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Dispersion analysis of ground roll using analytical velocity functions

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Seismic ground roll is a characteristic feature of area covered by loose loess sediments. On these areas mainly the velocity distribution is depth dependent and can be approximated by analytical functions. If the characteristic features of these sediments are the velocity dispersion and the absorption these can be considered to provide a visco-elastic rock model. We should like to support this hypothesis by analysing the results of our field experiments and modelling.

Keywords: dispersion, ground roll, velocity

1. Introduction

Since the beginning of the intensive seismic prospecting in Hungary, but particularly between 1953 and 1968, many experiments were carried out in an endeavour to learn about the nature of the most disturbing wave — *ground roll* — which causes the so called *no reflection (NR)* areas. These were the hilly part of Transdanubia, and several smaller local territories of the Great Hungarian Plain [ÁDÁM 1954, 1964; SZÉNÁS, ÁDÁM 1953; ÁDÁM, Sz.KILÉNYI 1963; ÁDÁM 1969]. After this rather long period and because of the technological–methodological development based on computerization, miniaturization of equipment, the *common depth point (*CDP) system and geophone arrays used in the field, ground roll became only a memento to the seismologists and not an object to study in detail. One of the most interesting publications in this respect was that of ANSTEY *[Whatever happened to ground roll?* 1986].

Since 1986 three other papers should be mentioned here: GABRIELS et al. [1987] investigating the dispersion characteristics of a sand layer series on a flat area of a beach; KRAGH et al. [1995] proposing the use of the elliptical nature of $(\mathbf{u}_x, \mathbf{u}_z)$ displacements or $(\mathbf{v}_x, \mathbf{v}_z)$ displacement velocity amplitudes of ground roll; SCHNEIDER, DRESEN [1994] who used the dispersion characteristics of ground roll to determine the depth variation of a shallow refuse pit. All

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three publications consider ground roll as different modes of Rayleigh type or at least *P-SV*-waves. Our aim is to give more information about the nature of ground roll with regard to dispersion.

2. Field investigation

During the last three years we had the opportunity to carry out field investigations on an area of *no reflections* or *very poor reflections*. Data acquisition was carried out by parameters (254 m long spread, 1 m geophone interval, vertically effective force as source) suitable for *f–k* analyses and determination of dispersion characteristics of the layer series consisting of loose sediments such as different kind of loesses and upper Pannonian clayey sands, etc.

A very important property of these sediments is the compressibility that involves the $V(z)$ depth dependence of any kind of seismic (*P*- and *S*-) wave velocities. The thickness of these dry formations is generally about 30–50 m above the watertable, but at some places much more. In consequence of these situations the seismograms were built up of disturbing *ground roll* waves (*Fig. 1*) suppressing and damaging all the reflection signals by their very large amplitudes, and generating different kinds of other noises (for example different kind of harmonics), too. The effect of depth dependent velocity variations is apparent by the *diving wave* character. On the seismogram the curvilinear character of different group of arrivals and the widening trains of disperse waves can be clearly seen. These also mean that 100 or more folds of common depth point spreads had or have to be used to obtain a fairly good time section. Because of the consistency of these loose sediments, the validity of a *visco-elastic solid model* is supposed. This was comprehensively analysed by RICKER [1953]. Judging from this model the main features of elastic waves are the *dispersion and frequency dependent attenuation*. DOBRIN [1951], TOLSTOY and USDIN [1953], ÁDÁM [1969] gave some proof of this behaviour. The present paper deals solely with the dispersion characteristics of *ground roll* but in somewhat more detail.

3. Fitting of seismic parameters

In order to describe the velocity–depth relation some simple analytical functions were used in seismic prospecting [BANTA 1941, WHITE 1963, KAUFMAN 1953, and, recently, AL-CHALABI 1997]. For example the more simple ones are

$$V(z)=V_0\,(1+kz)^{(1\ n)} \tag{1}$$

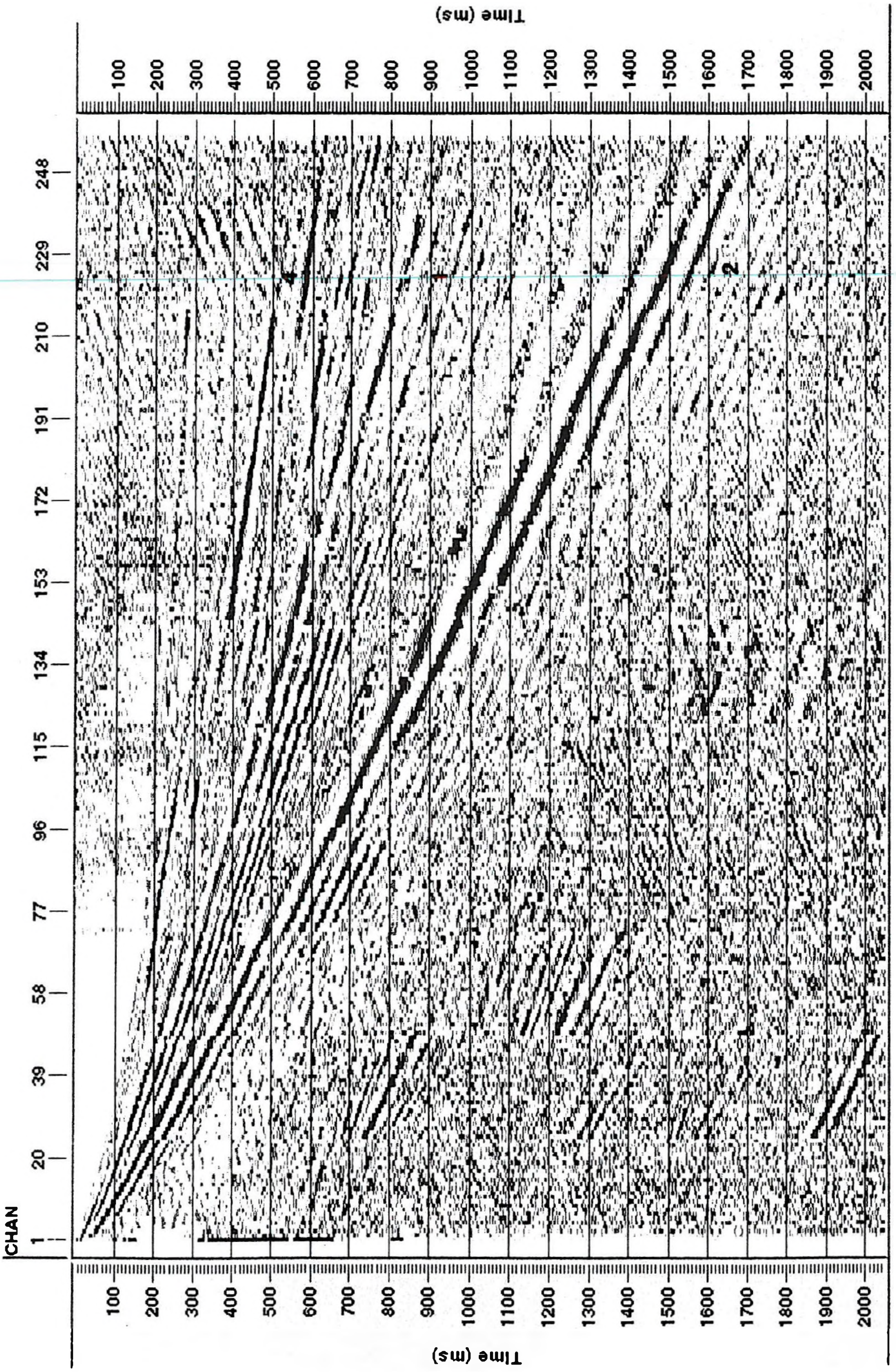

Fig. 1. Ground roll seismogram from Hungary's Udvari region (Tolna County)
1. ábra. Zavarhullám szeizmogram az Udvari (Tolna m.) területről

$$V\,(z)=A\,z^{(1\ n)} \tag{2}$$

$$V\,(z)=V_0 \exp\,(-kz) \tag{3}$$

(The dimensions are: V(m/s); A(1/s); n (dimensionless); z(m); k(1/m))

If we suppose the validity of one of these power functions, e.g. equation (2), the travel times can be approximated by a relatively simple equation, as follows:

$$t(x)=\frac{n\pi^{\frac{1}{2}}}{A}\,\frac{\Gamma\left(\frac{n-1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}\left\{\frac{x\Gamma\left(\frac{n}{2}+1\right)}{n\pi^{\frac{1}{2}}\Gamma\left(\frac{n+1}{2}\right)}\right\}^{\frac{n-1}{n}} \tag{2a}$$

where Γ is the well known gamma function, A is a parameter with dimension of 1/s, n is a dimensionless constant. The parameters of velocity function (2) are A and n and these can be computed from the travel time equation. For Fig. 1 this means at least three for the different groups of waves.

The *f–k* diagram (*Fig. 2*) has the characteristics of a power function, too, like *t(x)* travel-time on Fig. 1. Based on our own experience *the velocity function of type (2) is a good approximation and therefore the C(f) phase velocity–frequency relation of the ground roll* can in many cases be described quite well in the form of

$$C\,(f)=C_1 f^{-m} \tag{4}$$

where $m < 1$, C_1 is a constant *(Fig. 3)*.

From what is described above one can conclude that the depth dependence of seismic parameters (i.e. the $V_P(z)$, $V_S(z)$ propagation velocities and the $\rho(z)$ density) in such cases can also be described by simple analytical functions. The application of this kind of function has some advantages because

— the main features of data are represented

— the relations between *C(f)* and *V(z)* data are clearly shown

— the large computational efforts of inversion tasks can considerably be reduced.

According to the well-log data the density–depth function can be described in the form of

$$\rho(z)=\rho_v-(\rho_v-\rho_0)\exp\,(-Kz) \tag{5}$$

Where for the first layer $\rho_0=\rho(0)$, $\rho_v=\rho(\infty)$, K determines the gradient of the density function $\rho(z)$.

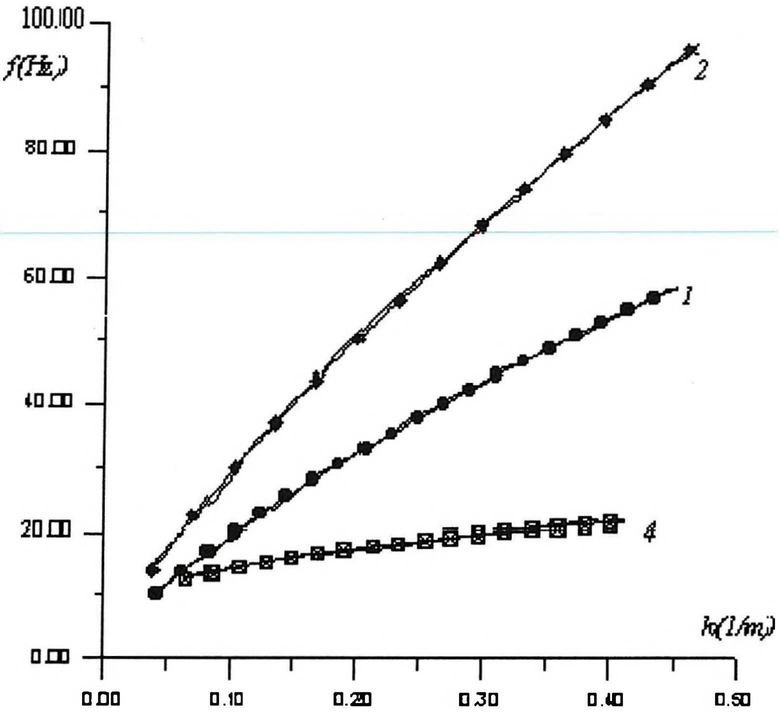

Fig. 2. Selected events from *f-k* diagram of the seismogram in Fig. 1
2. ábra. Az 1. ábra szeizmogram egyes jelenségeinek *f-k* diagramja

For determining the actual value of parameters at the sites examined we used the fitting of the calculated and the measured dispersion curves. The goodness of fit can be measured by the relative differences:

$$\Delta a = \sqrt{\frac{1}{n}\,\Sigma\left(\frac{c_m(f)-c_c(f)}{c_m(f)}\right)^2} \tag{6}$$

where n is the number of *data*, $c_m(f)$ and $c_c(f)$ are the measured and the calculated phase velocities at frequency f.

4. Dispersion calculation

Our N-layers dispersion calculations are based on the well-known algorithm of HASKELL [1953]. In this formulation the phase velocity curves can be determined by the (c, f) root pairs of

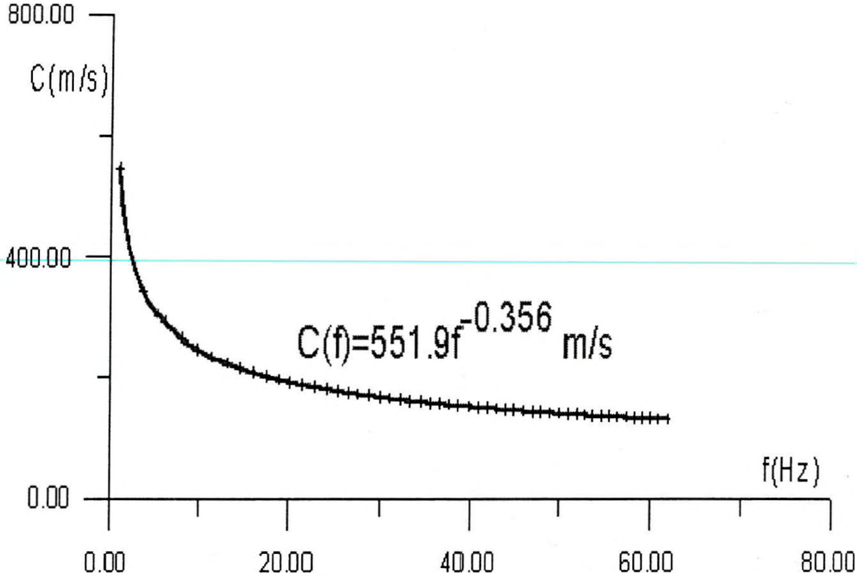

Fig. 3. *C (f)* phase velocity diagram of one *f–k* curve of Fig. 2.
3. ábra. Az egyik *f-k* diagramból számolt *C(f)* fázissebesség menet

$$F\ (c, f) = 0$$

where F is a relatively complex function constructed from the 4x4 layer matrices. Their elements are built from layer parameters.

Having fixed the frequency, the series of c roots give the phase velocities of fundamental and higher modes. Repeating the process at all frequencies of c_m data all c_c values can be determined.

Suitable approximation of continuous depth functions by a layered structure needs some consideration. In the case of N layers the depths of the lower- and uppermost layer interfaces (h_1, h_N) are determined by the extreme wavelengths of the dispersion data:

$$0.5\ \Lambda_{min} < h_1 < \Lambda_{min} \tag{7}$$

$$\Lambda_{max} < h_N < 2\ \Lambda_{max} \tag{8}$$

where Λ_{min} and Λ_{max} are the wavelengths of the examined dispersion data.

For the intermediate layers the ratio of layer thickness d_i to the layer depths h_{mi}, i.e. the value of

$$R = d_i / h_{mi}\ , \tag{9}$$

must be between 0.2 and 0.3, where

$$d_i = (h_i - h_{i-1}). \tag{10}$$

and the layer depth is defined as

$$h_{mi} = (h_{i-1} + h_i)/2 \, . \tag{11}$$

From these

$$h_i = h_{i-1} \; q \tag{12}$$

where

$$q = (R+2)/(2-R) \tag{13}$$

Using relations (7)–(11) the number of layers can be estimated as:

$$N = 5 + 10 \log (\Lambda_{max}/\Lambda_{min}).$$

In our practice the value of N is usually between 12 and 30.

The seismic parameter of layer i can be set by minimizing

$$\int_{h_{i-1}}^{h_i} (P(z) - P_i)^2 \, dz$$

5. Results of fitting

As mentioned above by preliminary investigations for the velocity–depth relationship the function of type (2) proved to be the best. The results of the fitting procedure can be seen in *Fig. 4a*.

Here we give the velocity and density parameters of the best fitting function for the 1st layer, (index $S1$ for transversal, index $P1$ for longitudinal) above the level of water saturation (down to 30 m)

A_{S1}=150 1 /s
n_{S1} = 3.55
A_{P1}= 300 1/s
n_{P1} = 3.65
$\rho_k = 1.7 \; 10^3$ kg/m^3
$\rho_v = 2.0 \; 10^3$ kg/m^3
K_1=0.12 1/m;

the parameters below 30 m are: V_S=420 m/s, V_P=1600 m/s, $\rho=2.3 \; 10^3$ kg/m^3.

6. Global sensitivity

During the fitting procedure we might have got data for the sensitivity of approximation of the various type of parameters. Below, we list the ratio of relative deterioration (increase) of Δa caused by the 1% change of parameters around its best P_0 values:

p	$d\Delta a/\Delta a$
V_{S1}	16.6%
n_S	4.9%
V_{P1}	3.2%
n_P	1.5%
ρ_k	0.4%
ρ_v	0.04%
k	0.3%

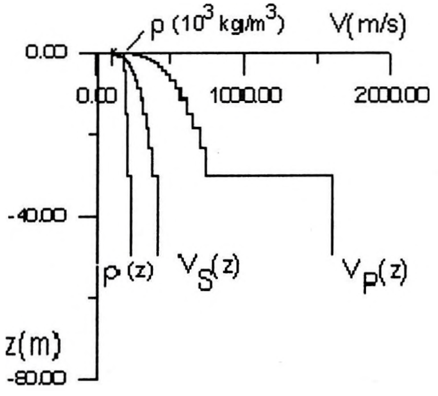

Fig. 4a. Layered model fitted to the dispersion curve

4a. ábra. A diszperziós görbéhez illesztett rétegmodell

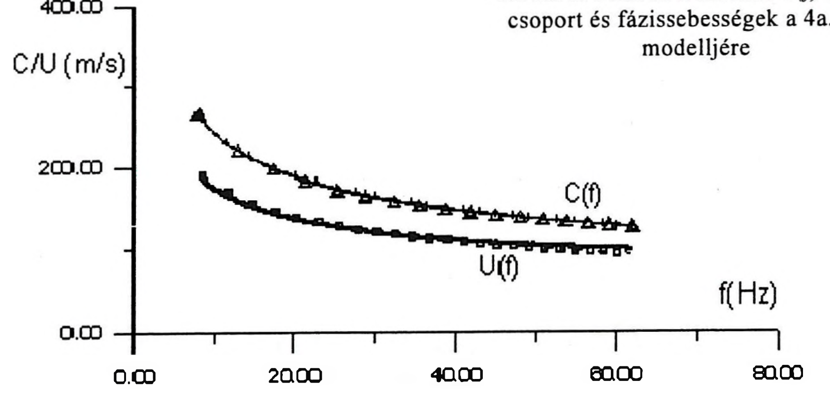

Fig. 4b. Measured and computed phase *C(f)* and group *U(f)* velocity for the model of Fig. 4a

4b. ábra. Mért és számított *U(f)* és *C(f)* csoport és fázissebességek a 4a. ábra modelljére i.e. the variation caused by a change in V_{S1} — for example — is fivefold that of the variation caused by the same relative change in V_{P1}.

These data are in accordance with the well-known fact that the dispersion curves are governed mainly by the *S*-velocity structure of the medium.

7. Investigation of differential sensitivity

Using the layered models described above it is easy to investigate the perturbations of dispersion curves caused by the alteration of seismic parameters of a single layer. Examination of these functions can provide insight into the contribution of the different seismic parameters at different depths to the structure of the *c(f)* function [NATAF et al. 1986]. In this investigation we have used a smooth model having 32 layers.

For a layer at an intermediate depth the perturbations belonging to the different relative changes of the V_S, V_P and ρ are shown in *Fig. 5*. It can be seen that this relatively thin layer (R=0.2) has a broad-band effect on the dispersion curve. On the curves — at least in the cases of small perturbations — well defined f_π 'peak frequencies' can be seen and using the equation

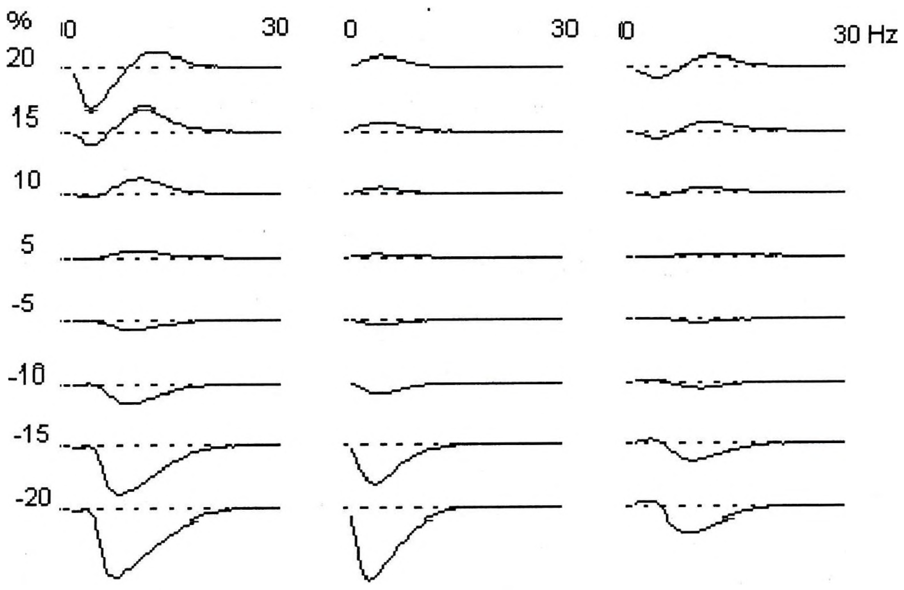

Fig. 5. Changes in the dispersion curve caused by the modification of layer parameters (V_P, V_S and ρ)

5. ábra. A diszperziós görbe megváltozása a rétegparaméterek (V_P, V_S, ρ) módosításának hatására

$$\Lambda_\pi = c_0\,(f_\pi)/f_\pi$$

'peak wavelength' can be calculated (here $c_0(f)$ is the nonperturbed dispersion curve). Taking the depth of the modified layer as 'peak wavelength-layer depth' (see equation (11)) it is found that there is a linear relationship between them, i.e. $\Lambda_\pi = a \cdot h_m$ (*Fig. 6*).

Because of the significant width of the perturbation functions (Fig. 5) the phase velocity determined at one frequency involves not only one layer but the neighbouring ones, too. Their influence decreases with the increasing 'distance' of the neighbouring layers. The half-peak interval of this effect — which is the depth resolution of the fitting or, generally speaking the inversion — has been found to be linear $W = b \cdot h_m$, too (Fig. 6).

By regression we obtained different values of a and b parameters for *P*- and *SV*-waves

$$a_P = 6.04 \qquad b_P = 1.86 \text{ for } P\text{-waves}$$

and

$$a_S = 2.17 \qquad b_S = 0.91 \text{ for } S\text{-waves.}$$

The value of $a_S = 2.17$ is in good agreement with the ratio

$$r = \Lambda / z$$

widely used in the simplified inversion of ground roll dispersion [MATTHEWS et al. 1996].

In this '*rule of thumb' inversion the $V_S(z)$* profile is approximated from the measured dispersion data simply by

$$V_S(z) = 1.1\, c_m(\lambda_m = r\, z), \qquad (14)$$

where $\lambda_m = c_m/f_m$ and the value of r is between 2 and 4.

Using formulae (2), (4) and (14) the relation of parameters of $C(f)$ and $V_S(z)$ can be determined from the following equations:

$$V_S(z) = Az^{1/n} = 1.1\, c_1^{(1/(m+1))} r^{(m/(m+1))} z^{(m/(m+1))} \text{ (m/s)}$$

$$A = 1.1\, c_1^{(1/(m+1))} r^{(m/(m+1))} \text{ (1/s)}$$

and

$$n = (m+1)/m$$

i.e. the initial parameters of the $V_S(z)$ function for the fitting can be estimated from the measured dispersion data.

With the values of $c_1 = 552$, $m = 0.355$ (Fig. 3) and $r = 2.17$ we have

$$n_S = 3.81 \quad \text{and} \quad A = 143 \text{ /s}$$

in good agreement with the data of final fitting (Fig. 4)

$$n_S = 3.55 \quad A = V_{S1} = 150 \text{ /s}$$

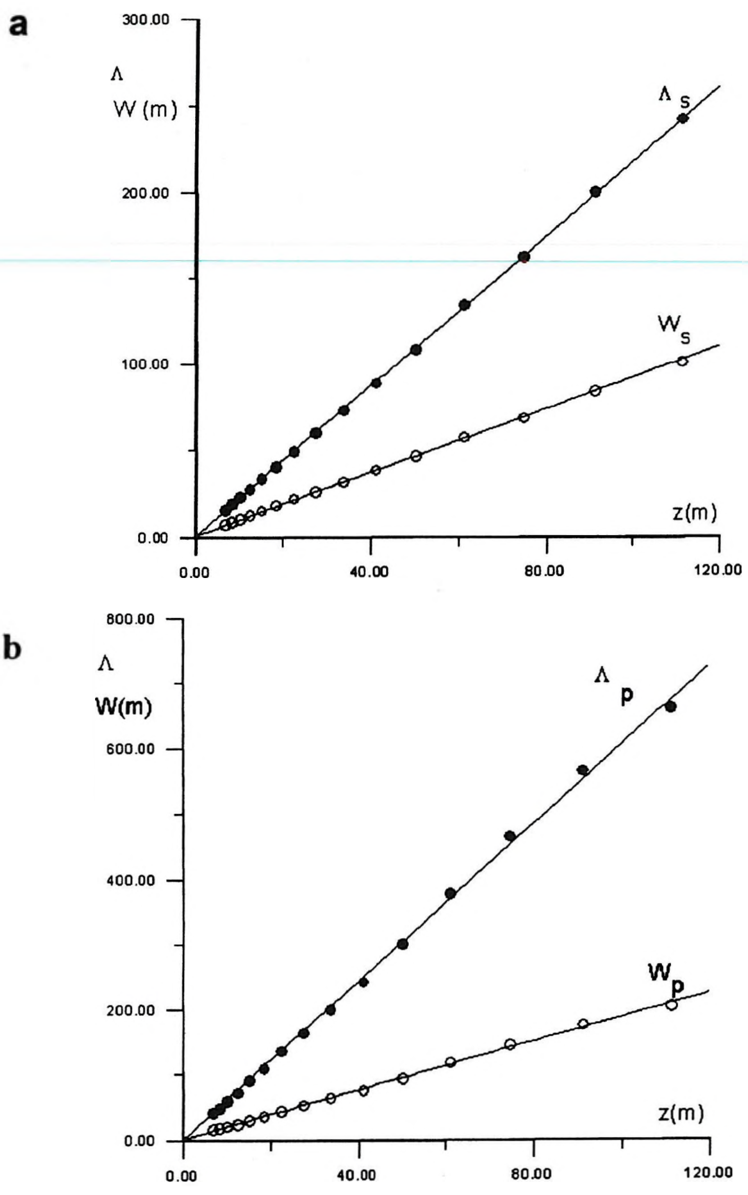

Fig. 6. Relation between the layer-depth and the perturbation parameters **a)** for *SV*-waves and **b)** for *P*-waves

6. ábra. A rétegmélység és a perturbációs paraméterek kapcsolata **a)** az *SV*-, **b)** a *P*-hullámra

8. Conclusions

Ground roll is a somewhat complicated and not very definite phenomenon in seismic prospecting. It differs from the direct or refracted first break arrivals because it has *a well defined apparent velocity change (diving wave) and dispersion* characteristics due to the very loose character of sediments. The approximation of depth dependent seismic parameters by simple analytical functions can be a useful tool in analysing ground roll dispersion caused by the inhomogeneous nature of loose sediments.

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A zavarhullámok diszperziójának analizise analitikai sebességfüggvények esetén

ÁDÁM Oszkár és HERMANN László

A szeizmikus felszníni zavarhullámok laza — főként lösz és harmadkori — üledékekben jönnek létre, amelyekben a *V(z)* vertikális sebességeloszlás analítikus függvényekkel is jól közelíthető. Ezek a képződmények többnyire a viszko-elasztikus kőzetmodell megjelenítőiként is felfoghatók, ha szeizmikus abszorpció és diszperzió jelensége is létezik. Ezek létezését kísérleti méréseink eredményeinek analízisével és modellezéssel kívánjuk igazolni.

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Oszkár Ádám for a photogprah and biography, see this issue, p. 18

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