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DIRECT INTERPRETATION OF MAGNETIC ANOMALIES DUE TO SPHERICAL SOURCES — A HILBERT TRANSFORM METHOD

N. SUNDARARAJAN*, B. UMASHANKAR*, N. L. MOHAN*
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Keywords: magnetic anomalies, spherical models, direct problem, Hilbert transform

1. Introduction

Point poles, magnetic doublets and spheres are some of the most important three-dimensional models in mining geophysics. Many methods are available in geophysical literature to interpret magnetic anomalies of ground and airborne magnetic data. [Henderson and Zietz 1948 and 1967, Smellie 1956, Gay 1965, Radhakrishna Murthy 1974, Rao et al. 1973]. These methods are subject to certain assumptions and are relatively cumbersome in their approach.

A more recent paper of Mohan et al. [1982] proposes a novel interpretation of spherical sources by means of spectral analysis, although it again involves tedious mathematical operations. In this paper, we present an elegantly simple mathematical procedure to extract the parameters of the sphere, namely the depth to the centre, the polarization angle and the radius. This process involves the computation of the first horizontal derivative of the vertical magnetic anomaly, and hence the vertical derivative by means of the Hilbert transform. Making use of these two derivatives, the parameters are obtained by means of simple mathematical expressions.

The application of the Hilbert transform in the interpretation of ground-magnetic anomalies has been gaining greater importance of late. [Nabighian 1972, Mohan et al. 1982, Sundararajan 1982 and Sundararajan et al . 1983,

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1985]. In all these papers the concept of amplitude of the analytic signal is used in precisely locating the origin. However, a mention can be made that this amplitude curve can also be used to delineate the sources from regional magnetic or gravity surveys. For all practical purposes, this method can also be realised by simple programming.

2. Vertical magnetic effect of a sphere

The geometry of the model is shown in *Figure 1*, with Z as the depth to the centre, R as the radius and Q as the magnetic polarization angle. The vertical magnetic effect of such a model is given by

$$V(x) = \frac{4}{3}\pi R^3 I \frac{(2Z^2 - x^2)\sin Q - 3xZ\cos Q}{(x^2+Z^2)^{5/2}} \tag{1}$$

where I is the intensity of magnetisation [Rao et al. 1973].

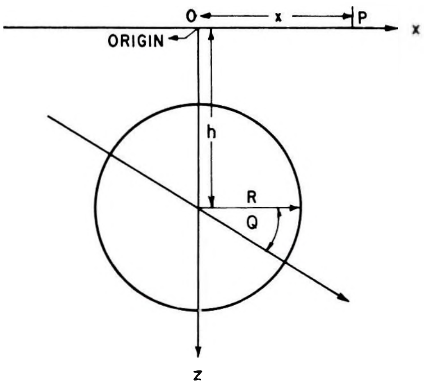

Fig. 1. Geometry of the spherical model

1. ábra. A modell geometriája

Рис. 1. Геометрия модели.

Differentiating equation (1) separately with respect to x and Z we obtain the first horizontal and vertical derivatives:

$$V_x(x) = \frac{4/3\pi R^3 I}{(x^2+Z^2)^{7/2}} \cdot [(x^2+Z^2)(2x\sin Q + 3Z\cos Q) + + 5x((2Z^2 - x^2)\sin Q - 3xZ\cos Q)] \tag{2}$$

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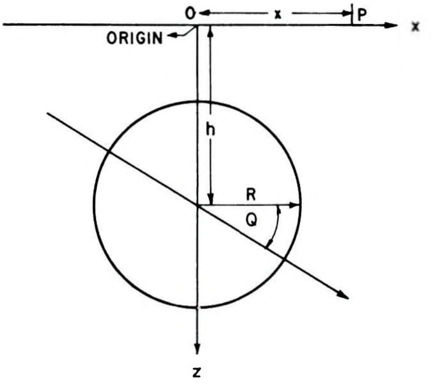

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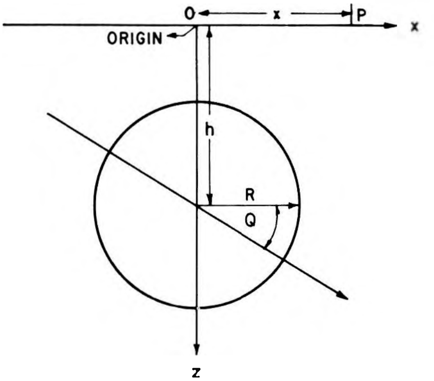

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According to NABIGHIAN [1972], the horizontal and vertical derivatives of a potential field forms a Hilbert transform pair. It can be represented here symbolically as:

$$V_x(x) \overset{\text{H}}{\longleftrightarrow} V_Z(x)$$

For mathematical convenience [SUNDARARAJAN 1982], either the positive or negative of the vertical derivative can be taken as the Hilbert transform of the horizontal derivative, since in both cases the magnitude of the field is the same, with a 180° phase difference to each other.

The relationship between the vertical and horizontal derivatives can be given in the form of Hilbert Transform equation as

$$V_Z(x) = V_Z(y) = \frac{1}{\pi} \cdot P \int_{-\infty}^{+\infty} \frac{V_x(x)}{x-y} \, dx \tag{4}$$

where P is Cauchy's principal value of the integral [THOMAS 1969].

This can be expressed in the form of convolution as:

$$V_Z(x) = V_x(x) * \frac{1}{\pi} x \tag{5}$$

where * denotes the convolution.

3. Interpretation

The location of the source—indispensible in geophysical interpretation—can be determined by solving a simple equation of the horizontal and vertical derivatives of the form

$$A(x) = [V_x(x)^2 + V_Z(x)^2]^{1/2} \tag{6}$$

The function $A(x)$ is termed the amplitude curve of the analytic signal in geophysical literature [NABIGHIAN 1972 and SUNDARARAJAN 1982]. The graph of $A(x)$ attains its maximum value over the causative body. This is true for 2-D and 3-D structures.

At $x=0$, equations (2) and (3) reduce to,

$$V_x(0) = 3K \frac{\cos Q}{Z^4} \tag{7}$$

$$V_Z(0) = 6K \frac{\sin Q}{Z^4} \tag{8}$$

where $K = \frac{4}{3} \pi R^3 I$.

Dividing equation (8) by equation (7) we obtain the angle of polarization as:

$$Q = \tan^{-1}\left(\frac{V_Z(0)}{2V_x(0)}\right) \tag{9}$$

From *Figs. 2* and *3* we see that the horizontal and vertical derivatives intersect at three distinct points. Therefore we can consider,

$$V_x(x) = V_Z(x) \quad \text{at} \quad x = x_1,\ x_2 \text{ and } x_3$$

where x_1, x_2 and x_3 are the abscissae of the points of the intersection of the derivatives, as cited above. Then, using equations (2) and (3) the polynomial equation will be

$$\begin{aligned} F(x) = {} & 3x^3(\sin Q + \cos Q) + 3x^2 Z(4\cos Q - 3\sin Q) - \\ & - 12xZ^2(\sin Q + \cos Q) - 3Z^3(\cos Q - 2\sin Q) \end{aligned} \tag{10}$$

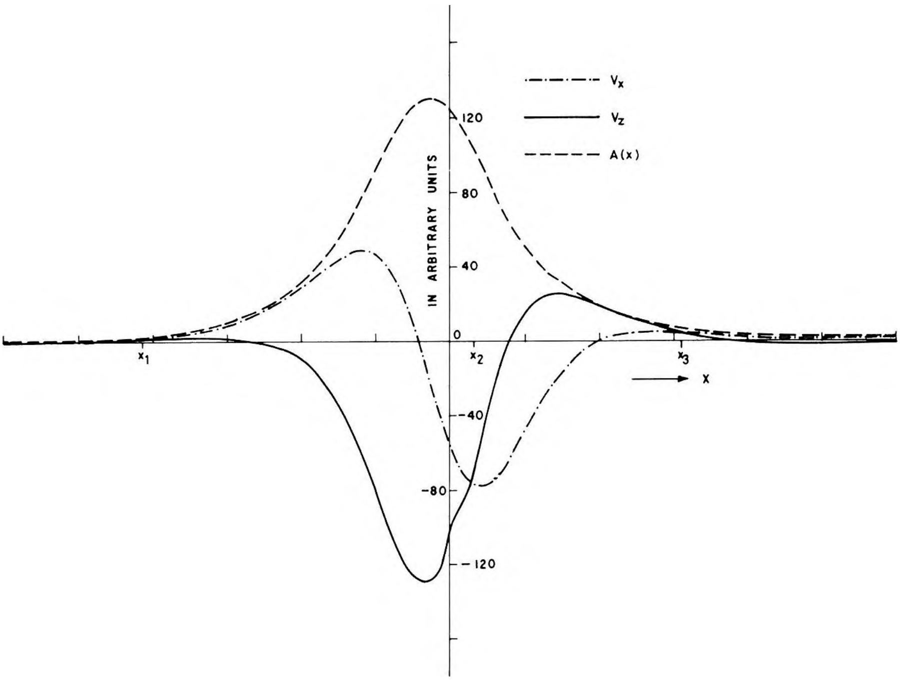

Fig. 2. The first horizontal (V_x) and vertical derivatives (V_z) of the vertical magnetic anomaly and their amplitude curve ($A(x)$) due to a sphere (Model I., for model parameters see Table I)

2. ábra. Egy gömbi ható (I. modell, paramétereit lásd az I. táblázatban) ΔZ anomáliájának első horizontális (V_x) és vertikális (V_z) deriváltja, valamint az ezekből képzett amplitudó görbe ($A(x)$)

Рис. 2. Первая горизонтальная (V_x) и вертикальная (V_z) производная аномалии ΔZ, обусловленной сферической возмущающей силой (модель I, параметры ее см.в табл.I), а также полученная по ним амплитудная кривая ($A(x)$).

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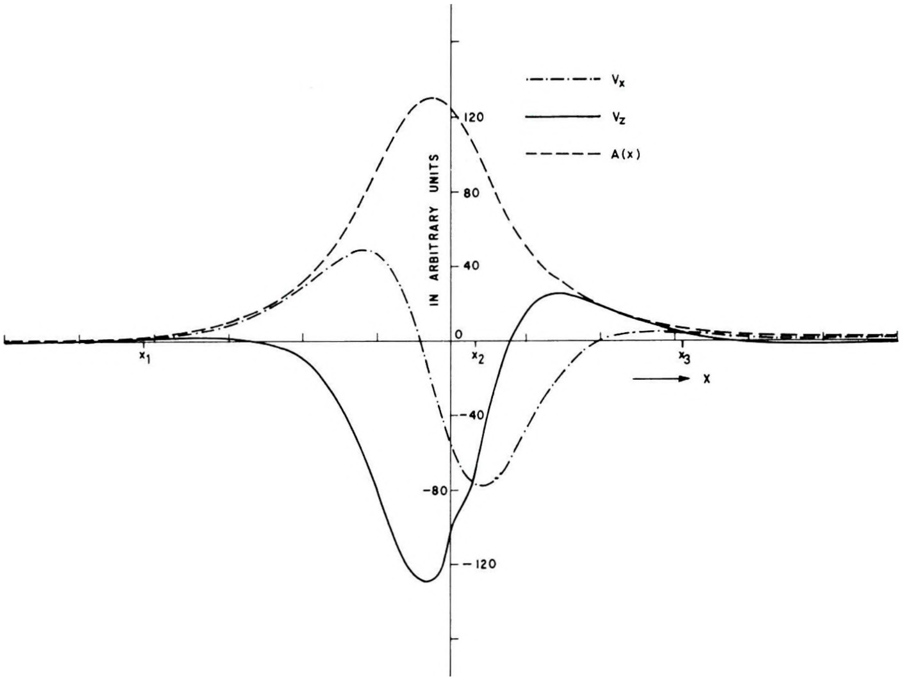

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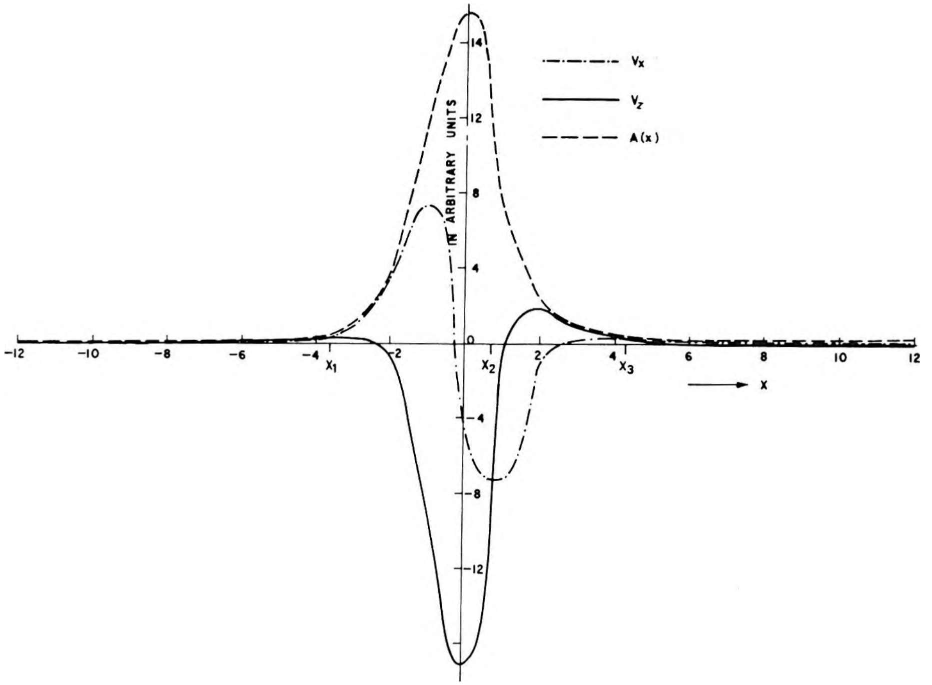

Fig. 3. The first horizontal (V_x) and vertical derivatives (V_z) of the vertical magnetic anomaly and their amplitude curve ($A(x)$) due to a sphere (Model II., for model parameters see Table I.)

3. ábra. Egy gömbi ható (II. modell, paramétereit lásd az I. táblázatban) ΔZ anomáliájának első horizontális (V_x) és vertikális (V_z) deriváltja, valamint az ezekből képzett amplitudó görbe ($A(x)$)

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This cubic equation in x could easily be solved for Z, i.e., the depth to the centre of the sphere is obtained as:

$$Z^3 = (A/B)\, x_1 \cdot x_2 \cdot x_3 \qquad (11)$$

Where x_1, x_2 and x_3 are the three real roots of equation (10), which also implies that equations (2) and (3) possess these roots. The constants A and B are given as:

$$A = \sin Q + \cos Q$$

$$B = \cos Q - 2 \sin Q$$

Since Q is already known, the depth Z to the centre of the sphere could easily be obtained from equation (11).

Squarring and adding equations (7) and (8) we get K as:

$$K = \frac{Z^4}{3}\left(\frac{V_x(0)^2 + V_Z(0)^2}{4-3\cos^2 Q}\right)^{1/2} \tag{12}$$

Thus, K yields either the radius (R) of the sphere or the intensity of magnetization (I) given as:

$$R = \left(\frac{3K}{4\pi I}\right)^{1/3} \tag{13}$$

$$I = \frac{3K}{4\pi R^3} \tag{14}$$

4. Theoretical examples

The procedure outlined above is demonstrated with two theoretical examples *(Table I)*. Using equations (2) and (3), the first horizontal and vertical derivatives of the magnetic field are computed and shown in Figures 2 and 3. These figures include the amplitude curve of the derivatives. It is observed that there are three distinct abscissae at the points of intersection of the horizontal and vertical derivatives.

The parameters, namely the magnetic polarization angle (Q), the depth to the centre of the sphere (Z) and the radius (R), are evaluated using equations (9), (11) and (13). The results are presented in Table I and it can be observed that the assumed and interpreted values agree very closely, thereby supporting the validity of the method.

	Parameters	Q^*	Z^*	R^*
MODEL I	Assumed values	45	2.00	1.00
	Evaluated values	45	1.96	1.00
MODEL II	Assumed values	60	2.50	0.75
	Evaluated values	60.14	2.49	0.75

(* in arbitrary units and + in degrees)

Table I. Theoretical examples

I. táblázat. Elméleti példák

Таблица I. Теоретические примеры.

5. Field examples

The technique under discussion is tested on two field examples, the first pertaining to the vertical component of the magnetic field in the Bankura area of West Bengal, India *(Fig. 4)*, and the second to the Louga anomaly in the USA (Fig. 6, after NETTLETON 1976). Both anomalies can be approximated by spherical models.

(a) The Bankura Anomaly, West Bengal, India

The total length of the Bankura anomaly is around 9.28 km and it is digitized into 100 equal parts at an interval of 92.8 meters. The first horizontal derivative is computed manually and then it is convolved with $(1/x)$ to obtain the discrete Hilbert transform. Also the amplitude curve is computed, using equation (6). The horizontal derivative, the discrete Hilbert transform and the amplitude curve are shown in *Fig. 5*. Using equations (9), (11) and (13) the parameters, namely the polarization angle (Q), the depth to the centre of the sphere (Z) and the radius of the sphere are evaulated. Thus, the results obtained *(Table II)* are compared with that of RAO et al. [1977], and with those obtained by the method of MOHAN et al. [1982].

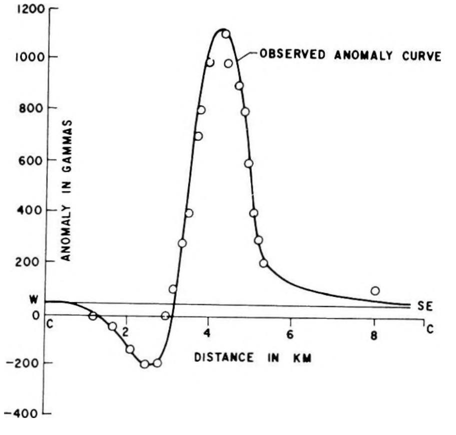

Fig. 4. The vertical component of the magnetic anomaly in the Bankura area of West Bengal, India

4. ábra. Nyugat Bengáliában Bankura területen (India) mért ΔZ anomália

Рис. 4. Аномалия ΔZ, замеренная в территории Банкура в Западной Бенгалии (Индия).

THE BANKURA ANOMALY, WEST BENGAL, INDIA

Parameters	Z (km)	R (km)	Q
Hilbert Transform method	1.252	1.099	41.52°
Spectral Analysis method [MOHAN et al. 1982]	1.312	0.993	41.50°
RAO et al. [1977]	1.32		

Table IIa. Field examples *IIa. táblázat.* Terepi példák *Таблица IIa.* Полевые примеры.

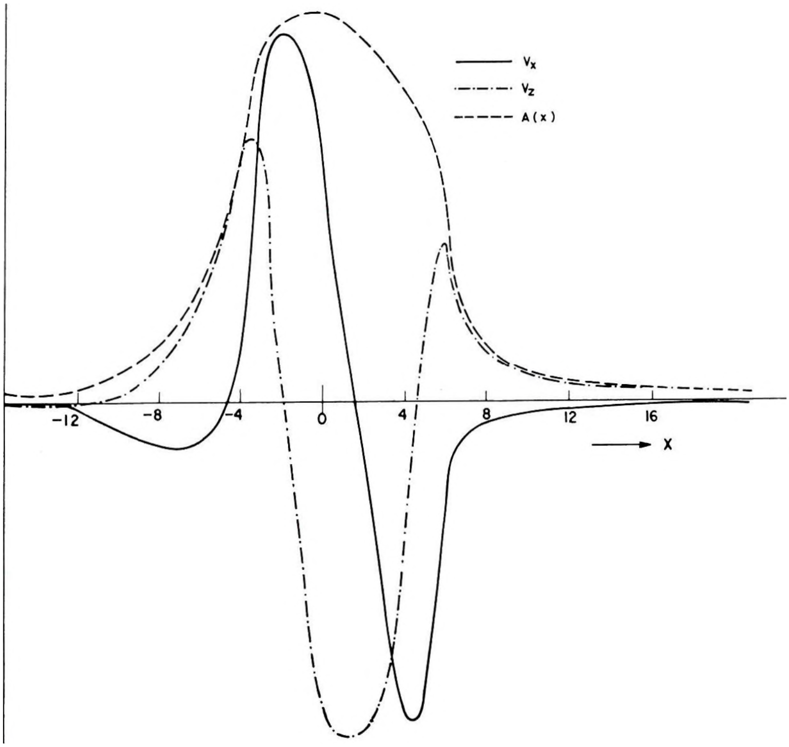

Fig. 5. The first horizontal derivative (V_x), the Hilbert transform (V_z) and their amplitude curve ($A(x)$) of the vertical component of the magnetic anomaly in the Bankura area of West Bengal, India

5. ábra. A Bankura ΔZ-anomália első horizontális deriváltja (V_x), ennek Hilbert transzformáltja (V_z) és az ezekből képzett amplitudó görbe ($A(x)$)

Рис. 5. Первая горизонтальная производная (V_x) аномалии ΔZ в Банкура, ее вид (V_z) по трансформации Гильберта и полученная по ним аномальная кривая ($A(x)$).

(b) The Louga Anomaly, USA

Figure 6 shows the profile of the vertical magnetic anomaly on a north–south line and a cross section of the probable source, a heavily magnetised spherical body, after NETTLETON [1976].. The entire length of the profile of around 65 km is digitised into 101 equal parts, and then the horizontal derivative is computed. As in the previous case the vertical derivative and the amplitude curve have been calculated and shown in *Figure 7.* The parameters are evaluated based on the procedure detailed in the text. The results obtained agree very well with that of NETTLETON [1976], presented in Table II. In addition, these results are also compared with those of MOHAN et al. [1986], who used the Mellin transform method for the integration of the gravity anomaly.

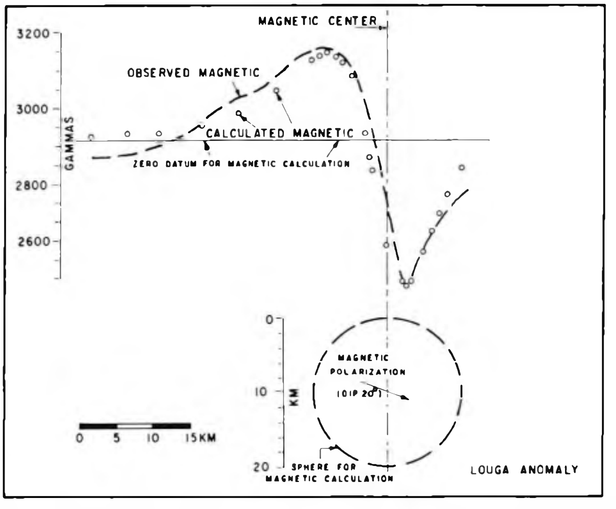

Fig. 6. The vertical component of the magnetic anomaly of Louga, USA, and the probable source [after NETTLETON 1976]

6. ábra. A Louga (USA) ΔZ-anomália és a valószínű ható [NETTLETON 1976 nyomán]

Рис. 6. Аномалия ΔZ в Луга (США) и вероятная возмущающая сила [по Неттлетону, 1976].

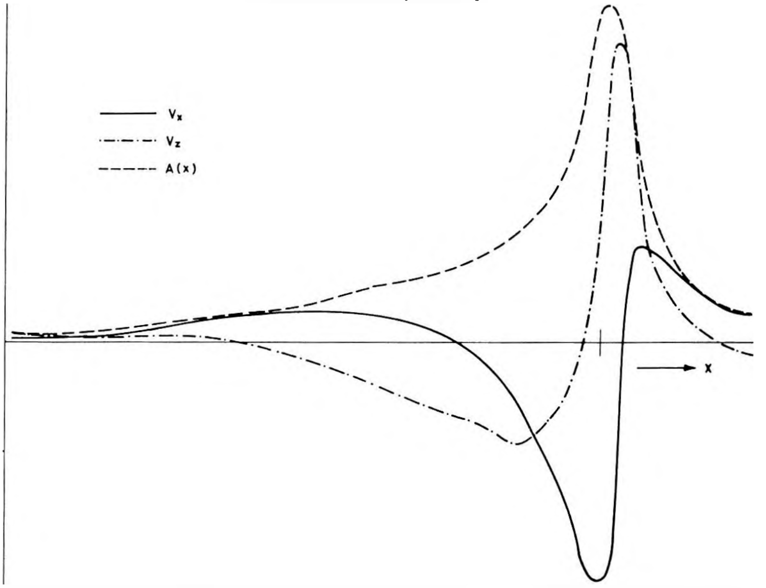

Fig. 7. The first horizontal derivative (V_x), the Hilbert transform (V_z) and the amplitude curve ($A(x)$) of the vertical component of the magnetic anomaly of Louga, USA

7. ábra. A Louga (USA) mágneses ΔZ-anomália első horizontális deriváltja (V_x), ennek Hilbert transzformáltja (V_z) és az ezekből képzett amplitudó görbe ($A(x)$)

Рис. 7. Первая горизонтальная производная (V_x), ее вид по трансформации Гильберта (V_z) и полученная по ним амплитудная кривая ($A(x)$).

THE LOUGA ANOMALY, USA

Parameters	Z (km)	Q
Hilbert Transform method of the authors	9.78	19.40°
The method of NETTLETON [1976]	9.88	20.00°
Mellin transform method [MOHAN et al. 1986]	9.31	

Table IIb. Field examples *IIb. táblázat.* Terepi példák *Таблица IIb.* Полевые примеры.

6. Conclusions

The method of interpretation of magnetic anomalies over 3-D sources by the Hilbert transform is very accurate, reliable, simple and effective in its approach. The amplitude curve of the analytic signal is extremely valuable for the interpretation of sources of arbitrary shape as well.

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GÖMB ALAKÚ HATÓK OKOZTA MÁGNESES ANOMÁLIÁK ÉRTELMEZÉSE – EGY HILBERT TRANSZFORMÁCIÓS MÓDSZER

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Gömb alakú hatók okozta mágneses anomáliák közvetlen értelmezésére dolgoztunk ki egy módszert a mágneses tér első horizontális és vertikális deriváltjai felhasználásával. A tér vertikális deriváltját Hilbert transzformációval számitottuk ki a horizontális deriváltból. A gömb paramétereit a deriváltak metszéspontjai abszcisszái függvényeként határoztuk meg. Két elméleti példán bizonyítjuk a módszer használhatóságát. Mérsékelten jó eredményeket nyertünk a Nyugat-Bengáliai Bankura és az egyesült államokbeli Louga területen, a gömb alakú ható okozta ΔZ-anomáliák értelmezése során. Ez az értelmezés alkalmazható ΔH- és ΔT-anomáliákra is. Gravitációs és természetes potenciál anomáliákat is értelmezhetünk hasonló módszerekkel. A módszer előnye, hogy könnyen programozható.

ИНТЕРПРЕТАЦИЯ МАГНИТНЫХ АНОМАЛИЙ, ВЫЗВАННЫХ СФЕРИЧЕСКИМИ ВОЗМУЩАЮШИМИ СИЛАМИ, МЕТОДОМ ТРАНСФОРМАЦИИ ГИЛЬБЕРТА

Х. СУНДАРАРАДЖАН, Б. УМАШХАНКАР, Н. Л. МОХАН и С. В. СЕСХАГИРИ РАО

Для непосредственной интерпретации аномалий выработан метод, основанный на использовании первых вертикальных и горизонтальных производных магнитного поля. Вертикальная производная рассчитывается из горизонтальной производной трансформацией Гильберта. Параметры сферы определяются как зависимости абсцисс точек пересечения производных. Возможность применения метода доказывается на двух теоретических примерах. Относительно хорошие результаты были получены для Банкура (Западная Бенгалия) и для Луга (США) при интерпретации аномалий ΔZ. Метод используем и для интерпретации аномалий ΔH и ΔT. Подобными методами возможна также интерпретация гравитационных аномалий и аномалий естественного потенциала. Преимущество метода – легкая возможность компьютеризации.