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RAY-TRACING MODELLING IN SEISMIC EXPLORATION

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Seismic modelling provides an inexpensive means of evaluating the potential effectiveness of a proposed data acquisition procedure. Using an approximate geological model based on information from previous work in the area, proposed source, receiver and recording parameters can be included in a seismic simulation to assess the preliminary design of a seismic survey. Seismic modelling can also be used to generate appropriate synthetic data sets for testing processing algorithms. The purpose of this paper is to present a general ray-tracing modelling package which has been written to compute finite-offset and walkaway VSP data as well as conventional seismic time sections; CRP gathers, seismograms, etc. in laterally varying two-dimensional (2-D) media. It requires sufficient speed and a means of providing a proper balance between speed and accuracy. To speed up computations, third-order parabolas are fitted to consecutive straight line segments of each interface in the vicinity of all breakpoints. Thus, in the calculation of intersections of raypaths and interfaces, linear equations have to be solved in most cases, whereas third-order equations are solved only when rays arrive in the neighbourhood of breakpoints. As a net result ray tracing becomes fast and accurate even with a personal computer. The examples that are given prove that even a relatively simple modelling method can provide valuable information for exploration geophysicists.

Keywords: ray tracing, seismic modelling, VSP, time sections, synthetic seismograms, CRP gather

1. Introduction

The typical geometry for a vertical seismic profile (VSP) consists of a single source on the earth's surface and a number of recorder positions in a vertical drill hole (offset VSP) or a single geophone in the well and several sources on the surface (walkaway VSP). Vertical seismic profiles have been the subject of many recent publications. Examples of numerical modelling of VSP data were presented in WYATT [1981] and THYBO [1985] who synthesized acoustic responses from sonic and density logs, and in URSIN and ARNTSEN [1985] who computed the acoustic response accounting for geometrical spreading and absorption. Synthetic VSPs have been used in iterative forward modelling [LEE and BALCH 1983], in iterative inversion for velocity and attenuation [DIETRICH and BOUCHON 1985; GRIVELET 1985] and in interpretation of CRP data [HARDAGE 1983, BALCH and LEE 1984].

One feature common of all the above-mentioned studies is that they deal with zero-offset VSP data in a one-dimensional context. This is the result of three factors: first, there is typically only one source position; second, the offset of the source is usually small; and third, the earth's structure is assumed to be a stack of horizontal layers. None of these restrictions is necessary. If the offset is not zero, the lateral variations of the geological structure should be taken into

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account. Similarly, in the computation of synthetic seismograms and conventional seismic time sections two- or three-dimensional modelling should be used. The purpose of this paper is to present a program package which has been written to model finite-offset and walkaway VSP data as well as conventional seismic time sections and seismograms in laterally varying two-dimensional models. In this topic the reader is referred to YOUNG et al. [1984], MCMECHAN [1985], SUPRAJITNO and GREENHALGH [1986] and others. Such modelling can be used directly as an interpretational tool and as a source of data for testing of inverse (imaging) processes.

2. Modelling by geometrical ray tracing

The geometrical ray-tracing method is chosen here to compute synthetic VSPs and surface seismic data. There are numerous modelling techniques commonly employed for the computer simulation of seismic wave propagation. The capabilities of these techniques vary according to the theoretical foundations and subsequent approximations upon which the algorithms are based. Ray-tracing modelling assumes that seismic energy propagates along straight line segments (rays), and that reflection and transmission processes occur at a mathemathical point when one of these rays intersects a boundary between two different rock units. Actually, seismic energy propagates through the earth as spherically spreading wavefronts, and reflection and transmission processes occur not at mathematical points but in large elliptical Fresnel zones. Each reflection point shown in the ray-tracing models in this paper marks only the centre of the Fresnel zone corresponding to a reflected raypath. The physical dimensions of these Fresnel zones must be kept in mind when interpreting ray trace models since a Fresnel zone defines the size of the reflector area which contributes to each reflection raypath. Particular attention should be given to situations where a Fresnel zone may be larger than the lateral dimensions of a structural or stratigraphic feature of a model. In such cases, if a raypath can exist mathematically, it generally yields the proper travel time, but the amplitude of the synthetic reflection will be incorrect since in the real earth the Fresnel zone would extend beyond one or more edges of the feature.

Diffracted energy can be defined as energy which propagates along raypaths other than those given by Snell's law of reflection and refraction. In real earth seismology, diffracted energy can, in some instances, be the dominant type of response recorded by geophones. However, since all energy propagation in ray-tracing modelling obeys Snell's law, the calculated results shown in this paper disregard diffracted energy. Diffraction effects can be ignored in modelling as long as the wavelengths contained in the seismic pulse are much shorter than the radius of curvature of any part of a reflecting surface occurring in the model. Thus, ray-tracing modelling in the vicinity of sharp curvature changes in an impedance boundary should be viewed with caution.

Apart from these disadvantages the ray-tracing technique has several advantages. Relatively simple, efficient ray-tracing algorithms can be developed for two- or three-dimensional media including structurally complex geological models. Sub-programs can be attached which compute partial amplitude information, geometrical spreading and absorption. User-defined types of multiple reflections can also be calculated easily. This method allows for flexible definition of source–receiver geometry and thereby facilitates modelling of any data acquisition configuration.

These features allowed us to prepare an efficient ray tracing program package on an IBM PC/AT personal computer, i. e. ray tracing could be realized without extremely fast computers. This tool of seismic modelling has proved to be valuable for determining the optimum VSP field geometry that can address a given structural or stratigraphic problem. Constructing ray-tracing models before starting a VSP experiment will result in fewer instances where VSP sources and geophones are positioned so that they record data which are incapable of achieving a given exploration objective. The modelling package also provides a framework for an economic analysis during the planning stage of a field experiment. For example, modelling can prevent the recording of more data than are needed to achieve an exploration objective. It can also indicate the amount of data that needs to be recorded in order to properly image a subsurface anomaly, and thus it allows an explorationist to make a reliable estimate of how much time and money will be required to record the data. And last but not least, it can demonstrate the structural and stratigraphic messages contained in VSP, CRP, or conventional seismic time sections.

3. Utilities of the program package

The geological model can be defined interactively on the computer screen. The layer boundaries consist of straight line segments, but in the subsequent calculations third-order parabolas are fitted to adjacent segments in the vicinity of each breakpoint. Each layer has constant P-wave velocity, bulk density and quality factor Q.

The user is able to get a preview with some possible raypaths for the given VSP geometry. Several ray types can be selected by the user, such as direct waves only, primary reflections, primary reflections from specified interfaces(s), all the second-order surface multiples and user-defined multiple reflections. The purpose of this calculation is to create a picture for a quick-look type interpretation. Because we do not prescribe for the raypaths to terminate at the exact receiver positions, it is not time-consuming to create previews. It is advisable to run a preview before subsequent detailed calculations in order to work out experimentally the suitable source–receiver configurations.

If a synthetic time section is to be calculated, those raypaths should be determined that start from the source(s) and terminate at the geophone(s). This calculation needs an iterative technique. It is time-consuming: if we use a complex geological model and the number of traces is a few hundred, the computation takes more than one hour. However, after such a computation the user is able to make a lot of different plots, such as the determined raypaths, the reflecting points of the primary refletions and the synthetic time section. In the last of these several options can be chosen. The computed quantity may be pressure, displacement, particle velocity or acceleration. In the last three cases both the vertical as well as the horizontal components can be determined. The impulse response can be convolved by four types of wavelets. Peak frequencies and lengths are defined by the user.

Absorption and/or geometrical spreading can also be included. The magnification of a given window of the time section is possible.

4. The algorithm

In the previous section it was mentioned that if one wishes to calculate synthetic time sections then those raypaths should be calculated that terminate at the exact receiver positions. This calculation needs iterations and, as a consequence, it is necessary to compute several times more raypaths than those appearing in the final results. It means that the efficiency of computation of a single raypath influences the running time of the whole ray-tracing program. Because the ray-tracing algorithm is given and cannot be modified, the computation time depends on how the layer boundaries are handled by the program.

The model-definition program actually accepts and stores the coordinates of some points of each consecutive interface. The simplest method of working with a layer boundary is to connect its points by straight line segments. In order to determine points of intersections of raypaths and the interface only linear equations have to be solved; this is fairly simple and can rapidly be done even with a personal computer. Unfortunately, even a small but abrupt change of an interface always causes an undesirable shadow zone in the reflection event *(Fig. 1a)*. In order to prevent the appearance of fictitious shadow zones the points of the interfaces can be connected by cubic spline functions. In this case, however, third-order equations have to be solved in order to compute the points of intersection of raypaths and the interface, and the determination of the angles of incidence requires the evaluation of second-order polynomials. For realistic geological models this calculation needs so much time on an IBM PC/AT personal computer that its implementation is not economical.

In our program package the following method is used. The defined points of an interface are connected by straight line segments, but in the calculations third-order parabolas are fitted to adjacent segments in the vicinity of each breakpoint. By this method the undesirable shadow zones are eliminated *(Fig. 1b)* and, in the calculation of the points of intersection, third-order equations need to be solved only in a few cases. This allows economical implementation of ray-tracing modelling on a personal computer such as the IBM PC/AT. Furthermore, it is not necessary to fit a parabola at every breakpoint so the geological model may include steep faults without any round-off. Spline interpolation does not possess this advantageous feature.

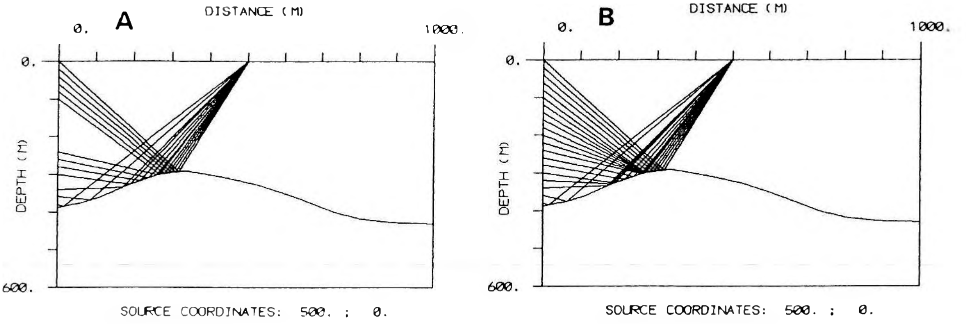


Fig. 1. If the layer boundary consists of straight line segments, two undesirable shadow zones occur (A). If third-order parabolas are fitted to adjacent line segments at the breakpoints, shadow zones disappear (B). The source-receiver geometries are the same in both parts of the figure

1. ábra. Ha a réteghatár egyenes szakaszokból áll, két nemkívánatos árnyékzóna jelenik meg (A). Ha azonban a töréspontokban harmadfokú parabolát illesztünk a szomszédos szakaszokhoz, az árnyékzónák megszűnnek (B). A mérési elrendezés mindkét ábránál azonos

Рис. 1. В случае границы раздела, состоящей из отрезков прямых, появляются две нежелательные теневые зоны (A). Однако если в точках сопряжения к смежным отрезкам подбирается многочлен третьей степени, теневые зоны исчезают (B). Схема измерения одинакова в обоих случаях.

5. Examples

This section presents a series of synthetic VSPs and surface seismic data. All models vary laterally (2-D) and, it is to be hoped, are characteristic in seismic exploration.

A low-velocity wedge is shown in *Fig. 2/a*. The calculated raypaths and the synthetic offset VSP for this model are shown in *Fig. 2/b* and *2/c*, respectively. (Here and in all the following examples, wiggle trace plots will show every geophone response, but raypath plots will show only every fourth raypath.) They contain the direct waves and the primary reflections only. Absorption and geometrical spreading are not included. The Ricker wavelet has a peak frequency of 30 Hz.

The four primary reflections originating from the direct wave branch can clearly be seen (Fig. 2/c). Reflections from the second and third interfaces (from the interfaces of the wedge) create two highly convergent branches. The slope of a reflection depends on the dip of the reflecting boundary. If the dip difference between two interfaces is considerable, the slope difference between the two upgoing reflection events on the VSP section is evident. Thus, the above-mentioned convergence implies a large dip difference.

The low-velocity wedge acts as a collector lens and can thus cause a shadow zone. This shadow zone can be found on the deepest reflection event between the 33rd and the 37th traces.

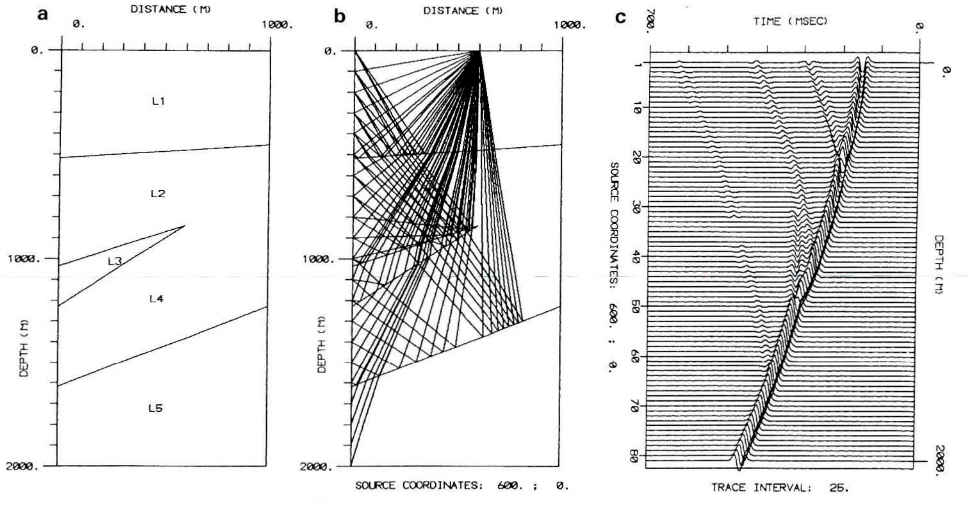


Fig. 2. Modelling for offset VSP geometry
a) Low-velocity wedge model. The layer parameters are shown in Table I
b) Calculated raypaths. The well is at the left-hand-side of the model, the source offset is 600 m. The distance between the equally spaced geophones is 25 m
c) The synthetic VSP section simulating pressure sensitive geophones with Ricker wavelets of 30 Hz peak frequency. Only direct waves and primary reflections are presented

2. ábra. Távoli (offset) VSP modellezése
a) Kissebességű kiékelődés. A rétegparamétereket az I. táblázat tartalmazza.
b) Számított sugárutak. A fúrólyuk a modell bal szélén helyezkedik el, a robbantópont–mélyfúrás távolság 600 m. Az egyenközűen elhelyezett geofonok közötti távolság 25 m.
c) A 2/b ábrán látható mérési elrendezésnek megfelelő szintetikus VSP szelvény, nyomásmérő geofonok és 30 Hz-es Ricker wavelet feltételezésével. Csak a direkt hullámokat és az elsődleges reflexiókat tartalmazza

Рис. 2. Моделирование удаленного (оффсетого) ВСП
а) Выклинивание с малой скоростью. Параметры пластов приведены в таблице I.
b) Расчитанные пути лучей при смещенной геометрии ВСП. Скважина находится на левом краю модели, величина смещения – 600 м. Расстояние между равномерно расположенными сейсмоприемниками равно 25 м.
с) Синтетический разрез ВСП, соответствующий схеме измерения, приведенной на рис. 2b. Содержит только прямые волны и действительные отражения. Предполагались чуствительные к давлению сейсмоприемники. Верхний предел частоты волны Рикера – 30 гц.

It is important to estimate the importance of multiples. It is evident from the plots of *Fig. 3,* where again absorption and geometrical spreading are not included, that a surface multiple may possess larger energy than a primary reflection. This suggests that the suppression or at least the identification of these multiples is very important in the interpretation of offset VSPs. The synthetic VSP sections are rather confused near the surface and at short travel times: the reflections from the upper two interfaces can hardly be separated from other

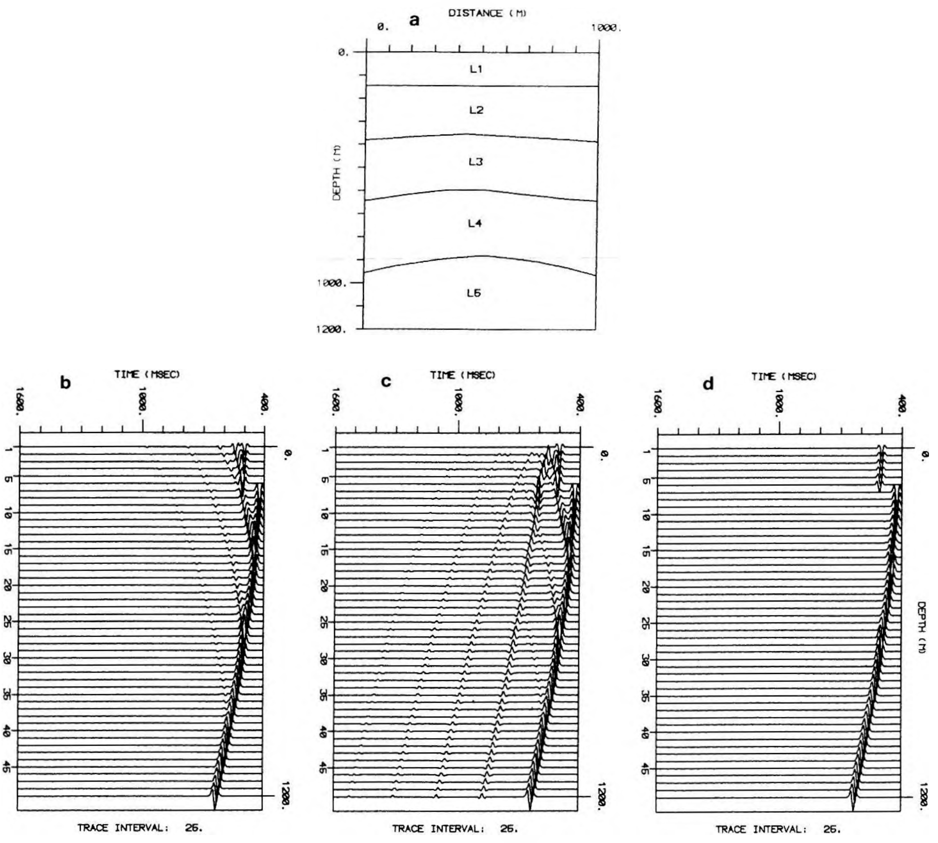


Fig. 3. Modelling with multiples
a) Geological model. The layer parameters are shown in Table II
b) Synthetic VSP section, conditions the same as in Fig. 2/b and 2/c. Only direct waves and primary reflections are shown
c) Synthetic VSP section with all the second-order surface multiples
d) Direct arrivals only

3. ábra. Modellezés többszörösökkel
a) Földtani modell. A rétegparamétereket a II. táblázat tartalmazza.
b) Szintetikus VSP szelvény, a 2/b ábrán bemutatott feltételekkel. Direkthullámok és elsődleges reflexiók
c) Szintetikus VSP szelvény az összes másodrendű felszíni többszörössel
d) Direkthullámok

***Рис.** 3.* **Моделирование с кратными волнами**
a) Простая геологическая модель с почти горизонтальными слоями. Параметры пластов приведены в табли.. ` II.
b) Синтетический разрез ВСП, соответствующий геологической модели, представленной на рис. 2b. Прямые волны и действительные отражения.
c) Синтетический разрез ВСП со всеми поверхностными кратными второго порядка.
d) Прямые волны.

Layer No.	Velocity (m/s)	Density (g/cm^3)
1	4000	2.80
2	5000	3.00
3	4000	2.80
4	5000	3.00
5	4500	2.90

Table I. Layer parameters of the geological model of Fig. 2/a

I. táblázat A 2/a ábrán bemutatott földtani modell rétegparaméterei

Таблица I. Параметры пластов геологической модели, представленной на рис. 2/a.

Layer No.	Velocity (m/s)	Density (g/cm^3)
1	1200	2.00
2	1800	2.10
3	2200	2.30
4	2600	2.40
5	2800	2.50

Table II. Layer parameters of the geological model of Fig. 3/a

II. táblázat A 3/a ábrán bemutatott földtani modell rétegparaméterei

Таблица II. Параметры пластов геологической модели, представленной на рис. 3/a.

Fig. 4. Modelling for walkaway VSP geometry
a) Unconformity reservoir. The layer parameters are shown in Table III
b) Synthetic walkaway VSP. The well is in the middle of the model, the geophone is at a depth of 700 m, the source spacing is 20 m, other conditions as in 2/c. Only direct waves and primary reflections are presented
c) Synthetic walkaway VSP for the modified model (L6=L7), geometry unchanged
d) Raypaths related to the synthetic section shown in Fig. 4/b

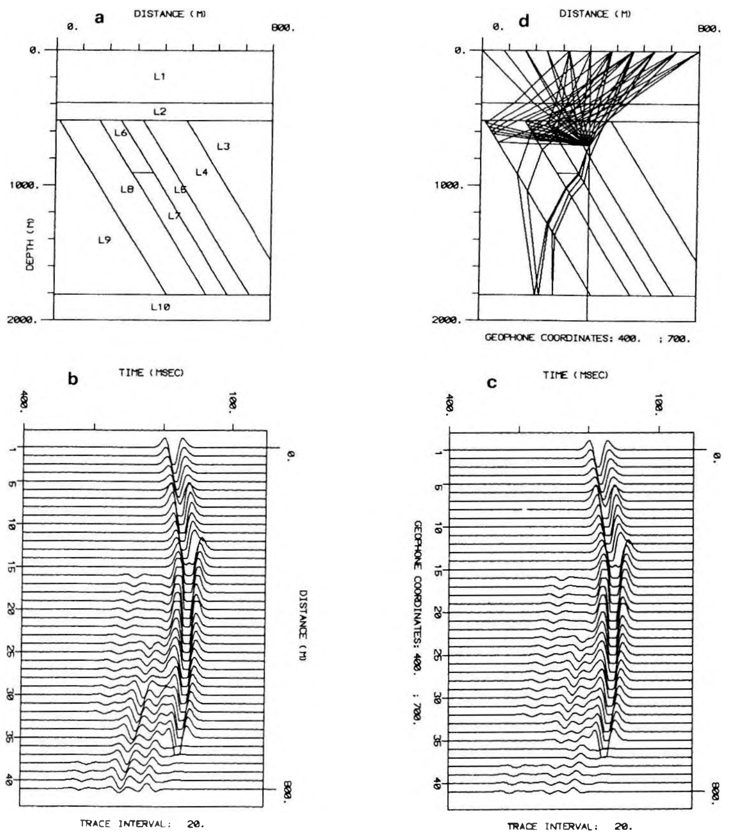
4. ábra. Távolodó (walkaway) VSP modellezése
a) Diszkordancia-csapda. A rétegparamétereket a III. táblázat tartalmazza.
b) Szintetikus távolodó VSP szelvény, nyomásérzékeny geofonok és 30 Hz-es Ricker wavelet feltételezésével. A fúrólyuk a modell közepén, a geofon 700 m mélyen helyezkedik el, a források közötti távolság 20 m. Csak a direkthullámokat és az elsődleges reflexiókat számítottuk.
c) A módosított modellnek megfelelő szintetikus távolodó VSP szelvény a 4/b ábrán leírt mérési geometria mellett.
d) A 4/b ábrán bemutatott szintetikus szelvényhez tartozó sugárutak

Рис. 4. Моделирование удаляющегося (walkaway) ВСП.
a) Ловушка приуроченная к несогласию. Параметры пластов приведены в таблице III.
b) Синтетический разрез ВСП с движущимся источником. Скважина находится в середине модели, глубина сейсмоприемника – 700 м, расстояние между источниками 20 м. Расчитывались только прямые волны и действительные отражения. Предполагались чуствительные к давлению сейсмоприемники. Верхний предел частоты волны Рикера – 30 гц.
c) Синтетический разрез ВСП с движущимся источником, соответствующий модифицированной модели, при геометрии измерения, представленной на рис. 4b.
d) Лучевые пути соответствующие синтетическому разрезу, представленному на рис. 4b.

events. In order to solve this problem we have calculated a synthetic VSP which contains the direct waves only (Fig. 3/d). Comparing these data with the other figures, the identification of the two reflections under consideration becames easier.

Modelling of an unconformity reservoir is shown in *Fig. 4/a.* Layer 6 contains hydrocarbons and, as a consequence, its P-wave velocity is lower and its bulk density less than those of layer 7. Two synthetic walkaway VSPs have been calculated: one for the original model *Fig. 4/b* and one for a slightly modified version of the model, in which the parameters of the 6th and 7th layers are the same (there is no hydrocarbon, *Fig. 4/c*). Figure 4/d shows the related raypaths. Only the direct waves and the primary reflections have been calculated. Absorption and geometrical spreading are not included and the Ricker wavelet again has a peak frequency of 30 Hz.

Detailed comparison of the two VSPs shows a salient difference. The decrease of the acoustic impedance in the reservoir locally enhances the reflectivities of its faces, giving increased amplitudes of the reflections (see the right-hand-side of the sections between 200 and 300 milliseconds). This



increased energy is similar to the bright spot well known as a potential hydrocarbon indicator.

Layer No.	Velocity (m/s)	Density (g/cm^3)
1	4000	2.80
2	5000	3.00
3	4000	2.80
4	4500	2.90
5	4700	2.95
6	3600	2.70
7	4000	2.80
8	4500	2.90
9	4900	3.00
10	5000	3.00

Table III. Layer parameters of the geological model of Fig. 4/a

III. táblázat A 4/a ábrán bemutatott földtani modell rétegparaméterei

Таблица III. Параметры пластов геологической модели, представленной на рис. 4/a.

A more complicated geological model is shown in *Fig. 5/a*. The related raypaths and the synthetic seismic time section can be seen in Figs. 5/b and 5/c, respectively. Absorption is included and the wavelet (a damped sine wave) has a peak frequency of 30 Hz. It is assumed that the recorded parameter is the z-component of the particle velocity. (In the previous examples pressure sensitive geophones were assumed.)

Fig. 5. Modelling a relatively complicated 2-D geological section
a) Geological model. The layer parameters are shown in Table IV.
b) Raypaths
c) Synthetic seismic time section, primaries only

5. ábra. Bonyolult 2-D földtani szelvény modellezése
a) A földtani modell. A rétegparamétereket a IV. táblázat tartalmazza
b) Sugárutak
c) Szintetikus időszelvény. Csak az elsődleges reflexiókat mutatjuk be

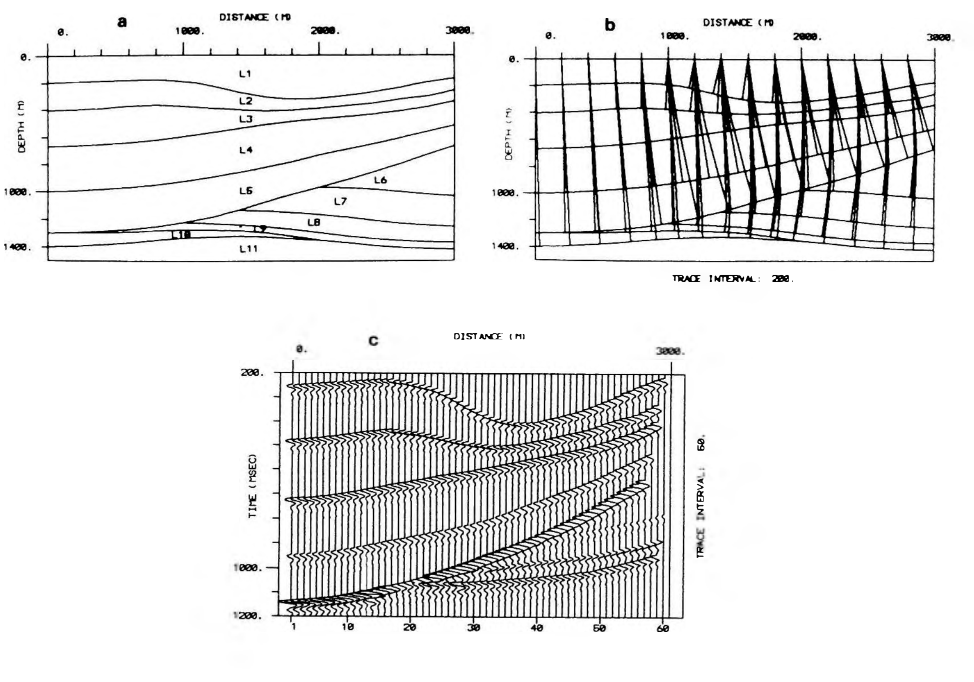
Рис. 5. Моделирование сложного двухмерного геологического разреза
a) Геологическая модель. Параметры пластов содержатся в таблице IV.
b) Лучевые пути
c) Синтетический времонной разрез. Представлены только действителные отражения.

Layer No.	Velocity (m/s)	Density (g/cm^3)	Q
1	1600	2.30	300
2	1800	2.50	450
3	2200	2.60	700
4	2900	2.60	800
5	3200	2.80	900
6	4200	3.10	1500
7	4500	3.10	1600
8	4000	2.90	1500
9	4800	3.10	1600
10	5000	3.10	2000
11	5200	3.20	2000

Table IV. Layer parameters of the geological model of Fig. 5/a

IV. táblázat Az 5/a ábrán bemutatott földtani modell rétegparaméterei

Таблица IV. Параметры пластов геологической модели, представленной на рис. 5/a.



Conclusions

The above discussed examples have proved that the modelling package described here can be used for all those tasks solved by modelling packages for big computers. The package also offers many more possibilities.

In addition to demonstrating the structural and stratigraphic messages contained in the data, ray trace modelling is also invaluable for determining the optimum field geometry. The energy ratio of specified multiples and primary reflections can also be investigated. One can estimate how the recorded data may be influenced by hydrocarbon saturation, if a potential petroleum trap is under consideration. Such modelling can also be used directly as a source of data for testing inverse methods or data processing techniques.

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SUGÁRVEZETÉSES SZEIZMIKUS MODELLEZÉS

WÉBER Zoltán

A szeizmikus hullámterjedés számítógépes modellezése a szeizmikus módszer fontos részévé vált sokirányú felhasználhatósága révén. Jelen dolgozat célja egy geometriai sugárkövetésen alapuló modellező programcsomag bemutatása, melynek segítségével mind a VSP különböző változatai, mind pedig stacking szelvények, illetve közös robbantópontokhoz tartozó szeizmogramok számíthatók horizontálisan is változó kétdimenziós modellekre. A megfelelő számítási sebességet úgy érjük el, hogy a számítások során az egyenes szakaszokból álló réteghatárok minden egyes töréspontjánál egy harmadfokú parabolát illesztünk a csatlakozó szakaszokhoz. Ezáltal a sugarak és a réteghatárok metszéspontjának meghatározásakor a legtöbb esetben csak egy lineáris egyenletet kell megoldani, míg harmadfokú egyenletek megoldására csak akkor van szükség, ha a sugarak a töréspont közvetlen közelében érkeznek a réteghatárra. A bemutatott példák bizonyítják, hogy még egy viszonylag egyszerű modellezési eljárás is értékes információkkal szolgálhat a geofizikus számára.

СЕЙМИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПО ГЕОМЕТРИЧЕСКОМУ ПРОСЛЕЖИВАНИЮ ЛУЧА

Золтан ВЕБЕР

Компьютерное моделирование распространения сейсмических волн стало важной частью сейсмического метода благодаря возможности многостороннего использования его результатов.

Целью настоящей работы является представление пакета программ моделирования, основанного на геометрическом прослеживании луча, а помощъю которого можно расчитать ВСП со смещением или движущимся источником, а также суммированные разрезы и сейсмограммы, относящиеся к общей точке взрыва для двумерных моделей с горизонтальными вариациями. Необходимая скорость расчетов достигается так, что в ходе расчетов в каждой отдельной точке сопряжения отрезков прямых, из которых состоят границы пластов, к сопряженному отрезку подбирается трехмерный многочлен. Благодаря этому, при определении точек пересечения границ слоев и лучей в большинстве случаев необходимо решить только одно линейное уравнение, а решать уравнения третьей степени нужно только тогда, если лучи приходят к границе пласта в непосретственной близости от точки сопряжения.

Представленными примерана доказывается, что даже относительно простая процедура моделирования может дать ценную информацию для геофизиков.