GEOPHYSICAL TRANSACTIONS 2000
Vol. 43. No. 1. p. 64

Erratum

Emő MÁRTON and Péter MÁRTON 1999. Tectonic aspects of a palaeomagnetic study on the Neogene of the Mecsek Mountains. Geophysical Transactions **42,** 3 – 4, pp. 159 – 180

Despite the general lack of unweathered Neogene outcrops in the Mecsek Mts., renewed efforts enabled palaeomagnetic sampling to be carried out at 29 localities of which 15 gave, after laboratory treatment, useful palaeomagnetic directions for the Ottnangian through Upper Pannonian.

Our results fall into four groups: (i) this group comprises the ignimbrites aligned with the northern margin of the Neogene sedimentary trough which exhibit counterclockwise rotated declinations averaging 60°; (ii) the younger sediments of the same trough with no rotation; (iii) Tertiary localities from the main Palaeozoic–Mesozoic body of the Mecsek Mts, where the Cretaceous alkali basalts and related rocks are characterized by declinations rotated to the east, exhibit similar easterly declinations; (iv) the declination of two Upper Pannonian localities from the surroundings of the Mecsek Mts. do not deviate significantly from the present north.

These results together with those from the Apuseni Mts. and the South Carpathians on one hand, and from the Slavonian Mts. on the other — all parts of the Tisza megatectonic unit — strongly suggest that this megatectonic unit was not yet a rigid body during the Tertiary as some tectonic models suggest.

The editor regrets the printing (software) error in the paper by E. Márton and P. Márton in the previous issue leading to the omission of the minus (–) sign in Tables I. and II. The corrected tables are enclosed.

Table I. Palaeomagnetic results for the igneous sites from the Mecsek Mts.
Explanation of symbols: Left column: identifying numbers (cf. text). n/n_0: useful collected number of samples. D and I: palaeomagnetic declination ($D°$) and inclination ($I°$) before tilt correction. k and $\alpha°_{95}$: statistical parameters after FISHER [1953]. k: precision, $\alpha°_{95}$: semi-angle of cone of confidence at the 95% confidence level. $D°_c$, $I°_c$: palaeomagnetic declination ($D°_c$) and inclination ($I°_c$) after tilt correction. *Dip*: bedding attitude, azimuth/magnitude of dip of bedding plane. *Remark*: *a*: result obtained by linearity analysis, *b*: result obtained from stable end points. Three results are shown for the Komló andesite (8). The first is from MÁRTON and MÁRTON [1969], the second is from MÁRTON [1986] and the third is from the present study. The last row shows an earlier result for an ignimbrite body from the Mid-Hungarian Zone [MÁRTON and MÁRTON 1989].

I. táblázat. A mecseki magmás kőzetek paleomágneses irányai.
Jelmagyarázat: bal oldali oszlop: azonosító számok (lásd szöveg), n/n_0: hasznos/gyűjtött minták száma, $D°$és $I°$: paeleomágneses deklináció és inklináció dőléskorrekció előtt; k és $\alpha°_{95}$: FISHER-féle [1953] statisztikai paraméterek; k: pontosság, $\alpha°_{95}$: a 95%-os megbizhatósági szintű konfidencia szög fele; $D°_c$, $I°_c$: paleomágneses deklináció és inklináció a dőléskorrekció után; *dip*: a tektonikai dőlés azimutja/a dőlés nagysága; *Remark*: *a*: linearitás analízissel kapott eredmény, *b:* stabil végpontokból kapott eredmény

⇨

	locality	n/no	$D°$	$I°$	k	$\alpha°_{95}$	$D°_C$	$I°_C$	k	$\alpha°_{95}$	dip	Remark
	Mecsek Mts. igneous rocks											
	Komló andesite* 1	10	82	+62	112	4						a
	Komló andesite* 2	5	71	+64	179	6						a
	Komló andesite* 3 7702–713	12/12	56	+59	164	3						a
8	Komló andesite mean	3	69	+62	143	10						
22	Kisbesztece ign. 7299–314	11/16	7	–67	67	6	105	–52	67	6	315/49	a
4	Horváthertelend ign. 7663–679	16/17	124	–53	33	7	84	–33	33	7	(220/40)	a
6	Vörösvölgy ign.	8/10	139	–33	15	15	99	–80	15	15	146/50	a
3	Balinca ign. 7417–425	9/9	67	–61	491	2	111	–59	491	2	351/26	a
7	Máza-Szászvár ign 6372–381	10/10	118	–57	35	8	118	–57	35	8	horizontal	a
	Mecsek ignimbrites overall mean	5	108	–62	8	28	113	–60	45	12		
	Mid-Hungarian Zone											
	Sárszentmiklós ignimbrite*	12/–	315	+55	47	7						

	locality	n/n_0	$D°$	$I°$	k	$\alpha°_{95}$	$D°_C$	$I°_C$	k	$\alpha°_{95}$	dip	Remark
9	Feked 7973–983	6/11	59	+55	22	17	62	+6	22	17	220/6	b
13	Mecsekjánosi 7602–611	5/10	191	+11	23	16	5	+79	23	11	10/89	a
12	Magyaregregy 7591-601, 7880–891	15/23	59	+74	120	3	355	+47	120	3	330/40	a
14	Orfű 7345–362	16/18	36	+58	43	6	19	+57	43	6	304/11	a
18	Komló-Kökönyös 7330–41	7/12	357	+53	126	5	350	+54	126	5	252/5	b
16	Husztót 7426–37	7/12	355	+63	25	12	4	+68	36	7	82/5 168/14	a
24	Danitzpuszta 7573–590	9/18	90	+60	19	12	118	+25	60	19	145/35	b
27	Bátaszék 7950–972	10/11	187	–61	136	4	201	–60	136	4	100/8	a
28	Kakasd 7892–901	5/10	193	–41	75	9	201	–46	75	9	135/10	a
	13, 12, 14, 18, 16 overall mean	5/5	24	+79	3	50	1	+61	34	13		

Table II. Palaeomagnetic results for sedimentary localities from the Mecsek Mts. *Explanation of symbols*: as for Table I.
II. táblázat. A mecseki üledékes kőzetek paleomágneses irányai. *Jelmagyarázat:* l. I táblázat alatt.

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GEOPHYSICAL TRANSACTIONS

EÖTVÖS LORÁND GEOPHYSICAL INSTITUTE OF HUNGARY

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VOL. 43. NO. 1 MARCH 2000. (ISS N0016-7177)

TARTALOMJEGYZÉK

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Analysis of ground roll measurements based on a WKB solution of motion

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During the past 20–30 years seismic ground roll lost a great deal of their significance because of the extensive use of geophone arrays, common depth points, etc. observation systems. Although with the help of these systems seismograms appear to be a great deal better the distortion effect of wave-guide generated ground roll has not eliminated from their frequency characteristics. This is connected to the viscoelastic phenomenon by velocity dispersion as well as absorption of *P* and *S* body waves. In this paper we deal only with velocity dispersion.

Keywords: ground roll, WKB solution, seismograms, P-waves, S-waves, velocity

1. Introduction

Ground roll — as is well known — was so-named because of its connection with reflection measurements. Because of its large amplitude ground roll can suppress the useful signals almost completely unless this disturbance is removed from seismograms. Detailed investigations relating to ground roll are contained in ÁDÁM's papers [1968, 1969] in which it was stated that the phenomenon can be considered as a mode of guided waves, due to the character of sedimentation of the near surface (mainly loess-like) layers with vertically increasing *P* and *SV* velocity distributions that can be considered continuous functions of the vertical co-ordinate. DOBRÓKA's [1987, 1988] and FANCSIK's [1995, 1997] publications have proved that the WKB approach can be applied for to describe the wave guide in an inhomogeneous medium. For this reason it seems to be useful to consider whether certain propagation characteristics of ground roll are explainable on the basis of the WKB solution of the motion equation valid in a vertically inhomogeneous medium. We shall examine this question in detail on the basis of experiments performed in the vicinity of Nagytilaj village (Zala County, Hungary) on the area of a loess plateau and compare the results with a modelling method described here. Acknowledge-

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Manuscript received: 20 May, 1998.

ment is made to the Hungarian Scientific Research Fund (OTKA) grant No. 0155850.

2. Wave guide in vertically inhomogeneous medium

The ground roll mechanism is discussed on the basis of the full form in an inhomogeneous medium of the following motion equation:

$$\rho\frac{\partial^2\vec{s}}{\partial t^2}=\mu\Delta\vec{s}+(\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\vec{s}+2(\mathrm{grad}\,\mu,\mathrm{grad})\vec{s}+ \\ +\mathrm{grad}\,\mu\times\mathrm{rot}\,\vec{s}+\mathrm{grad}\,\lambda\,\mathrm{div}\,\vec{s} \qquad (1)$$

where λ and μ are Lamé's constants, ρ is the density.

In the case of two-dimensional wave propagation, by means of the displacement potentials ($\vec{s}=\nabla\varphi+\mathrm{rot}\,\vec{\psi}$, φ is the scalar potential; $\vec{\psi}=\{0,\psi,0\}$ is the vector potential) this relation is reducible to easily solvable form by the WKB method [FANCSIK 1995, 1997]:

$$\begin{aligned}\Delta\varphi+k_\alpha^2\varphi&=-\frac{2}{\xi}\frac{\mathrm{d}\mu}{\mathrm{d}z}\frac{\partial\psi}{\partial x}\\ \Delta\psi+k_\beta^2\psi&=\frac{2}{\mu}\frac{\mathrm{d}\mu}{\mathrm{d}z}\frac{\partial\varphi}{\partial x}\end{aligned} \qquad (2)$$

(k: wave number; $\xi=\lambda+2\mu$). The wave propagation was supposed in the (x, z) plane (then only the x and z direction components of the displacement vector exist), and the density of the medium was considered constant. The equations are valid only for fulfilling the following inequalities:

$$\begin{aligned}&|\mathrm{grad}\,\varphi|\gg\left|\frac{2}{\rho\omega^2}\{(\mathrm{grad}\,\varphi,\mathrm{grad})\mathrm{grad}\,\mu-\mathrm{grad}\,\varphi\,\Delta\mu\}\right|\\ &|\mathrm{rot}\,\vec{\psi}|\gg\left|\frac{2}{\rho\omega^2}(\mathrm{rot}\,\vec{\psi},\,\mathrm{grad})\mathrm{grad}\,\mu\right|\end{aligned} \qquad (3)$$

where ω is the angular frequency.

The λ, μ Lamé coefficients depend only on the z co-ordinate and in the case of two dimensions we can disregard the derivative from the y direction. Equations $k_\alpha=\omega/\alpha(z)$ and $k_\beta=\omega/\beta(z)$ are the wave numbers of body waves (where $\alpha(z)$, $\beta(z)$ are respectively the velocities of the longitudinal and the transversal body wave). The displacement components of these waves fall in the (x, z) plane too. According to the inhomogeneities of layer series the relations of equations (2) are valid, the P- and SV-waves became coupled waves. Naturally the horizontally polarized transversal (SH) wave appears too, but from our present point of view it is not of interest.

The conditions of inequalities given by equations (3) are trivially fulfilled if dependence of the Lamé coefficients is assumed to be linear in space, that is $\mu=\mu_0(1+az)$, $\lambda=\lambda_0(1+az)$, where μ_0, λ_0 are values of the Lamé coefficients in the $z=0$ plane, and a is the parameter determining the rate of inhomogeneity (namely equations (3) contain only second derivatives). In this case the velocities of waves in the medium will vary according to the 1/2 power of the z depth and the gradient of $V(z)$ is continuously decreasing. In this way we shall get such a velocity distribution which has a decreasing gradient with depth and will approach the often experienced velocity courses caused by the compression within the near-to-surface geological structures [ÁDÁM 1968].

On the basis of the WKB solutions of equations (2) the components of the displacement vector in a vertically inhomogeneous medium will be as follows [FANCSIK 1997]:

$$u_x=\left(\mathrm{i}ka(z)A\mathrm{e}^{-\int^{z}p(y)\mathrm{d}y}+qb(z)C\mathrm{e}^{-\int^{z}q(y)\mathrm{d}y}+\mathrm{i}ka(z)B\mathrm{e}^{\int^{z}p(y)\mathrm{d}y}-qb(z)D\mathrm{e}^{\int^{z}q(y)\mathrm{d}y}\right)\mathrm{e}^{\mathrm{i}kx}$$

$$u_z=\left(-pa(z)A\mathrm{e}^{-\int^{z}p(y)\mathrm{d}y}+\mathrm{i}kb(z)C\mathrm{e}^{-\int^{z}q(y)\mathrm{d}y}+pa(z)B\mathrm{e}^{\int^{z}p(y)\mathrm{d}y}+\mathrm{i}kb(z)D\mathrm{e}^{\int^{z}q(y)\mathrm{d}y}\right)\mathrm{e}^{\mathrm{i}kx} \tag{4}$$

where

$$p=\sqrt{k^2-k_\alpha^2},\quad q=\sqrt{k^2-k_\beta^2},\quad a(z)=\sqrt{p_0/p},\quad b(z)=\sqrt{q_0/q},$$

k is the wave number to the x direction and we disregarded from the above-mentioned coupling effect; A, B, C, D are arbitrary coefficients. The members with the zero subscript relate to one of the fixed points of the layer, e.g. to the $z=0$ point of co-ordinates. The fulfilment of these equalities according to the WKB solution implies new conditions; with the notations

$$P(z,f)=\left|\frac{1}{p^{5/2}}\left(\frac{3}{4}\left(\frac{1}{p}\frac{\mathrm{d}p}{\mathrm{d}z}\right)^2-\frac{1}{2}\frac{1}{p}\frac{\mathrm{d}^2p}{\mathrm{d}z^2}\right)\right|$$

and

$$Q(z,f)=\left|\frac{1}{q^{5/2}}\left(\frac{3}{4}\left(\frac{1}{q}\frac{\mathrm{d}q}{\mathrm{d}z}\right)^2-\frac{1}{2}\frac{1}{q}\frac{\mathrm{d}^2q}{\mathrm{d}z^2}\right)\right|$$

They will take the following shape

$$\begin{aligned} P(z,f) &<< 1 \\ Q(z,f) &<< 1 \end{aligned} \tag{5}$$

On interpreting equations (4) one can see that in a vertically inhomogeneous medium, such P- and/or SV-waves can develop which, for example, starting from a free surface will return to the free surface in case of certain incidence angles. These are *diving-waves,* that characterize loose sediments. *Figure 1* models an often occurring near surface geological situation from the point of view of a wave-guide. In the figure a vertically inhomogeneous, low velocity layer lies on a rock body that can be considered as an infinite half space in which the longitudinal and the vertical body wave velocities, depending on the given location and density, are constant. Consider H the thickness of this layer. This layer having lower SH-wave velocity (by chance P-wave velocity) compared with the half space is the wave guide channel in which — if the channel was homogeneous — a guided wave built of reflected waves from

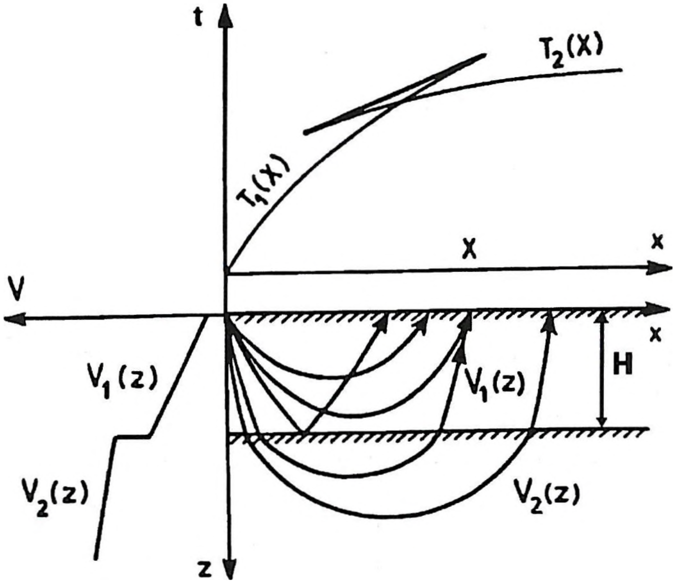

Fig. 1. Theoretical ray combination in the case of diving waves. Two layers with different *V(z)* characteristics

1. ábra. Elméleti sugárkombináció bemerülő hullámok esetén. Két réteg eltérő *V(z)* karakterisztikákkal the surface and the basement would propagate parallel with the surface. According to the above interpretation, but for an inhomogeneous case, besides the reflected waves such waves (*P*- and/or *SV*-waves) also appear in the upper layers that will not reach the surface of the lower half space and in this way they return to the free surface. Thus the wave guide developing in the inhomogeneous medium differs from the wave guide developing in a homogeneous, two-layer case when it is not necessary to take into account the wave propagation as a diving wave or the frequent reflections (on the surface) of these waves.

In order to examine the wave-guide in an inhomogeneous medium, the two-layer model of Fig. 1 should also be taken into account in the following. In the layer of finite thickness, displacements given by equations (2) are valid. Moreover, consider the lower half space as an inhomogeneous one in which the following functions will give the displacement components (marking the par-ameters relating to the second layer with a prime):

$$\begin{aligned} u'_x &= \left(\mathrm{i}kE\,\mathrm{e}^{-p'z} + q'F\,\mathrm{e}^{-q'z}\right)\mathrm{e}^{\mathrm{i}kx} \\ u'_z &= \left(-p'E\,\mathrm{e}^{-p'z} + \mathrm{i}kF\,\mathrm{e}^{-q'z}\right)\mathrm{e}^{\mathrm{i}kx} \end{aligned} \tag{6}$$

E, F are arbitrary constants.

In this layer only the *P-SV*-waves travelling outward in the direction of the z-axis were taken into account at the resolution (which is given on the basis of the regularity condition). The functions

$$p' = \sqrt{k^2 - k_\alpha'^2}, \quad q' = \sqrt{k^2 - k_\beta'^2}$$

are direction cosines. In a layered medium the waves given by displacement components (2) and (6) should fulfil the boundary conditions:

- — the normal components of the stress will disappear on the surface;
- — at depth *H*, on the boundary of the two layers the appropriate displacement components (of x and z direction) and normal stress are continuous;
- — in the lower half space in case of $z \to \infty$ the regular solution of wave equation can be get.

If we write these conditions for the integration constants *A, B, C, D, E, F* as unknown coefficients we shall get a homogeneous, linear equation system. On the basis of this system of equations we arrive to the dispersion relation of *P-SV*-waves propagating in a vertically inhomogeneous medium as follows:

$$\det\left(\underline{\underline{MN}}\right) = 0 \tag{7}$$

where

$$\underline{\underline{M}} = \begin{pmatrix} \mathrm{i}ka & qb & \mathrm{i}ka & -qb & +\mathrm{i}k & -q' \\ -pa & \mathrm{i}kb & pa & \mathrm{i}kb & p' & +\mathrm{i}k \\ -2\mathrm{i}kp\mu a & -\mu\left(k^2+q^2\right)b & -2\mathrm{i}kp\mu a & -\mu\left(k^2+q^2\right)b & 2\mathrm{i}kp'\mu' & -\mu'\left(k^2+q'^2\right) \\ \left(p^2\xi-k^2\lambda\right)a & -2\mathrm{i}kq\mu b & \left(p^2\xi-k^2\lambda\right)a & 2\mathrm{i}kp\mu b & +\left(p'^2\xi'-k^2\lambda'\right) & 2\mathrm{i}kq'\mu' \\ -2\mathrm{i}kp\mu e & -\mu\left(k^2+q^2\right)/f_s & -2\mathrm{i}kp\mu/e & -\mu\left(k^2+q^2\right)/f_s & 0 & 0 \\ \left(p^2\xi-k^2\lambda\right)e & -2\mathrm{i}kq\mu bf_s & \left(p^2\xi-k^2\lambda\right)/e & 2\mathrm{i}kq\mu/f_s & 0 & 0 \end{pmatrix}$$

$$\underline{\underline{N}} = \mathrm{diag}\ 1/e \quad 1/f_s \quad e \quad f_s \quad g \quad h)$$

$$e=\exp\left(\int_0^H p(y)\,\mathrm{d}y\right), \quad f_s=\exp\left(\int_0^H q(y)\,\mathrm{d}y\right), \quad g=\exp(+p'H), \quad h=\exp(+q'H),$$

$$\xi=\lambda+2\mu, \quad \xi'=\lambda'+2\mu'.$$

The values of a and b are of the WKB amplitude correction at depth H given in connection with equation (4).

On analysing the results of field registration by means of equation (7) we investigate the conformity of phase velocities supplied by the field registrations and determined from modelling.

3. Comparison of results of ground roll measurements and results of modelling

The experimental ground roll measurements were carried out in the above mentioned region near to Nagytilaj, where the surface waves were generated by vertical force (by shot) on the surface; the observation was by means of geophones of horizontal and vertical orientation and linear phase digital system-equipment corresponding to the direction of the seismic line. *Figure 2* shows the seismogram where different wave groups can be distinguished. In order to determine the dispersion relations of the waves *f–k* analyses have been performed the results of which can be found in *Fig. 3*. The c_1, c_2 and c_3 phase velocity courses which can be determined on the basis of the curves marked f_1, f_2 and f_3 are indicated respectively by empty squares, empty triangles and empty circles in *Fig. 4*. We represented the *f–k* curves and phase velocities of those waves which support the largest ratio of the energy propagating in the wave-guide.

In order to apply dispersion relation (7), a geological model of the wave-guide will be required. In the experimental measurement for clarifying the geological relations and the physical parameters, velocity and density profil-

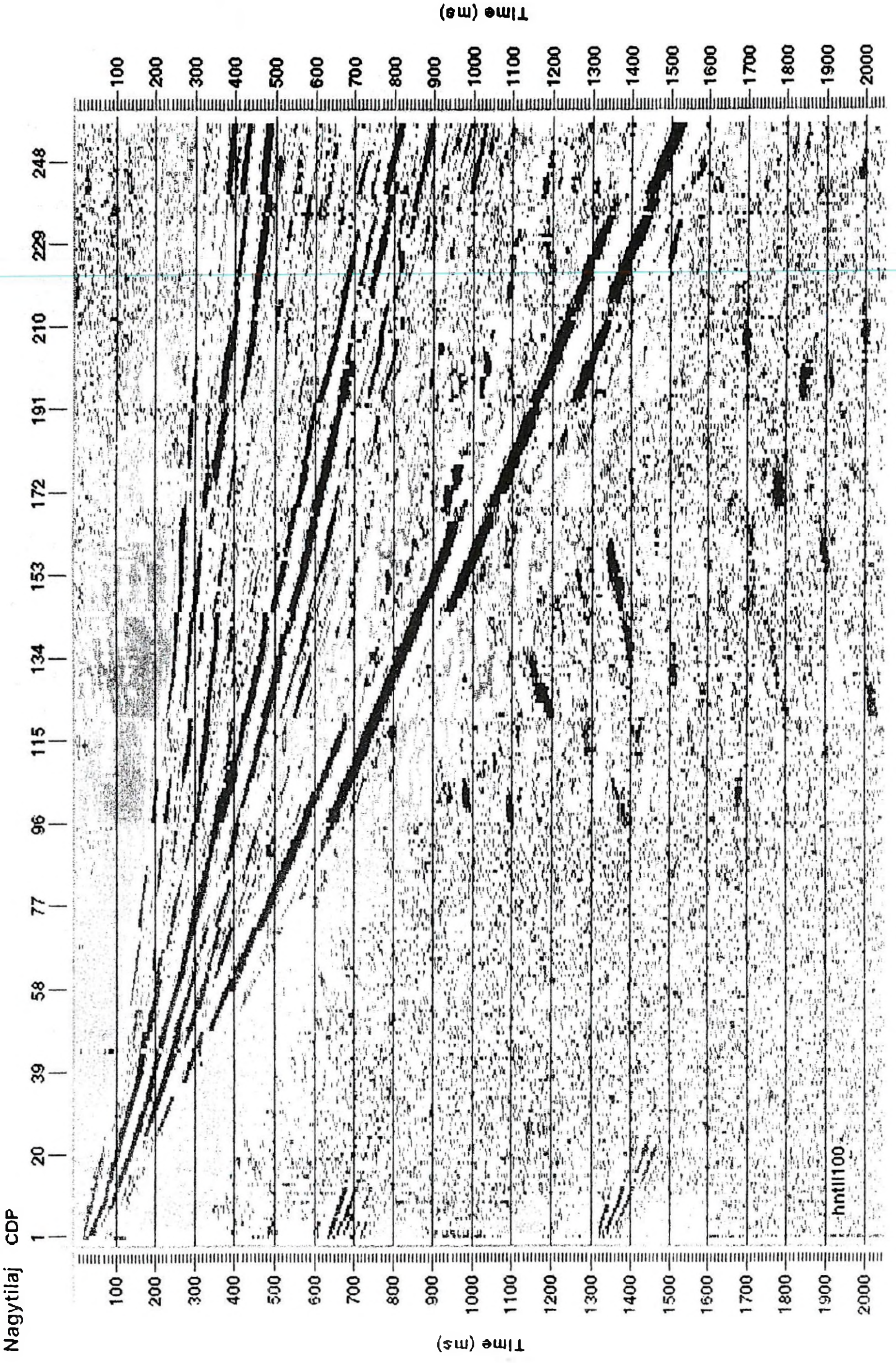

Fig. 2. Seismogram from Nagytilaj area observed with 254 vertical geophones, Δx=1m; L=254 m

2. ábra. Szeizmogram Nagytilajból. 254 geofon, Δx=1m, L=254 m

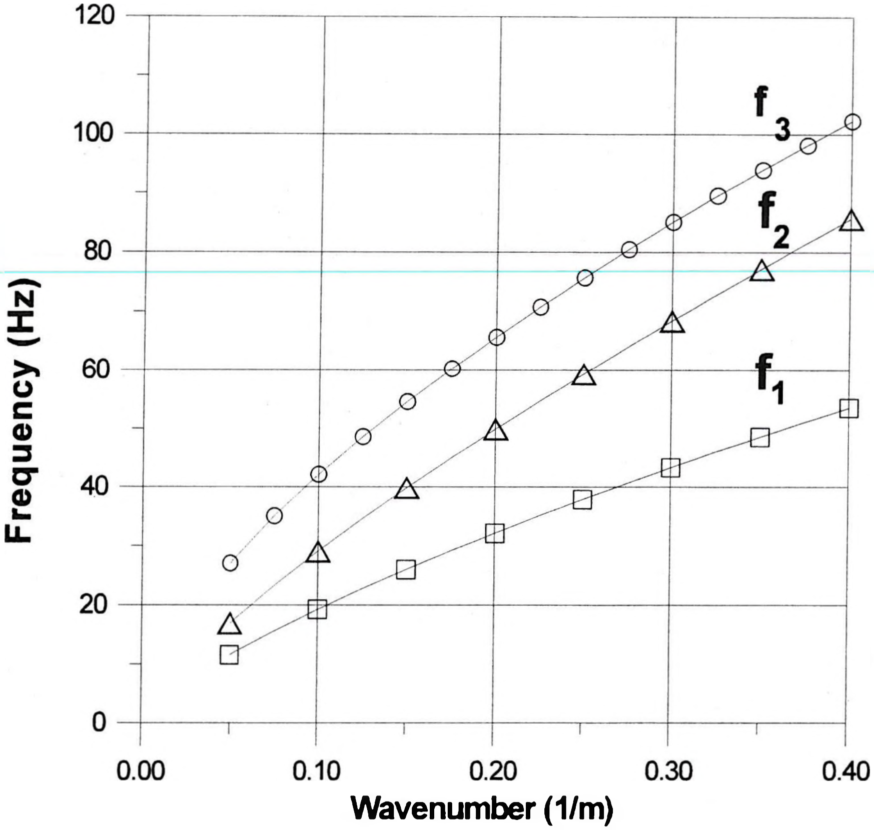

Fig. 3. Results of *f–k* analysis for the different time–distance curves
3. ábra. Az *f–k* analízis eredményei különböző idő–távolság görbékre

ing were performed as a means of determining the distribution by depth of the *P*- and *S*-waves as well as the density. In addition, data from the first break registrations were processed.

Assessment of the results of seismograms indicated that the thickness of the layers is approximately 40 m, meaning a significant *P*- and *S*-wave velocity gradient. Below this thickness, from the aspect of propagation of the longitudinal wave there is however a consolidated layer. The transversal wave velocity in this layer can be considered as a location-dependent one. The parameters of the model of the wave-guide are given in *Table I*.

V_P (m/s)	V_S (m/s)	Density (kg/m^3)	Thickness of layer (m)
$228\sqrt{1+0.3z}$	$115\sqrt{1+0.3z}$	2000	40
1900	1100	2300	–

Table I. Model of wave-guide formed on the basis of the field measurements. V_P means the longitudinal body wave velocity, V_S the transversal body wave velocity. The parameters of the upper layer are shown, and the bottom line gives the values of the half space considered to be consolidated

I. táblázat. A terepi mérések alapján kialkított hullámvezető modell. V_P a longitudinális, V_S a transzverzális térhullám sebessége. A második sor a felső réteg paramétereit, az alsó sor a konszolidáltnak tekintett féltér értékeit adja

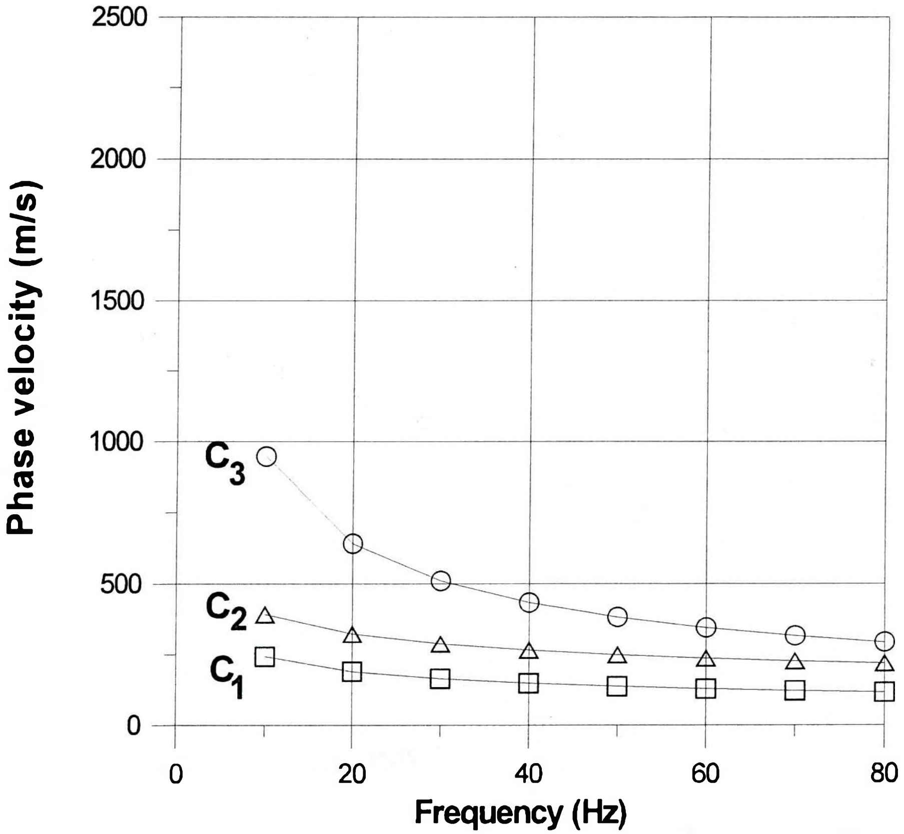

Fig. 4. Dispersion curves computed from curves of Fig. 3

4. ábra. A 3. ábra görbéiből számított diszperziós görbék

This form of $V = V_0\sqrt{1+az}$ is the velocity approximation determined from the field measurements (the validity of the WKB approximation will be proved later). In *Fig. 5* the velocity–depth curve supplied by this function for transversal body wave velocity is presented. (In the figure we also presented the $V=Az^{1/n}$ form velocity equation that proved good in practice [ÁDÁM 1968] and that was applied in this region too and the parameters of which were A=113/s, n=2.82 on the basis of observation). In the lower half space the transversal wave velocity was regarded as constant on the one hand because the variability of V_S is lower in this range than that experienced in the upper layer

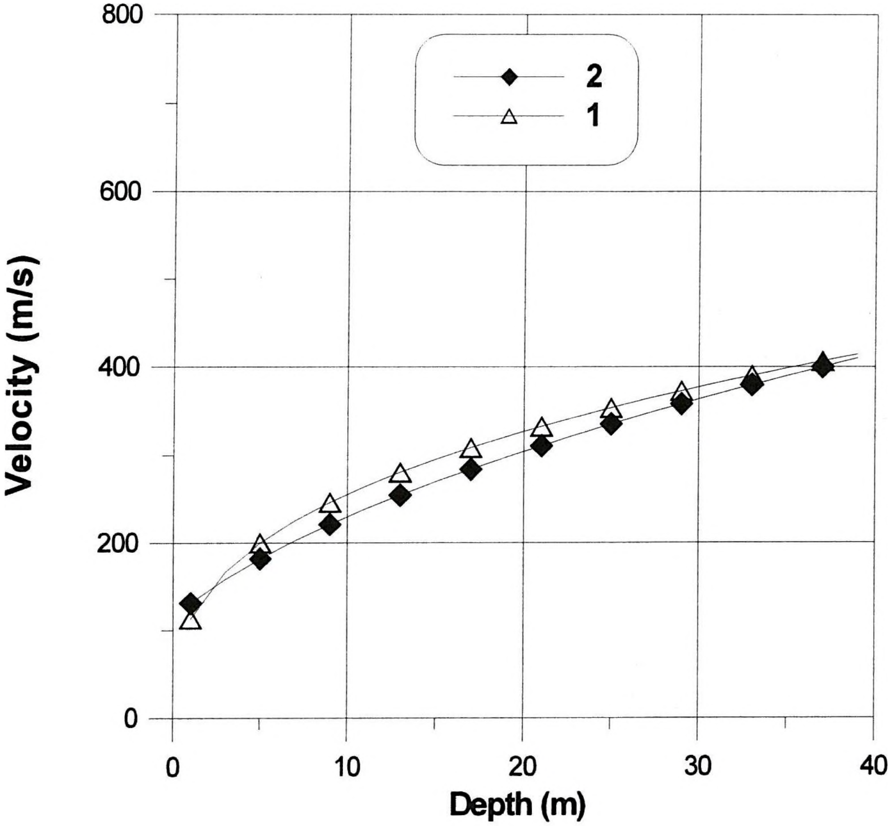

Fig. 5. V(z) velocity–depth curves. *Curve 1*: $V = V_0\sqrt{1+az}$ simplified from well velocity survey ; *curve 2*: computed from surface measurement $V=Az^{1/n}$

5. ábra. V(z) sebesség–mélység diagram.

1. görbe: fúrólyukban mért sebességfüggvény egyszerűsített formája $V = V_0\sqrt{1+az}$; *2. görbe*: felszíni mérésekből számított görbe $V=Az^{1/n}$

and on the other hand the wave guide connects to the near-to-surface layer that shows more significant inhomogeneity [ÁDÁM 1987], namely the propagation characteristics of the guided wave connecting to the near-to-surface are influenced into a smaller extent by the velocity relations of the deeper ranges.

On the basis of the model given above the dispersion curves plotted by using dispersion relation (7) can be found in *Fig. 6* (curves marked with filled circles, triangles and squares) where also the dispersion curves derived from the ground data shown in Fig. 2 are indicated (with empty circles, triangles and squares). It can be seen that the curves from the measurements and the modelling lie together essentially in the examined frequency range. A difference can be observed mostly for the middle velocity branch. The degree of fit relating to a pair of curves can be characterized numerically too, for example

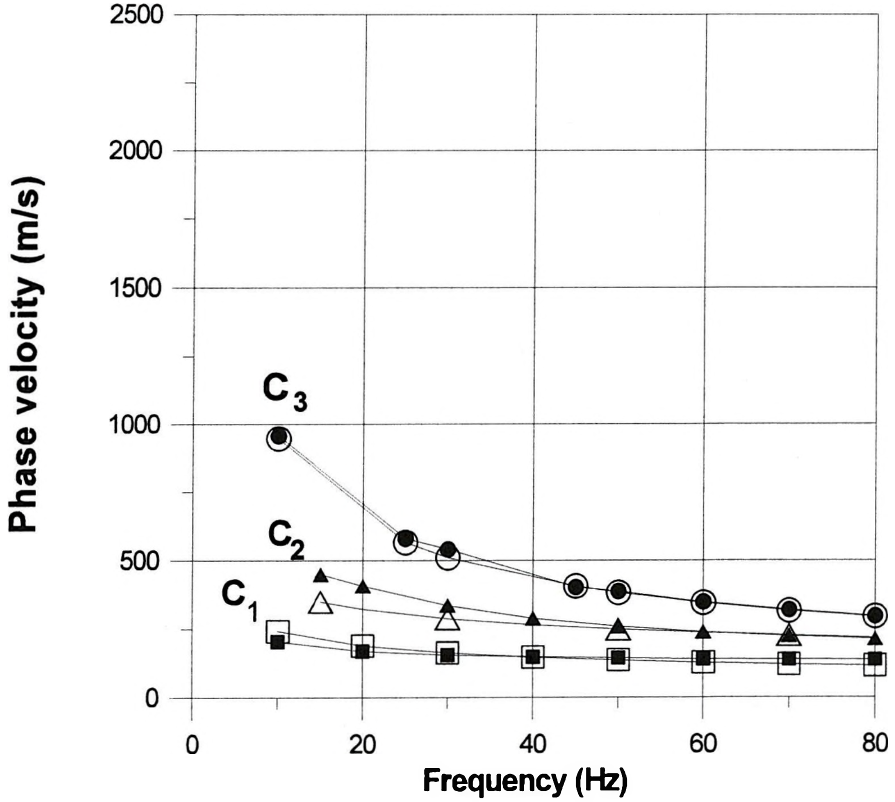

Fig. 6. Measured and computed phase velocities
6. ábra. Mért és számított fázissebességek

by means of the relative errors of the two curves according to the following formula:

$$\varepsilon_a = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{c_i^{(m)} - c_i^{(sz)}}{c_i^{m}}\right)^2} \quad , \tag{8}$$

where ε_a is the 'standard deviation' of relative errors of the plotted $c_i^{(m)}$ and calculated $c_i^{(sz)}$ phase velocities for the same N frequencies as the parameter characterizing the average fit. Using this formula the degree of fit for the single curves is as follows (*Table II*):

Velocity curve	ε_S(%)
c_3	2.4
c_2	15
c_1	11

Table II. The degree of fit of the phase velocities plotted on the basis of velocity curves marked c_1, c_2, and c_3 presented in Fig. 4 as well as the dispersion relation formula (7) and the parameters of Table I according to formula (8)

II. táblázat. A 4. ábrán megadott c_1, c_2 és c_3 jelű sebsséggörbék, valmint a diszperziós egyenlet (7) és az I. táblázat paraméterei alapján számított fázissebességek illeszkedésének mértéke a (8) egyenletnek megfelelően

Bearing in mind the measuring errors as well as the accuracy of the two-layer model (that simplifies the reality) formed on the basis of the above statements, the results of the modelling can be considered satisfactory.

The results of the WKB modelling could be accepted if the conditions of the WKB approximation are fulfilled in the frequency range examined. Since the validity of inequalities (3) is trivial due to the above selection of shape of the velocity functions only the validity of the pair of inequalities (5) should be examined.

Regarding the above requirement by means of the results in *Figures 7* and *8* we would like to demonstrate the response of $P(z, f)$ and $Q(z, f)$ as functions of frequency and of depth relating to unity (marking unity with a thick line in the figures) in the case of velocity curve c_1 (marked with squares in Figure 4) at three different depths. On the basis of the figures it is evident that the results of the WKB modelling are valid for higher frequencies than 20 Hz. It is mentioned that the consequence will also be the same for the further two phase velocity functions.

With regard to the ground roll registrations measured in the Nagytilaj region: on the basis of the results obtained from dispersion equation (7) we can state that ground roll can be considered as a higher mode of *P-SV* guided waves. The lowest velocity mode (marked with circles in Figures 3 and 4) is the first overtone following the Rayleigh type wave propagating in the medium. (In this case, by Rayleigh type wave we mean the dispersion fundamental harmonic which at low frequency approaches the Rayleigh wave velocity of the half space considered consolidated and by increasing the frequency it approximates the Rayleigh wave velocity of the layer of finite thickness.) On the basis of the velocities from dispersion analysis and the displacement functions given by (4) it can also be stated that the displacement components of the ground roll are built of displacement components of *SV*-waves propagating as a diving one and as of inhomogeneous *P*-waves. The neglecting of coupling did not lead to significant errors.

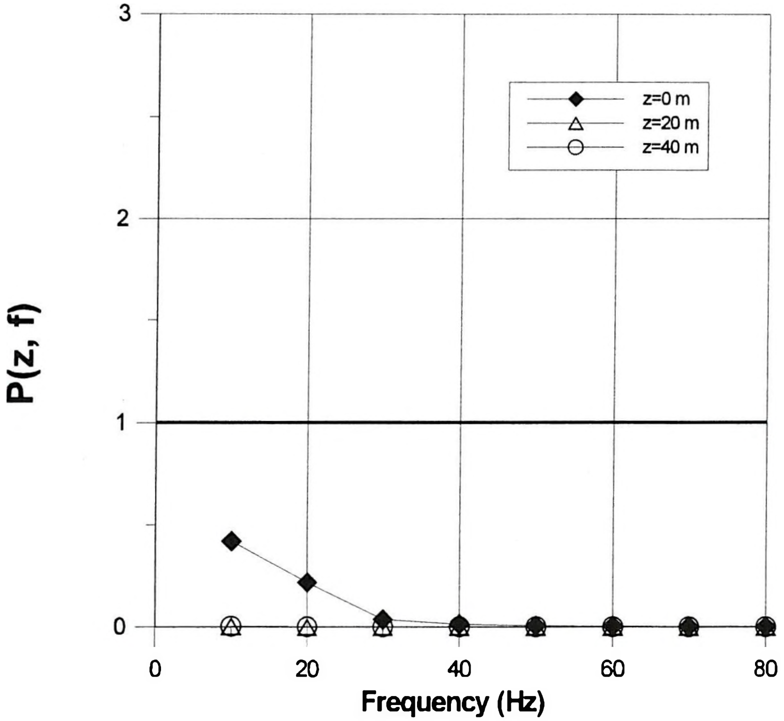

Fig. 7. Value of *P (z, f)* as a measure of the approximation
7. ábra. P (z, f) értéke mint a közelités mértéke

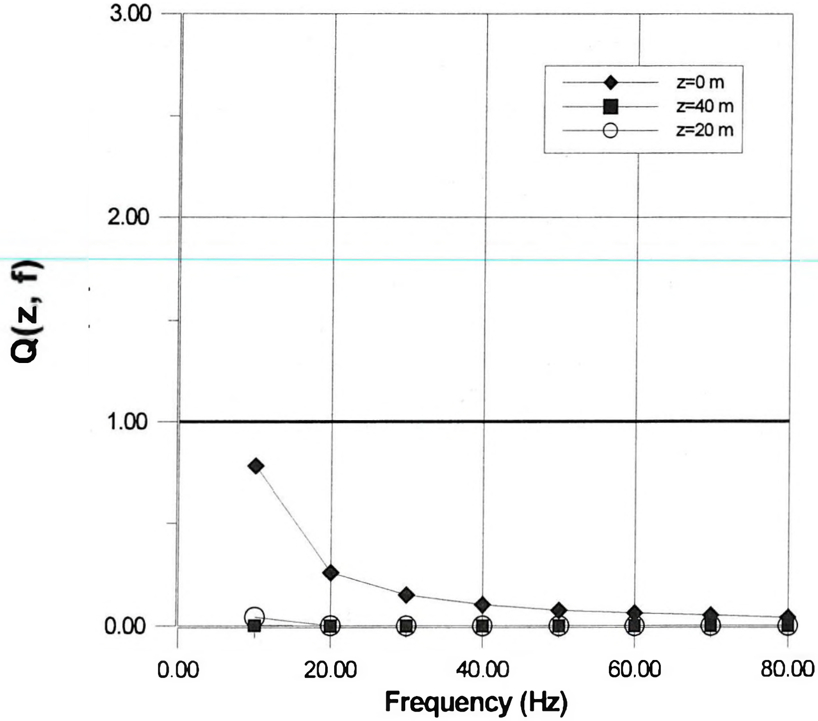

Fig. 8. Value of $Q\ (z, f)$ as a measure of the approximation
8. ábra. $Q(z, f)$ értéke, mint a közelítés mértéke

4. Summary

On the basis of the WKB approximation of the motion equation in an inhomogeneous medium we gave the dispersion relation of *P-SV*-waves propagating in such a two-layer, near-to-surface structure where the layer of finite thickness situated on a half space considered homogeneous was considered vertically inhomogeneous. Using these results we derived the phase velocities of the *P-SV*- waves relating to that wave guide model whose determination occurred on the basis of seismic and well-log measurements. Comparing our curves with the phase velocity curves — derived from dispersion analysis — of ground roll measurements performed in the above-mentioned region we are able to state appropriate coincidence in the examined frequency range. However this frequency range was determined by the terms relating to the validity of the WKB approximation. From the results the following consequence was drawn. In the given area, surface ground roll is the higher mode of such *P-SV* guided waves whose displacement components are built of displacement components of *SV*-waves propagating as immersion waves as well as of inhomogeneous *P*-waves.

The above presented WKB modelling of the dispersion relation of ground roll as a *direct solution* gives the possibility that in future the examination of seismic parameters of near-to-surface structures should be performed by the inversion technique based on this measuring method, a method that may prove to be of great importance in resolving various environmental, geotechnical or geophysical engineering tasks.

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Felszíni zavarhullám mérések analízise a mozgásegyenlet WKB megoldása alapján

FANCSIK Tamás és ÁDÁM Oszkár

Az elmúlt 20–30 évben a szeizmikus zavarhullám (seismic ground roll) jelensége annyiban veszített jelentőségéből, amennyiben a hosszú geofon csoportok és nagy fedésszámú észlelési rendszerek segítségével nagyrészt elnyomták. Bár az ezekkel az észlelési rendszerekkel felvett időszelvényeket kevésbé terhelik a hullámvezetőkben keletkező zavarhullámok, de hatásuk a frekvencia karakterisztikákban továbbra is él. Ehhez kapcsolódnak a viszko-elasztikus kőzetmodellre jellemző abszorpció és diszperzió, amelyek mind a *P*, mind az *S* hullámot terhelik. Ebben a dolgozatban csak a *sebesség diszperzióval* foglalkozunk.

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