GEOPHYSICAL TRANSACTIONS 1994
Vol. 39. No. 1. pp. 35-44

LABORATORY METHOD FOR DETERMINING THE COMPLEX DIELECTRIC PERMITTIVITY OF LOOSE ROCKS (STANDING WAVE METHOD)

Pjotr FIJAS*, Bernhard FORKMANN*, Ivo RAPPSILBER**

To determine the dielectric permittivity of loose rocks a laboratory method employing the standing wave method was tested. This procedure is based on the impedance transformation equation for a partial medium-filled coaxial line [von HIPPEL 1954]. By measuring the complex dielectric permittivity of different loose rock samples, it was possible to establish a systematic trend in relation to frequency, water content and the structure of the sediments. The measurements were realized in the frequency range of ground-penetrating radar from 60 to 1000 MHz. If the dielectric permittivity is converted into characteristic propagation parameters, it is possible to describe the propagation relation of high-frequency electromagnetic waves in certain sediments.

Keywords: complex dielectric permittivity, dispersion, ground-penetrating radar, standing wave method

1. Theoretical foundations

The boundary surfaces between two different media (conductor/nonconductor) can be used to transmit electromagnetic waves. The coaxial line is one of the significant types of such wave conductors. From the geometrical condition for transmitting waves, the distance between the conductors being an integer multiple of the half vacuum wavelength projected to the normal of the conductor surface, follows for the loss-free wave propagation in an air-filled wave guide [BADEN FULLER 1974]:

---

* TU Bergakademie Freiberg, Institut für Geophysik, D-09599, Freiberg
** Geologisches Landesamt Sachsen-Anhalt, D-06118, Halle
Manuscript received: 14 Februray, 1994

$$\lambda_H = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}} \tag{1}$$

λ_H—waveguide wavelength,
λ_0—vacuum wavelength,
λ_c—limiting wavelength (TEM-wave).

If a dielectric $\varepsilon_r \neq 1$ is put into the waveguide, Eq. (1) is valid in a modified form:

$$\lambda_{H\varepsilon} = \frac{\lambda_\varepsilon}{\sqrt{1 - \left(\frac{\lambda_\varepsilon}{\lambda_c}\right)^2}} \tag{2}$$

λ_ε—wavelength in the medium,
$\lambda_{H\varepsilon}$—waveguide wavelength in the medium.

With

$$\lambda_\varepsilon = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

the following equation results for the dielectric permittivity [SCHILD 1981]:

$$\varepsilon_r = \left(\frac{\lambda_0}{\lambda_{H\varepsilon}}\right)^2 + \left(\frac{\lambda_0}{\lambda_c}\right)^2 \tag{3}$$

The consideration of the real, hence not loss-free, dielectrics requires the introduction of the complex dielectric permittivity

$$\varepsilon_r = \varepsilon' - j\varepsilon''$$

and the complex wave number

$$\gamma = \alpha + j\beta$$

Hence Eq. (3) is extended to [SCHILD 1981]:

$$\varepsilon' = 1 - \left(\frac{\lambda_0}{\lambda_H}\right)^2 - \left(\frac{\lambda_0}{2\pi}\right)^2 (\alpha^2 - \beta^2) \tag{4}$$

$$\varepsilon'' = \frac{\alpha\,\beta\,\lambda_o^2}{2\pi^2} \tag{5}$$

The approach to determining the wave number parameters α and β is possible using the impedance transformation in a waveguide [MEGLA 1961, UNGER 1980]:

$$\frac{\tanh\left[(\alpha+j\,\beta)\,d\right]}{(\alpha+j\,\beta)\,d} = \frac{1}{j\,\beta_0\,d}\;\frac{m-j\tan(\beta_0\,z_0)}{1-j\,m\tan(\beta_o\,z_0)} \tag{6}$$

m—adapting factor,
d—length of sample,
β_0—imaginary part of the wave number in the air-filled part of the coaxial line,
z_0—distance of the first voltage minimum to the surface of the sample.

Eq. (6) is a complex-valued transcendental function. Its evaluation is difficult. SCHILD [1981] recommends that the equation be decomposed into real and imaginary parts and the results be found for ε' and ε'' by an iteration algorithm.

Another problem is connected with the fact that Eq. (6) contains trigonometric functions. Some types of solution result from the periodicity of the function tanh.

2. Methodology of measurements

For the measurements, a coaxial line with a tuning out probe, an attachable container for the sample material, and a selective SMV 8.5 microvoltmeter were available. The wave impedance was 70 Ω. All plug connections and connecting cables were adjusted. At the end of the sample container, short circuiting was realized by a metal plate (*Fig. 1*).

The selective microvoltmeter allows one simultaneously to feed into the coaxial line electromagnetic waves with certain frequencies. These are guided along the measuring line and reflected at the short circuit end of the sample container. Thus a standing wave is formed in the coaxial line. Inside the sample the wave is guided with a shorter wavelength. By the transition from sample to air and the superposition with reflected waves from the surface of the sample, a 'change' is produced in the wave picture.

The resulting pattern of the electric field strength depends on the dielectric properties of the sample and reflects the special propagation reaction in the given sample material.

With the help of the capacitive tuning out probe the field distribution can be determined. The target quantities ε' and ε'' can be calculated (Eqs.4, 5) using

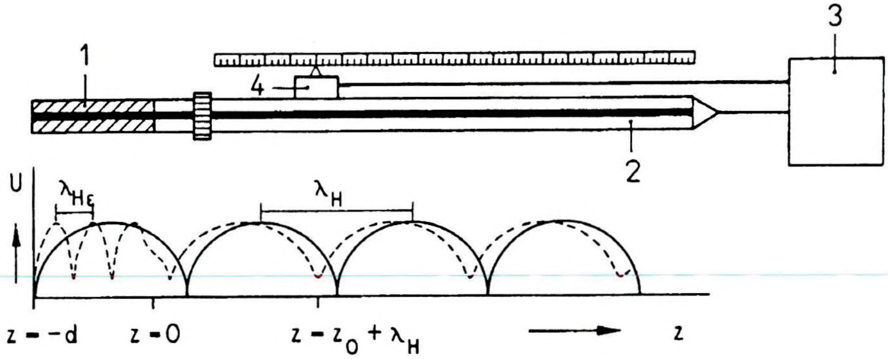

Fig. 1. Measurement place schematically with field strength pattern. 1— sample container with samples; 2—coaxial line; 3—SMV 8.5; 4—capacitive tuning out probe; continuous line — field strength pattern without sample; dashed line — field strenth pattern with sample

1. ábra. A mérési elrendezés sematikus vázlata a térerősség eloszásával. 1—mintatok mintával; 2—coaxiális vonal; 3—SMV 8.5; 4 — kapacitiv hangoló egység. (A folytonos vonal a minta nélküli, a szaggatott vonal a mintával előálló térerősségeloszlást mutatja)

Eq. (6) by means of a computer program based on the values of the measured maximum and minimum voltage, the position of the voltage minimum with respect to the surface of the sample, the sample length, and the frequency of the fed wave. The computer program also enables one to carry out the iteration procedure in different intervals and thus to calculate different solutions. The correct solution can be found by carrying out measurements with two different sample lengths.

Only one solution corresponding to the measurement with greater sample length is in agreement with one solution corresponding to the measurement with the smaller sample length, thus representing the valid solution. By performing the measurement in a wide frequency range, the correct solution can be seen from the course of the dispersion curve. All the values of this curve have to lie in an interval between ε=1 (air) and ε=81 (water), and they must show a certain course with increasing frequency according to previous investigations [Böhm 1985, Forkmann 1983]. The different solutions converge at high frequencies. It was necessary to carry out the measurements at two different wavelengths with some samples only where local dispersion effects were observed (Figs. 4, 5) at higher frequencies and higher dielectric permittivities. Different sediments were prepared as samples [Rappsilber 1991] with different water content (all with 0, 1, 2, 4, 8 and some with 12, 16 mass percentage). A definite total mass (sediment and water) was filled into the sample container and compressed to a definite length. Thus the density was found. The sample container was attached to the measurement equipment. The measurements took place at 18 different frequencies between 62 and 1000 MHz.

3. Error analysis

The following input data for the computer program
m — adapting factor (from maximum and minimum voltage),
f — measuring frequency,
z_0 — distance of the first voltage minimum to the surface of the sample,
d — sample length
are erroneous, on the one hand due to accidental errors, on the other hand because of the systematic errors of the measuring devices. The error propagation law cannot be used because Eq. (6) is of transcendental type. Therefore LEHMANN [1989] carried out an error simulation for estimated normal distributions of the four input quantities. Apart from the above, other sources of errors had to be taken into consideration:

- inaccuracies in the determination of sample density,
- erroneously determined water contents,
- uneven sample-air boundary surface,
- possible air gaps between sample and inner or outer conductor [ROST 1979, HUANG, SHEN 1983].

The following error intervals were found:
$\Delta\varepsilon' = \pm 5\%$
$\Delta\varepsilon'' = \pm 8\%$

4. Results

A 3-dimensional representation should be used to illustrate the results of local effects in the dispersion curve of the dielectric permittivity, depending on the water content. This is shown in *Figs. 2, 3* for a sand sample, where the error bars are marked in accordance with the remarks made in Section 3.

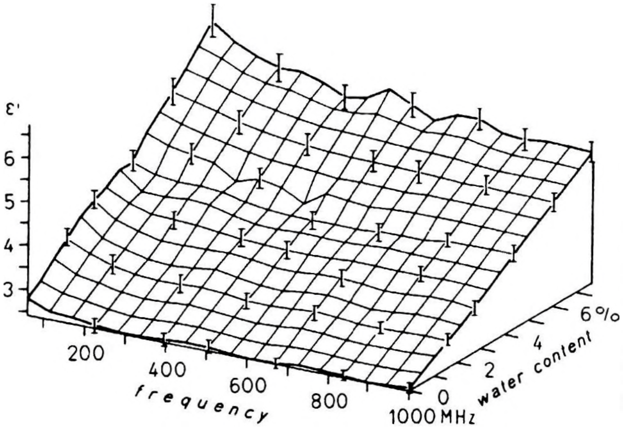

Fig. 2. Real part of the dielectric permittivity of a sand sample (density 1500 kg/m^3)

2. ábra. Egy homok minta dielektromos állandójának reális része (sűrűség 1500 kg/m^3)

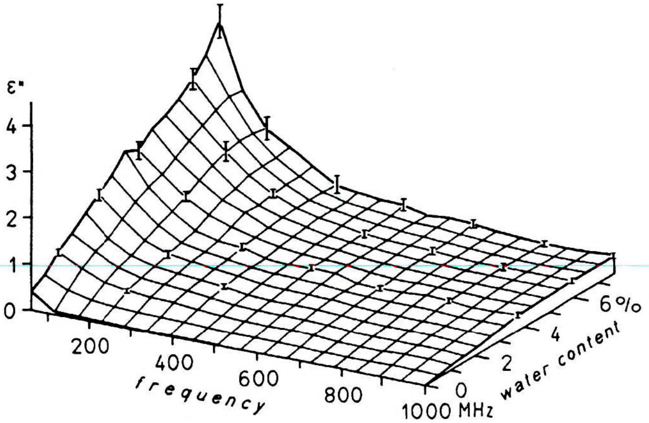

Fig. 3. Imaginary part of the dielectric permittivity of a sand sample (density 1500 kg/m^3)
3. ábra. Egy homok minta dielektromos állandójának képzetes része (sűrűség 1500 kg/m^3)

From similar representations for a number of sediment samples, an attempt was made to find a systematic pattern in the dependence of the dielectric permittivity on several parameters. The dispersion curves of the dielectric permittivity generally show the same behaviour: a stronger decrease in the range from 62 to 200 MHz and a flowing into a plateau area up to 1000 MHz. This behaviour was expected and is in agreement with the measurement results of other authors [SCHILD 1981, BÖHM 1985].

The characteristic increase of the dielectric permittivity with increasing water content is explained at least qualitatively by various mixing formulae. The ε-values obtained can be compared with the results of DELANEY and ARCONE [1984].

Comparing the results of samples with different grain-size distribution it follows that higher values of the dielectric permittivity, both in the real and in the imaginary part, were obtained for smaller effective grain diameters d_w. This may be proved by comparing Figs. 2 and 3 (d_w=0.35 mm) with *Figs. 4* and *5* (d_w=0.01 mm).

The values from the measurement with a glass sphere sample are interesting when compared with similar grain-size distributions of natural samples. Higher values of the dielectric permittivity were obtained for the glass sphere sample with higher water contents. This is due to the lower water bond of grains with relatively small inner surface, in contrast to the stronger water bond with non-regular grain form and, at the same time, a larger inner surface.

Characteristic dispersion effects were obtained for clay and silt samples. Just to give an example, the real and imaginary part of the dielectric permittivity of a clay sample are represented in Figs. 4 and 5.

This observation indicates a complicated relaxation effect of the water being more strongly bonded in fine-grained sediments. The relaxation of free

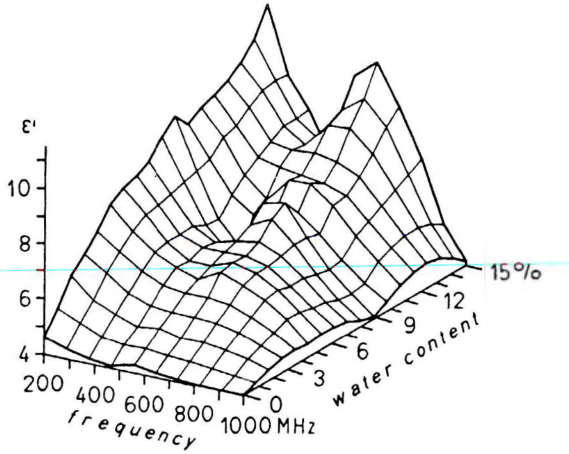

Fig. 4. Real part of the dielectric permittivity of a clay sample (density 1500 kg/m^3)
4. ábra. Egy agyag minta dielektromos állandójának reális része (sűrűség: 1500 kg/m^3)

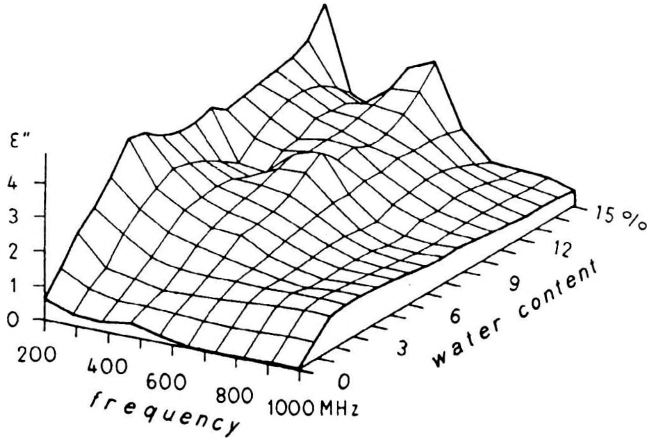

Fig. 5. Imaginary part of the dielectric permittivity of a clay sample (density 1500 kg/m^3)
5. ábra. Egy agyag minta dielektromos állandójának képzetes része (sűrűség: 1500 kg/m^3)

water lies in the GHz-range [HOEKSTRA, DELANEY 1974], but FORKMANN [1983] already assumed that because of specific bonding mechanisms of water in porous loose sediments a shift of the relaxation range to lower frequencies is to be expected.

5. Consequences for the ground-penetrating radar

The penetration depth τ and the phase velocity v are important propagation parameters for high-frequency electromagnetic waves applied to ground-penetrating radar (FORKMANN, PETZOLD 1989]. Both are directly dependent on the complex dielectric permittivity:

$$\tau = \frac{c\sqrt{2}}{\omega\sqrt{\sqrt{\varepsilon'^2 + \varepsilon''^2} - \varepsilon'}} \qquad (7)$$

$$\nu = \frac{c\sqrt{2}}{\sqrt{\sqrt{\varepsilon'^2 + \varepsilon''^2} + \varepsilon'}} \qquad (8)$$

Note that the ground-penetrating depth is defined as the distance along which the amplitude of the wave is decreased by e-times, whereas for the assessment of the range depth of a measurement system the performance factor and other equipment parameters have to be taken into account [FORKMANN, PETZOLD 1989]. The penetration depth mainly serves for comparing the propagation properties of different earth substances. Thus, transformation of dielectric permittivity values determined in the laboratory into propagation parameters contributes to the assessment of the applicability or the determination of actual performance parameters of ground-penetrating radar. As an example, the penetration depth is shown for a clay sample according to Figs. 4 and 5 (*Fig. 6*).

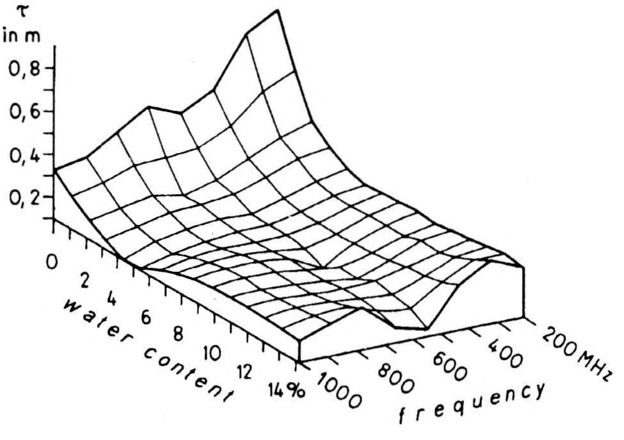

Fig. 6. Penetration depth for the clay sample
6. ábra. Behatolási mélység az agyag mintában

This example demonstrates the selective reduction of the penetration depth for this clay sample as a function of the frequency and water content, thus indicating the special features of wave propagation in cohesive loose sediments.

6. Conclusions

The laboratory determination of the dielectric permittivity of hard rocks in general is realized by placing the rock sample in a so called material condenser, and this does not cause any essential measuring problem. However, a loose rock sample working as a dielectric has to be filled into a coaxial line. Thus there are problems in maintaining in situ conditions. The use of the standing wave method shows that reliable results can be achieved for researching the dispersion curves of the dielectric permittivity as a function of the water content and the structure of various sediments. The utilization of the given laboratory method led to the error intervals being significantly smaller than the dispersion effects. On the other hand some dispersion effects especially of cohesive loose rocks point to the special nature of the water bond in these sediments. Their effects on the propagation of high-frequency electromagnetic waves are of considerable practical interest for ground-penetrating radar.

REFERENCES

BADEN FULLER A. J. 1974: Mikrowellen. Friedr. Viehweg & Sohn, Braunschweig

BÖHM C. 1985: Bestimmung der komplexen Dielektrizitätszahl zwischen 100 und 1000 MHz an Lockergesteinen. Diploma thesis, Bergakademie Freiberg

DELANEY A., ARCONE S. A. 1984: Dielectric measurements of frozen silt using time domain reflectometry. Cold region science and technology **9**, pp. 39-46

FORKMANN B. 1983: Strukturuntersuchungen mit elektromagnetischen Wellen (Gesteinsradar) für Aufgaben in der Betriebserkundung in der Braunkohlenindustrie. Research report, Bergakademie Freiberg

FORKMANN B., PETZOLD H. 1989: Prinzip und Anwendung des Gesteinsradars zur Erkundung des Nahbereiches. Freiberger Forschungshefte C 432. Deutscher Verlag f. Grundstoffindustrie Leipzig

von HIPPEL A. R. 1954: Dielectrics and waves. J.WILEY & Sons, New York

HOEKSTRA P., DELANEY A. 1974: Dielectric properties of soils at UHF and microwave frequencies. J. Geophys. Res. **79**, 11, pp. 1699-1708

HUANG F. S., SHEN L. C. 1983: Analysis of error due to presence of gaps in the measurement of rock samples in a coaxial line. Geophysics, **48**, pp. 206-212

LEHMANN F. 1989: Fehlersimulation zur Berechnung der komplexen Dielektrizitätskonstante auf der Grundlage experimenteller Ergebnisse. Research work, Bergakademie Freiberg

MEGLA G. 1961: Dezimeterwellentechnik. Verlag der Technik, Berlin, 128 p.

RAPPSILBER I. 1991: Labormethodik zur Messung der komplexen Dielektrizitätskonstante nach dem Stehwellenverfahren. Diploma thesis, Bergakademie Freiberg

ROST A. 1979: Messung dielektrischer Stoffeigenschaften. Akademie-Verlag, Berlin, WTB, Band 242, 126 p.

SCHILD D. 1981: Die Messung der komplexen Dielektrizitätskonstante von Gläsern über einen breiten Temperaturbereich im Mikrowellengebiet. Thesis, Technische Hochschule Ilmenau

UNGER H. G. 1980: Elektromagnetische Wellen auf Leitungen. Hüthig-Verlag, Heidelberg

---

LABORATÓRIUMI MÓDSZER LAZA KŐZETEK KOMPLEX DIELEKTROMOS ÁLLANDÓJÁNAK MEGHATÁROZÁSÁRA (ÁLLÓ HULLÁMOK MÓDSZERE)

Pjotr FIJAS, Bernhard FORKMANN, Ivo RAPPSILBER

A laza kőzetek dielektromos állandójának meghatározására az állóhullám módszert vizsgálták. Ez az eljárás egy, közeggel részben feltöltött koaxiális kábel impedancia transzformációs egyenletén alapszik [von HIPPEL 1954]. A különböző laza kőzetminták komplex dielektromos állandójának mérésénél szisztematikus kapcsolatot találtak a frekvenciafüggés és az üledékes kőzet víztartalma, szerkezeti jellemzői között.

A méréseket 60 és 1000 MHz közötti frekvenciatartományban hajtották végre. A dielektromos állandót a terjedésre jellemző paraméterekké transzformálva a nagyfrekvenciás elektromágneses hullámok terjedési összefüggéseit írhatjuk le az üledékes kőzetekben.