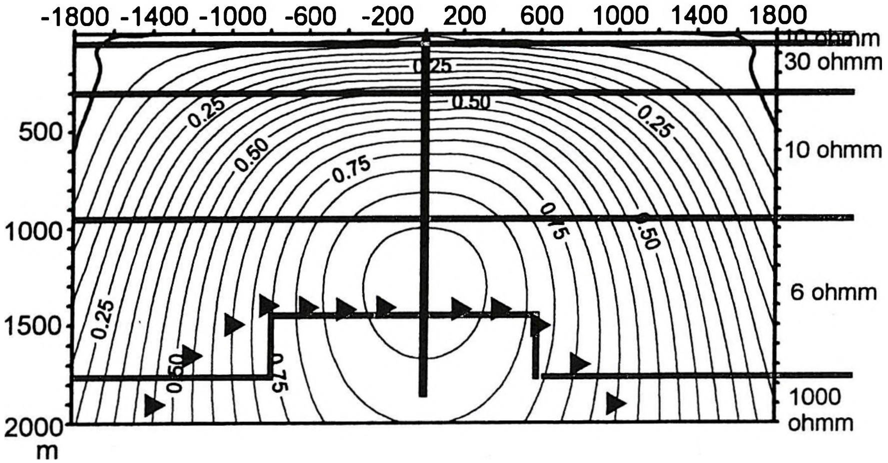


Fig. 11. Horizontal electric dipole occurrence probability section calculated from the vertical magnetic amplitude due to the lower inhomogeneity along the SW–NE profile

11. ábra. A vertikális mágneses térerősségből számított dipólus-előfordulási valószinűség metszete az alsó inhomogenitás esetében a DNY–ÉK szelvény alatt great depth, and it is shifted upwards. The upper and lower inhomogeneities have distinct indications in *Fig. 12*. The exact positions of the maxima along the vertical axis under the observation points are marked by triangles. Asymmetry between the eastern and western side becomes more accentuated. Above the side of larger extension of inhomogeneities — where the field strength is also larger — the superposition of effects gives rise to distortion in the position of the probability maximum. Omitting values at small separation between –600 and +600 m (where the effect of the upper inhomogeneity is the strongest) the result — marked by circles — will be somewhat better.

It is to be noted that 2-D inversion suffers from several ambiguities. Very characteristic patterns of the contour maps are not taken into account The real areal subsurface charge- or dipole-distribution — which is asymmetrical to the profile with dynamically changing strength and orientation — is substituted by dipoles perpendicular to the profile. Therefore, it is reasonable anyway to investigate the efficiency of the 3-D version.

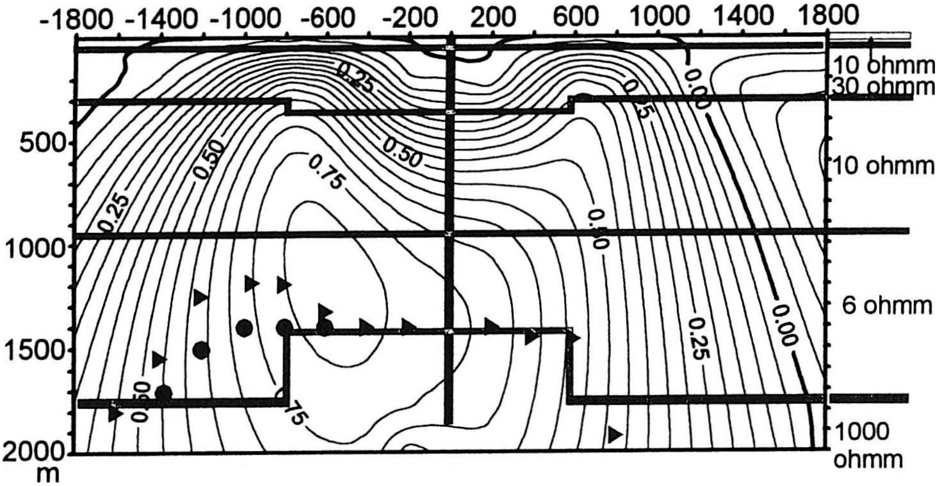


Fig. 12. Horizontal electric dipole occurrence probability section calculated from the vertical magnetic component due to the upper and lower inhomogeneities along the SW–NE profile

12. ábra. A vertikális mágneses térerősségből számított dipólus-előfordulási valószinűség a felső és alsó inhomogenitás esetére a DNY–ÉK szelvény alatt

5. Information gained by frequency sounding

Rather large separations are required to detect deep inhomogeneities by geometrical sounding. The resolving capability and reliability of the data gained by geometrical sounding will become worse with the increase of the separation and depth. Therefore I investigated what kind of information abundance can be gained by frequency sounding especially in the resolving capability of deep inhomogeneities at shorter separations.

Figure 13 shows a separation–frequency section of the vertical magnetic amplitude for the lower inhomogeneity along the N–E profile through the maximum. At each frequency a single, isolated maximum indicates the uplift of the basement. This maximum due to the eastern border of the horst grows and moves outwards as the frequency decreases, this means that the depth of the penetration increases. In this case — as was mentioned earlier — the depth of the inhomogeneity can be quite well estimated from the separation dependence of the stationary field strength.

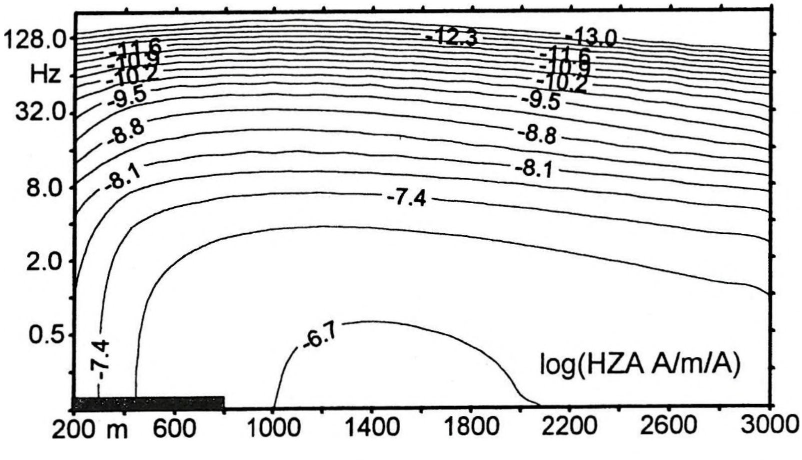


Fig. 13. Separation–frequency section of the vertical magnetic amplitude due to the lower inhomogeneity along the profile through the maxima in Fig. 7.

13. ábra. A vertikális mágneses térerősség távolság–frekvencia metszete az alsó inhomogenitás esetében a 7. ábra maximumain átmenő szelvényre

As far as the model with two inhomogeneities is concerned, we found that the superposition of the secondary fields generated by the two subsurface sources results in a rather surprising pattern (*Fig. 14*). The upper prism appears with a distinct maximum at 4–32 Hz. The field

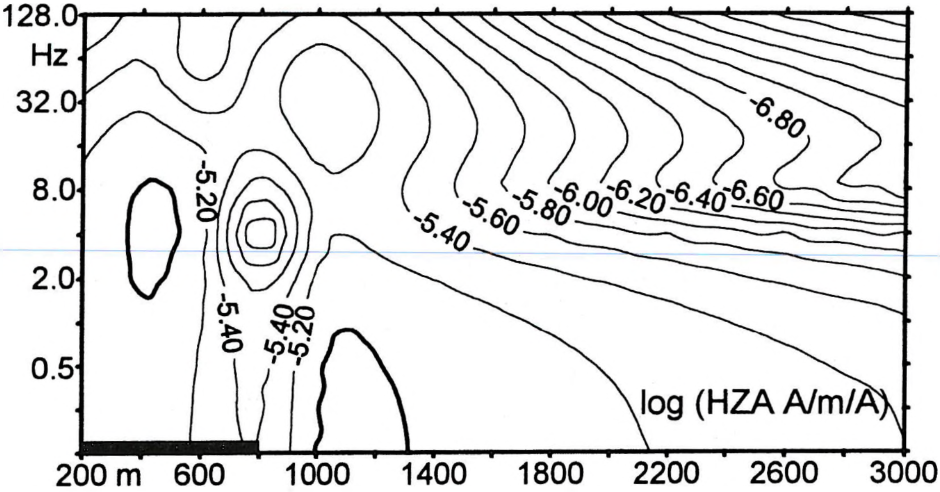


Fig. 14. Separation–frequency section of the vertical magnetic amplitude due to the upper and lower inhomogeneities along the profile through the maxima

14. ábra. A vertikális mágneses térerősség távolság–frekvencia metszete a felső és alsó inhomogenitás esetére a maximumokon átmenő szelvényen

strength has been increased significantly and its separation dependence — at least in the region close to the casing — is basically determined by the upper inhomogeneity. The lower inhomogeneity is indicated at 0.1– 0.5 Hz by a second, separated maximum. It is noteworthy that the frequency dependence of the amplitude shows, even at stations near to the casing, the presence of the two prisms. In the case of geometrical sounding the trend change — caused by the lower inhomogeneity in the separation dependence of the amplitude — would appear clearly only at much larger separations.

Moving outwards along the profile on the separation–frequency section — as well as in the frequency sounding curves (*Fig. 15*) — a well correlated sharp minimum zone takes shape — in the frequency-range 4–8 Hz — between the indications of the two prisms. To avoid false interpretation it is important to emphasize that this pattern of the separation–frequency section — and of the frequency sounding curves — of the secondary fields is caused by inhomogeneities in the vicinity of the casing and does not indicate local layering.

The sharp local extreme values and the surprising variability found in the frequency dependence of the secondary components hint at new

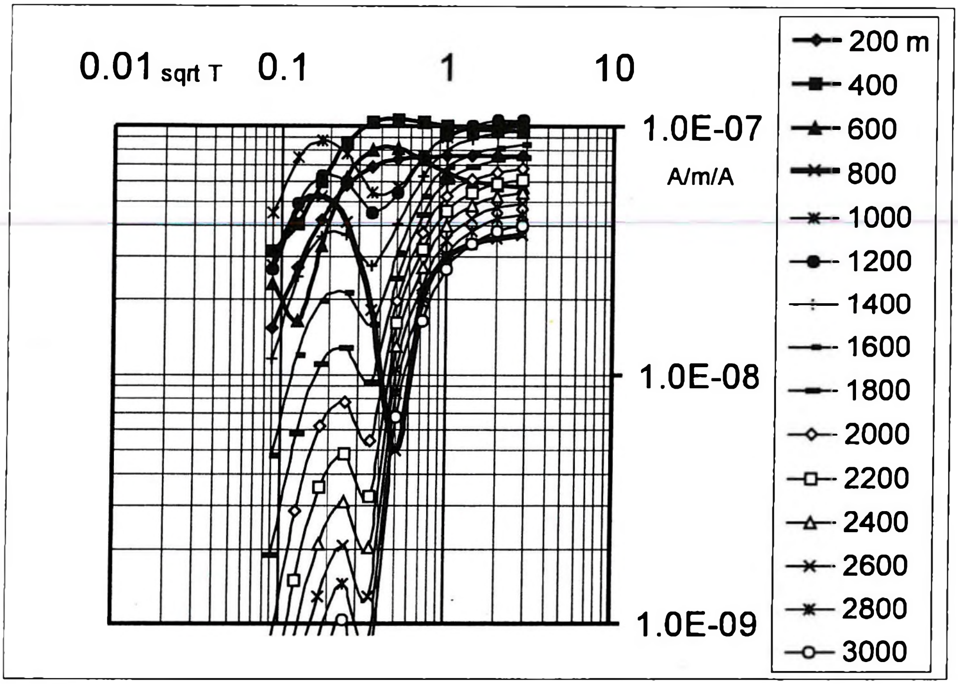


Fig. 15. Vertical magnetic amplitude frequency sounding curves along the profile through the maxima

15. ábra. A vertikális mágneses összetevő frekvencia-szondázási görbéi a maximumon átmenő szelvényen

possibilities in the lateral and vertical delineation of inhomogeneities. Besides, it seems quite reasonable that much shorter profiles are needed using frequency sounding than with the geometrical one. However, we have to add that there is a need for a thorough study of the features of the indications because they depend strongly on the shape of the inhomogeneity and on the orientation of the profile.

6. Conclusions

Based on the above analysis we can conclude that the secondary electromagnetic components have very characteristic, very dynamic and — for the interpretation — very useful indications of the lateral inhomogeneities. At the same time very detailed measurements are needed to detect all their local peculiarities. Perfect inversion could be realised only by 3-D inversion taking into account the changeable areal distribution of the field-strength and the amount of data.

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The paper was read at the 3rd Inversion Seminar in 2002. My intention was to use it as a means of greeting Professor Dr. Ferenc Steiner on his 70th birthday, motivated by our nearly 50-years' friendship and common work.

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Az anomális elektromágneses összetevők inverziós szempontú elemzése

TAKÁCS Ernő

A laterális földtani változások határvonalának felderítéséhez az elektromágneses tér anomális komponensei kedvező sajátságúak. Ez a tény különösen akkor hasnzálható ki, ha a tér forrásának primér térerősségében hiányzik valamelyik összetevő és így a megfelelő anomális komponens közvetlenül mérhető. Emiatt a határvonalak indikálásának lehetőségén túllépve érdemes az anomális ösysszetevők inverziójával foglalkozni, ami forrásuk — a hozzájuk kötődő elektromos töltések, illetve dipólusok — felszín alatti eloszlásának meghatározását jelenti.

A tanulmány egyrészt azt vizsgálja, hogy a töltéseknek és dipólusoknak milyen eltérő sajátságai vannak az anomális elektromos és mágneses komponensek létrehozásában. Másrészt szimulált adatokon teszteli Patellának eredetileg a természetes potenciál adatok inverziójára kidolgozott eljárását.

ABOUT THE AUTHOR

Ernő Takács (1927) graduated from the Technical University of Heavy Industry — Sopron/Miskolc — as a mining engineer specialized in prospecting. In 1951 he joined the Department of Geophysics that had been founded that year. He was head of the Department from 1983 to 1991. He is now a professor emeritus. During the years 1956–59 he worked as telluric party-chief of the Chinese–Hungarian Geophysical Expedition.

His main scientific interest is in the development of electromagnetic methods. He was a pioneer in introducing tellurics, magnetotellurics, and several modifications of dipole frequency sounding in Hungary. He is honorary member of the Association of Hungarian Geophysicists and served two terms as chairman of the Scientific Committee of Geophysics of the Hungarian Academy of Sciences.