GEOPHYSICAL TRANSACTIONS 1988
Vol. 34. No. 4. pp. 263–282

A STUDY OF THE VARIATION OF TIDAL NUMBERS WITH EARTH STRUCTURE

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Studies in earlier works on the determination of Love numbers were based on specific Earth models. This paper, however, aims at the systematic investigation of Love-number variations as a function of varying certain elements of the Earth model. We also examine the possible regional variations of Love numbers and their combinations in the case of a 3D Earth model based on seismology. From these model investigations we concluded that the difference between observation results and the gravity Love-number combination theoretically determined for the PREM cannot be explained by lateral inhomogeneities of the 3D model.

Keywords: Earth tides, Earth models, Love numbers, radial inhomogeneity, lateral inhomogeneity

1. Brief historical outline and our present conception about the interior of the Earth

In the course of the 4 years between the XVIIIth and XIXth congress of the International Union of Geodesy and Geophysics (held in Hamburg and in Vancouver, respectively), a fundamental change has taken place in our conception about the interior of the Earth.

In order to only approximately demonstrate this development it is worthwhile glancing at the improvement of the conception formed by science on the interior of our planet. 1987 was the 300th anniversary of one of the greatest scientific works ever published, Newton's Philosophiae Naturalis Principia Matematica. This was a turning point in the history of science. It is evident that preceding this time—in ignorance of the law of gravity—one cannot speak about any scientifically grounded idea relating to the interior of the Earth. After Newton, one had to wait for more than 50 years till it could be stated, on the basis of Bouguer's Chimborazo experiment in 1738, that the surface rocks are substantially less dense than the Earth generally i.e. density is increasing from the surface towards the centre of our planet. This was actually the first step towards the understanding of our planet. The real mean density values were obtained only considerably later on the basis of the experiments carried out on the Schiehallion hill and in its surroundings in Scotland in 1774 and of Cavendish's laboratory measurements of 1798.

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Manuscript received: 8 March, 1988

Radial density distribution was given for the first time by the theories of Legendre (1793) and Laplace (1825) based on Clairaut's formula (1743) which we now know was based on completely unrealistic assumptions. The core of the Earth appears only in Radou's (1855) and Wiechert's (1897) models.

From the beginning of the XXth century the development and application of seismology in the research of the structure of our planet enabled us to form a detailed conception about the interior of the Earth. Applying Williamson's and Adam's theory set up in 1923 and the seismic data accumulated so far, Bullen created his first model—the A model—in 1936. In the same year Lehmann discovered the solid inner core. Oldham had already proved in 1906 that the outer core was fluid. Bullen's A model together with Gutenberg's velocity data, the so called GBA model, enabled the forming of a rather good overall conception on the structure of the planet that acquited itself well until the early 60s, when the first successful and good quality records of free oscillations afforded possibility of checking the model created earlier. On the basis of free oscillations and the rapidly accumulating seismic data base, a great number of new Earth models appeared in the late 60s, which in fact gave a similar picture of the interior of the Earth as the Bullen model. This new wave of the radially symmetric 1D models can be divided into two main groups:

- optimum models made by using all possible data e.g. model B 497 [DZIEWONSKI–GILBERT 1972] or model B1 of JORDAN and ANDERSON [1974];
- reference models from which in addition to the expectations concerning good approximation of the observations and minimum deviation from the optimum models, primarily simple handling is required. Up to now there has been no final generally accepted reference model, but DZIEWONSKI and ANDERSON'S [1981] Preliminary Reference Earth Model (PREM) is widely used.

Thus we arrived—in our historical review—to 1983 and to the year of the UGGI congress in Hamburg. In the course of the 296 years from the appearance of Newton's Principia, planetary geophysics had reached the stage of having a reliable picture on the radial distribution of density and elastic parameters. Naturally a model like this leaves several questions without answers. If the internal features of our planet had only spherical symmetry, the Earth would be completely lifeless from the viewpoint of tectonics. As it is not so, for the investigation of tectonic and even shorter period effects, the lateral variations of the physical parameters need to be known. Geophysicists have been seeking after this for a long time but a real break-through could be achieved only now, when in the years following the Hamburg UGGI conference an ever increasing number of publications has appeared describing 3D planetary structures instead of one-dimensional models supposing only radial inhomogeneity. The study of lateral inhomogeneities in the interior of the Earth gathered a decisive impetus from two sources, (i) seismic networks set up in the 70s, yielding digital data, and (ii) the appearance of big computers in the early 80s that are able to handle simultaneously the enormous data base. These two facts explain how, between 1983 and 1987, 3D Earth research became a central topic. Compared to the past we have today substantially improved picture about the inhomogeneities in both the mantle and the core. The lateral inhomogeneities of the upper mantle were investigated in detail by WOODHOUSE and DZIEWONSKI [1984]. In the upper-most 50 km of the upper mantle, lateral inhomogeneities of ±8% appear, whereas from 250 km on only those of ±2.5% and in the immediate vicinity of the transition zone velocity anomalies detectable perpendicularly to the radious amount to 2%. The detected inhomogeneities roughly correspond to the extent of radial velocity changes. In the PREM model e.g. crossing the Mohorovičić discontinuity, the velocity contrast is 15% whereas at depths of 200 and 670 km it is 6 and 7%, respectively.

The velocity anomalies of the upper mantle are connected to the surface elements of global tectonics. This is true primarily for the mantle up to a depth of 250 km, where the most substantial heterogeneities can be found. The roots of the continental shields can be characterized by positive velocity anomalies of 4% and they penetrate down to a depth of 200 km. From here the extent of the velocity anomaly gradually decreases and fully disappeares in the transition zone between 400 and 670 km. The mid-oceanic ridges and the subduction zones of the western part of the Pacific Ocean can be characterized by negative velocity-anomalies traceable up to 350 km depth. On the basis of DZIEWONSKI's model [1984], the 3D model of the lower mantle presents the following picture: in the vicinity of the transition zone and in the D'' layer covering the core–mantle boundary, the lateral velocity anomalies reach 3%, but in the greater part of the lower mantle these do not exceed 1% i.e. laterally the lower mantle is substantially more homogeneous than the upper mantle. There appears to be no connection between velocity anomalies in the lower mantle and surface tectonics. For understanding the processes within the mantle, it is rather important to investigate the nature of transition zone C. It is a question to be decided whether the definite seismic discontinuity at 670 km is a mineralogical phase boundary through which the material of the mantle can flow or if it is a boundary between different materials not allowing such penetration. The question can be solved by investigating the velocity anomalies at both sides of the 670 km discontinuity as follows: If the anomalies on both sides are similar, then the first assumption is more probable. Otherwise one should deduce that the composition of the lower- and upper mantle differs. DZIEWONSKI and WOODHOUSE [1987] compared the lateral distribution of these velocity anomalies and found them, apart from a few exceptions (N-Siberia and the middle part of the Pacific Ocean) to be similar. CREAGER and JORDAN [1986] also proved that there are anomalies crossing the 670 km discontinuity, i.e. transition zone C separating the upper and lower mantle is probably of a phase boundary character and as such it does not hinder material flow. Lateral velocity variations throughout the Earth mantle can be explained by temperature, rheological features and density distribution. Studying the 3D model of the upper mantle shows that positive velocity anomalies can be connected to low temperatures and vice versa (i.e. negative anomalies relate to areas of higher temperatures) [DZIEWONSKI–WOODHOUSE 1987].

Interpreting the anomalies of the lower mantle seems to be a more difficult task. There is an opinion that the reason for geoid undulations can be found in the lower mantle [Hager et al. 1985], although several observations show that horizontal variations of seismic velocities connected to the geoid anomalies can be detected in the Earth's crust and in the uppermost part of the mantle [Masters et al. 1982, Stark et al. 1983].

In the interior of the Earth the most drastic changes of physical parameters can be observed at the core–mantle boundary. The anomalies of this discontinuity amount to ± 8 km, as compared to the regular hydrostatic surface [Creager–Jordan 1986, Morelli et al. 1986]. The topography of the core–mantle boundary shows that the shape of the Earth's core cannot be considered as a hydrostatic surface in the strict sense of the word. Stevenson [1987] showed, at the same time, that—with highly good approximation—the exterior liquid part of the core is laterally homogeneous.

2. Dependence of the Love numbers and their combinations on the Earth's structure

The new 3D model on the Earth's interior is based almost exclusively on seismic results. Naturally it would be good if the conception formed on lateral inhomogeneity could be supported by other, independent observations. The distribution of Love numbers and their combinations obtained from Earth tide observations is one method for such investigations. In this paper we wish first of all to clarify the connection of Love numbers and their combinations with the structure of our planet, and to what extent they contribute to the making of our picture on lateral inhomogeneity more complete.

Theoretical Love numbers (*h*, *k* and *l*) have already been studied for different spherically symmetrical Earth models by many authors [Takeuchi 1950, Molodensky 1953, 1961, Altermann–Jarosch–Pekeris 1959, Longman 1962, 1966, Alsop–Kuo 1964, Kuo–Erwing 1966, Farell 1972, 1973, Varga 1974, Denis 1974, 1979, Wilhelm 1978]. The most important result of these calculations is that the Earth tide varies only to a small extent, provided that the structure of the mantle in the radially symmetric Earth models is only varied between realistic limits determined by seismology.

This paper aims—differently from earlier investigations—at the systematic study of Love-number variations upon changing certain elements of the Earth model. The method is to vary the physical parameters describing the PREM model in the Earth mantle or in some of its spherical layers, and to study the effects on Love numbers. Our calculations are based on Molodenskii's inhomogeneous differential equation system [1953, 1961], which was solved by the fourth order Runge–Kutta method, choosing an integration step of $\Delta r/a = 0.001$ ($a = 6371$ km, r is the distance from the Earth's centre). On the basis of the latest seismic data, we supposed the core–mantle boundary (CMB) to be at the relative depth of $r/a=0.547$, in spite of the fact that in the PREM (the basis of our calculations) $r/a=0.546$.

We have investigated not only the dependence of h, k and l on the Earth structure, but we have also considered their combinations. The following combinations were determined:

— gravity tidal factor: $\delta = 1+h-3/2k$
— tilt factor: $\gamma = 1+k-h$
— vertical extensometric factor: $\Sigma_V = ah'+2h$
— horizontal, 2-dimensional deformation factor: $\Sigma_H = 2h-6l$
— dilatation factor: $\Theta = \Sigma_V+\Sigma_H$
(where h' is a derivative relative to the Earth's radius)

As compared to the PREM, wave velocities α and β as well as the compressibility modulus (κ) and the shear modulus (μ) were changed:

a) As a first step we performed the changes as compared to the PREM for the whole mantle within the rather extreme limits of ± 20%. *Table I/a* shows the effect of varying P-wave velocities (α) (with unvaried β and ϱ, where ϱ is the density function). It can be seen that changing the value of α has negligible effect on the values of k, h, δ and γ but substantially modifies the value of l and those of the three deformation factors (Σ_V, Σ_H, and Θ).

Varying the S-wave velocity (β)—also within the $\pm 20\%$ extreme limits—only affects the value of l to any significant extent (Table I/b). Tables I/c and I/d show the effect of varying the elastic constants κ and μ. The results of the above calculations are illustrated by graphs in *Figs. 1/a* through *h*. It can be seen that the variation of Love numbers and their combinations describing the Earth's reaction in the course of varying the extent of perturbation is on the one hand not linear and on the other, not symmetrical compared to the original case ($\varepsilon=0$) i.e. to the PREM. Results show that while the dependence of the gravity tidal factor—which can be most reliably recorded by traditional earth-tide observation methods—on the mantle structure is not remarkable, the extensometric components greatly depend on the mantle structure. Unfortunately these can be observed with less accuracy—primarily due to calibration problems—and thus the lateral earth tide variation cannot be effectively examined with them.

b) How Love numbers and their combinations depend on the perturbation of the wave velocities at different depths is an interesting question. The results of an investigation into this can be seen in *Table II*. For the investigation a spherical layer of 0.05 relative thickness (~ 319 km) located in various depths was assumed. Tables II/a–d show the position of the upper boundary of the spherical layer. Thus the first and uppermost layer can be found between the depth limits of 1.00–0.95, whereas the lowest layer characterized by a relative depth of 0.6, practically lies on the core–mantle boundary. The respective average depths are as follows: 159, 478, 796, 1115, 1433, 1752, 2071, 2389 and 2708 km. The velocities (α and β) and the elastic constants (κ and μ) were uniformly changed by 10%. Varying the P-wave velocity (Table II/a), greater effects were again obtained for the deformation factors than for Love numbers

a)

ε	Δk, %	Δh, %	Δl, %	$\Delta\delta$, %	$\Delta\gamma$, %	$\Delta\theta$, %	$\Delta\Sigma_H$, %	$\Delta\Sigma_V$, %
−0.20	8.48	28.95	−64.59	12.15	−22.25	79.38	94.48	68.50
−0.15	4.43	14.90	−32.71	6.22	−11.42	40.87	48.14	35.64
−0.10	2.31	7.58	−16.30	3.14	−5.86	20.80	24.31	18.28
−0.05	0.96	3.07	−6.45	1.26	−2.33	8.42	9.78	7.44
+0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
+0.05	−0.31	−2.22	4.69	−0.91	1.68	−6.10	−7.02	−5.43
+0.10	−1.22	−3.89	8.20	−1.60	2.96	−10.71	−12.31	−9.56
+0.15	−1.64	−5.20	10.90	−2.14	3.94	−14.32	−17.41	−12.81
+0.20	−2.00	−6.26	12.90	−2.56	4.74	−17.21	−19.30	−15.43

b)

ε	Δk, %	Δh, %	Δl, %	$\Delta\delta$, %	$\Delta\gamma$, %	$\Delta\theta$, %	$\Delta\Sigma_H$, %	$\Delta\Sigma_V$, %
−0.20	26.90	22.38	50.88	1.17	−8.03	6.00	2.46	8.55
−0.15	18.67	15.21	36.34	0.64	−5.28	3.51	1.24	5.14
−0.10	12.27	9.90	23.56	0.36	−3.38	1.64	0.34	2.58
−0.05	5.85	4.63	11.49	0.12	−1.54	0.45	−0.13	0.86
+0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
+0.05	−5.30	−3.97	−10.90	0.00	1.19	0.41	0.89	0.06
+0.10	−10.00	−7.27	−21.57	0.16	2.00	1.85	2.78	1.17
+0.15	−14.01	−9.43	−32.36	0.59	2.16	4.61	6.03	3.58
+0.20	−18.09	−11.46	−43.73	1.15	2.16	9.18	11.20	7.72

c)

ε	Δk, %	Δh, %	Δl, %	$\Delta\delta$, %	$\Delta\gamma$, %	$\Delta\theta$, %	$\Delta\Sigma_H$, %	$\Delta\Sigma_V$, %
−0.20	1.22	3.89	−8.21	1.61	−2.96	10.28	10.28	8.74
−0.15	0.84	2.74	−5.74	1.13	−2.07	7.21	8.69	6.15
−0.10	0.55	1.71	−3.52	0.70	−1.30	4.52	5.43	3.86
−0.05	0.25	0.80	−1.64	0.33	−0.61	2.13	2.55	1.82
+0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
+0.05	−0.22	−0.70	1.52	−0.29	0.55	−1.91	−2.28	−1.64
+0.10	−0.42	−1.37	2.93	−0.56	1.04	−3.64	−4.34	−3.14
+0.15	−0.61	−1.94	4.22	−0.81	1.49	−5.21	−6.20	−4.50
+0.20	−0.77	−2.49	5.28	−1.02	1.88	−6.64	−7.89	−5.74

d)

ε	Δk, %	Δh, %	Δl, %	$\Delta\delta$, %	$\Delta\gamma$, %	$\Delta\theta$, %	$\Delta\Sigma_H$, %	$\Delta\Sigma_V$, %
−0.20	13.01	10.52	24.97	0.40	−3.61	1.82	0.42	2.83
−0.15	9.13	7.24	18.05	0.21	−2.41	1.03	0.07	1.73
−0.10	6.01	4.76	11.72	0.13	−1.58	0.47	−0.12	0.09
−0.05	2.89	2.27	5.74	0.05	−0.74	0.13	−0.15	0.33
+0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
+0.05	−2.60	−1.96	−5.39	−0.01	0.59	0.08	0.33	−0.09
+0.10	−5.17	−3.89	−10.67	0.00	1.16	0.38	0.86	0.04
+0.15	−7.33	−5.34	−15.59	0.09	1.51	0.91	1.60	0.41
+0.20	−9.67	−7.00	−20.63	0.15	1.94	1.67	2.56	1.02

(h, k and l) which reached maximal mean values when the anomaly was in the third layer from the surface. There is a maximum change in δ and γ Love–number combinations for the second layer (at the average depth of 478 km).

Perturbating the shear-wave velocity the greatest changes for k, h, γ and Σ_H can be observed when the layer is on the core–mantle boundary (Table II/b). The dilatation (Θ) and the vertical extensometric component (Σ_V) are sensitive primarily to the variations of the near-surface velocities. l is the most sensitive to the varying of β when it is done in the 3rd and 4th layer from the surface. The gravity factor is equally sensitive to such perturbations on the surface and on the core–mantle boundary, but with the opposite sign. Perturbating the elastic constants it can be stated that:

— varying the compressibility modulus (κ) by 10% (Table II/c) similar influence is obtained when it is carried out in the 1st to the fourth layer. Perturbation in the lower part of the mantle has practically no effect;

— varying the shear modulus (μ) (Table II/d)—similarly to varying β—the situation is more complicated. The k, h and γ values depend primarily on the changes carried out in the lower mantle. l and the horizontal extensometric component (Σ_H) are the most sensitive to the variations of κ (and β) in the 0.95–0.80 relative depth range. The other two extensometric components (Θ, Σ_V) depend primarily on surface perturbations. The gravity tidal factor is the less dependent on μ and β.

⇦ *Table I.* Love number variations versus varying elastic parameters α, β, κ and μ within the limits of ±20%. ε is the extent of variation, $\varepsilon = 0$ corresponds to the PREM

a) Variations of P-wave velocity: $\alpha = \alpha_0(1+\varepsilon)$
b) Variations of S-wave velocity: $\beta = \beta_0(1+\varepsilon)$
c) Variations of compressibility modulus: $\kappa = \kappa_0(1+\varepsilon)$
d) Variations of shear modulus: $\mu = \mu_0(1+\varepsilon)$

⇦ *I. táblázat.* A Love számok változásai az α, β, κ és μ rugalmas paraméterek ±20% határok közötti változtatásai függvényében. ε a változtatás mértéke. $\varepsilon = 0$ a PREM esetének felel meg.

a) a longitudinális hullámsebesség változtatásai: $\alpha = \alpha_0(1+\varepsilon)$
b) a transzverzális hullámsebesség változtatásai: $\beta = \beta_0(1+\varepsilon)$
c) az összenyomhatósági modulus értékének változtatásai: $\kappa = \kappa_0(1+\varepsilon)$
d) a nyírási modulus értékének változtatásai: $\mu = \mu_0(1+\varepsilon)$

⇦ *Таблица I.* Изменения чисел Лава в зависимости от изменений упругих параметров α, β, κ и μ в пределах ±20%—степень изменений; $\varepsilon = 0$ соответствует случаю PREM

a) Изменение скоростей продольных волн: $\alpha = \alpha_0(1+\varepsilon)$
b) Изменение скоростей поперечных волн: $\beta = \beta_0(1+\varepsilon)$
c) Изменение модуля сжимаемости: $\kappa = \kappa_0(1+\varepsilon)$
d) Изменение модуля скалывания: $\mu = \mu(1+\varepsilon)$

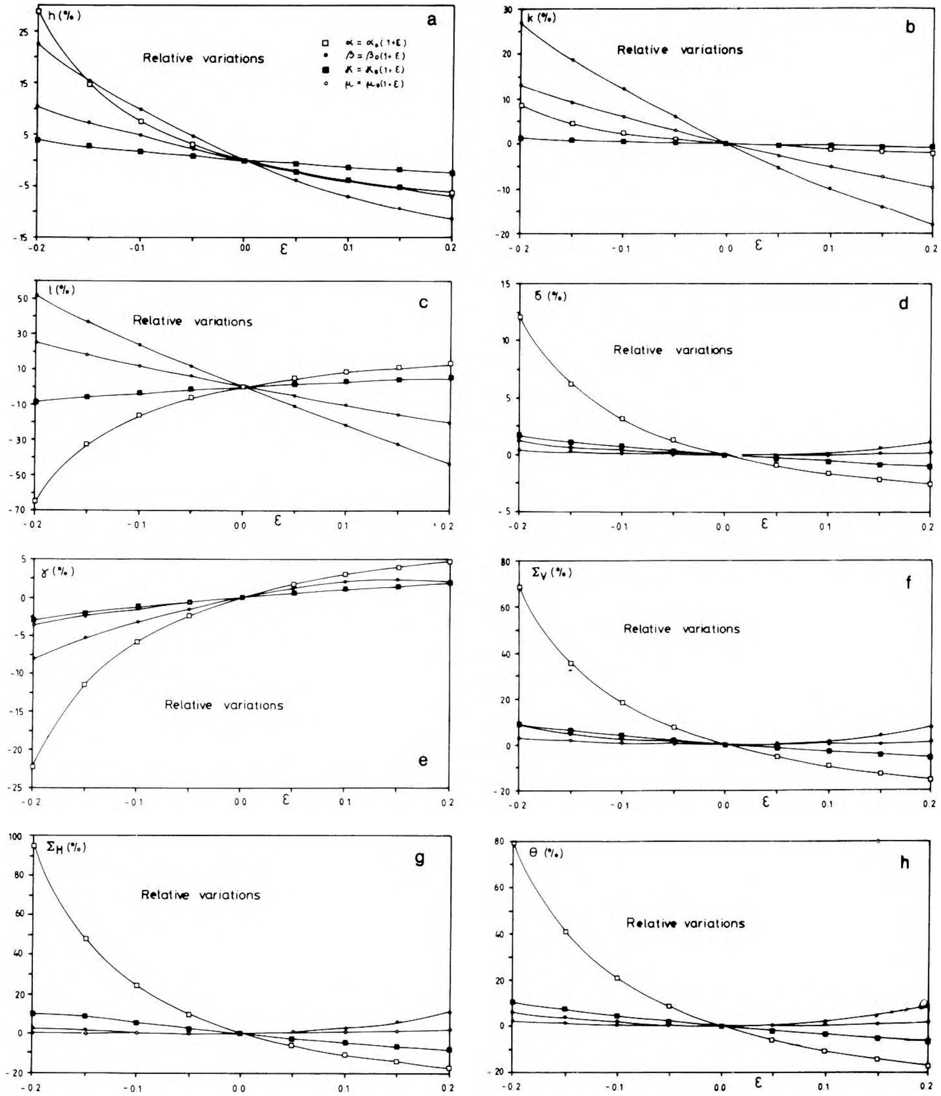

a
h (%)
Relative variations
□ α = α₀(1+ε)
• β = β₀(1+ε)
■ κ = κ₀(1+ε)
○ μ = μ₀(1+ε)
25
15
5
-5
-15
-0.2
-0.1
0.0
ε
0.1
0.2
b
k (%)
Relative variations
30
20
10
0
-10
-20
c
l (%)
Relative variations
50
30
10
-10
-30
-50
-70
d
δ (%)
Relative variations
15
10
5
0
-5
e
γ (%)
Relative variations
5
0
-5
-10
-15
-20
-25
f
Σ_V (%)
Relative variations
80
60
40
20
0
-20
g
Σ_H (%)
Relative variations
100
80
60
40
20
0
-20
h
θ (%)
Relative variations
80
60
40
20
0
-20

Fig. 1. Variations of the second order Love numbers on the surface caused by varying α, β, κ and μ in the Earth's mantle, within the limits of ±20%. $\varepsilon=0$ corresponds to the case of PREM

a) Variation of Love number h
b) Variation of Love number k
c) Variation of Love number l
d) Variation of Love number combination $\delta = 1+h-3/2k$
e) Variation of Love number combination $\gamma = 1+k-h$
f) Variation of Love number combination $\Sigma_V = ah'+2h$
g) Variation of Love number combination $\Sigma_H = 2h-6l$
h) Variations of Love number combination $\Theta = \Sigma_V+\Sigma_H = ah'+4h-6l$

1. ábra. A másodfokú Love számok változásai a felszínen α, β, κ és μ ±20% határok közötti változtatásának hatására a földköpenyben. $\varepsilon=0$ a PREM esetének felel meg

a) A h Love szám változásai
b) A k Love szám változásai
c) Az l Love szám változásai
d) A $\delta = 1+h-3/2k$ kombináció változásai
e) A $\gamma = 1+k-h$ kombináció változásai
f) A $\Sigma_V = ah'+2h$ kombináció változásai
g) A $\Sigma_H = 2h-6l$ kombináció változásai
h) A $\Theta = \Sigma_V+\Sigma_H = ah'+4h-6l$ kombináció változásai

Рис. 1. Изменения чисел Лава второго порядка на поверхности, обусловленные изменениями α, β, κ и μ в мантии в пределах ±20%. $\varepsilon=0$ соответствует случаю PREM

a) Изменения числа h Лава
b) Изменения числа k Лава
c) Изменения числа l Лава
d) Изменения комбинации $\delta = 1+h-3/2k$
e) Изменения комбинации $\gamma = 1+k-h$
f) Изменения комбинации $\Sigma_V = ah'+2h$
g) Изменения комбинации $\Sigma_H = 2h-6l$
h) Изменения комбинации $\Theta = \Sigma_V+\Sigma_H = ah'+4h-6l$

c) Subsequently, we also investigated the relation between the Love-number variations and the Earth core structure. This investigation was not extended to the extensometric components Σ_H, Σ_V and Θ, since these hardly depend on the structural variations of the lower mantle either (see Table II). Concerning the core, we tried to clarify the following problems:

— dependence of the Love numbers on the density distribution in the core
— dependence of the Love numbers on the density contrast at the CMB
— to what extent Love numbers depend on the uncertainty of the CMB position
— to what extent Love numbers depend on the outer core being a potentially not-ideal fluid.

Table II. Love number variations in relation to the PREM model versus increasing the elastic parameters by 10% in spherical layers of 5% thickness of the Earth radius (a) and located at different relative depth (r_{up}/a)

a) Increasing P-wave velocities (α) by 10%
b) Increasing S-wave velocities (β) by 10%
c) Increasing compressibility modulus (κ) by 10%
d) Increasing shear modulus (μ) by 10%.

⇨

II. táblázat. A Love számok variációi a PREM modellhez viszonyítva a rugalmas paraméterek értékének 10%-kal történő növelésével a gömbhéjak belsejében. A héjak a földsurág 5%-át kitevő vastagságúak különböző relatív mélységeknél (r_{up}/a)

a) a longitudinális hullámsebesség 10%-kal történő növelése
b) a transzverzális hullámsebesség 10%-kal történő növelése
c) az összenyomhatósági modulus értékének 10%-kal történő növelése
d) a nyírási modulus értékének 10%-kal történő növelése

⇨

Таблица II. Изменения чисел Лава по сравнению со случаем PREM при увеличении значений упругих параметров на 10% внутри сферических оболочек, мощность которых составляет 5% от радиуса Земли при различных относительных глубинах (r_{up}/a)

a) Увеличение скоростей продольных волн на 10%
b) Увеличение скоростей поперечных волн на 10%
c) Увеличение модуля сжимаемости на 10%
d) Увеличение модуля скалывания на 10%

⇨

a)

r_{up}/a	Δk, %	Δh, %	Δl, %	$\Delta\delta$, %	$\Delta\gamma$, %	$\Delta\Theta$, %	$\Delta\Sigma_H$, %	$\Delta\Sigma_V$, %
1.00	−0.19	−0.92	1.64	−0.42	0.74	−6.30	−2.69	−8.89
0.95	−0.29	−1.01	1.99	−0.43	0.80	−1.62	−3.11	−0.55
0.90	−0.32	−0.95	2.23	−0.39	0.72	−1.59	−3.16	−0.47
0.85	−0.32	−0.71	1.76	−0.29	0.54	−1.22	−2.45	−0.33
0.80	−0.16	−0.47	1.29	−0.19	0.35	−0.80	−1.64	−0.20
0.75	−0.06	−0.24	0.82	−0.10	0.19	−0.45	−0.93	−0.10
0.70	0.00	−0.08	0.35	−0.04	0.07	−0.17	−0.36	−0.03
0.65	0.00	0.00	0.00	0.00	0.00	−0.02	−0.06	0.00
0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

b)

r_{up}/a	Δk, %	Δh, %	Δl, %	$\Delta\delta$, %	$\Delta\gamma$, %	$\Delta\Theta$, %	$\Delta\Sigma_H$, %	$\Delta\Sigma_V$, %
1.00	−0.09	0.55	−1.17	0.33	−0.54	7.07	1.81	10.85
0.95	−0.19	0.41	−2.81	0.30	−0.46	1.07	2.68	−0.09
0.90	−0.45	0.11	−3.63	0.24	−0.30	0.86	2.72	−0.48
0.85	−0.80	−0.37	−3.63	0.12	−0.03	0.31	1.96	−0.89
0.80	−1.12	−0.84	−3.28	0.01	0.23	−0.32	0.95	−1.24
0.75	−1.48	−1.27	−2.81	−0.09	0.48	−0.95	−0.15	−1.52
0.70	−1.80	−1.67	−2.23	−0.18	0.71	−1.57	−1.28	−1.77
0.65	−2.09	−2.06	−1.64	−0.26	0.91	−2.16	−2.87	−2.00
0.60	−2.57	−2.62	−1.17	−0.37	1.20	−2.90	−3.58	−2.41

c)

r_{up}/a	Δk, %	Δh, %	Δl, %	$\Delta\delta$, %	$\Delta\gamma$, %	$\Delta\Theta$, %	$\Delta\Sigma_H$, %	$\Delta\Sigma_V$, %
1.00	−0.06	−0.29	0.59	−0.14	0.23	−1.93	−0.87	−2.69
0.95	−0.10	−0.32	0.70	−0.14	0.26	−0.53	−1.02	−0.18
0.90	−0.10	−0.31	0.70	−0.12	0.23	−0.51	−1.01	−0.15
0.85	−0.10	−0.23	0.59	−0.10	0.17	−0.39	−0.79	−0.11
0.80	−0.06	−0.14	0.47	−0.06	0.12	−0.26	−0.54	−0.07
0.75	−0.03	−0.08	0.23	−0.03	0.06	−0.15	−0.31	−0.03
0.70	−0.03	−0.03	0.12	−0.02	0.03	−0.05	−0.12	−0.01
0.65	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

d)

r_{up}/a	Δk, %	Δh, %	Δl, %	$\Delta\delta$, %	$\Delta\gamma$, %	$\Delta\Theta$, %	$\Delta\Sigma_H$, %	$\Delta\Sigma_V$, %
1.00	−0.04	0.24	−0.46	−0.15	−0.23	3.30	0.79	5.11
0.95	−0.10	0.18	−1.29	−0.14	−0.20	0.47	1.22	−0.06
0.90	−0.22	0.02	−1.64	−0.10	−0.13	0.37	1.23	−0.25
0.85	−0.38	−0.19	−1.76	−0.05	0.00	0.11	0.89	−0.45
0.80	−0.55	−0.42	−1.64	0.00	0.13	−0.18	0.43	−0.61
0.75	−0.70	−0.62	−1.29	−0.04	0.23	−0.47	−0.09	−0.75
0.70	−0.87	−0.82	−1.06	−0.09	0.35	−0.77	−0.62	−0.87
0.65	−1.03	−1.00	−0.82	−0.13	0.43	−1.05	−1.14	−0.98
0.60	−1.25	−1.27	−0.59	−0.18	0.58	−1.41	−1.73	−1.18

In *Table III* deviations of the Love number combinations (from the PREM model) for two different core models with theoretically assumed, completely invalid density distributions are presented. Although there is a considerable difference between the homogeneous hydrostatic core and the real case, it is hardly reflected in the δ and γ values (i.e. the density distribution in the core does not remarkably affect Love-number combinations). On the contrary, the core density at the core–mantle boundary affects the h, k, l, δ and γ values *(Fig. 2)*. In reality, however, this density may vary between rather narrow limits 9.9–10.2 g/cm^3 [PRESS 1970]. Accordingly, the variations for the investigated quantities may be as follows:

$$\Delta k = 0.67\%,\ \Delta h = 0.40\%,\ \Delta l = 0.43\%,\ \Delta\delta = 0.22\%,\ \text{and}\ \Delta\gamma = 0.18\%.$$

Since the variations given by PRESS relate to the maximum possible density variation, it can be concluded that the effect of the core on the surface is not remarkable in this respect.

Although the average position of the CMB is known rather precisely undulations up to 10 km may be envisaged. The influence of relative depth of the CMB on Love numbers can be seen in *Fig. 3*. The variations belonging to a 10 km undulation are not very large:

$$\Delta k = 0.43\%,\ \Delta h = 0.21\%,\ \Delta l = 0.10\%,\ \Delta\delta = 0.06\%\ \text{and}\ \Delta\gamma = 0.04\%.$$

Core model	Δk(%)	Δh(%)	Δl(%)	$\Delta\delta$(%)	$\Delta\gamma$(%)
Constant core density	−3.01	−1.52	4.76	0.37	0.03
Hydrostatic density distribution	−0.46	−0.40	−0.24	−0.02	0.13

Table III. Dependence of Love numbers (and their combinations) on the selected core model, (in relation to the respective PREM values)

III. táblázat. A Love számok és kombinációik változásai a választott mag-modell függvényében (a megfelelő PREM értékekhez viszonyítva)

Таблица III. Изменения чисел Лава и их комбинаций как функция выбранной модели Земли (отнесенные к соответствующим значениям в случае PREM)

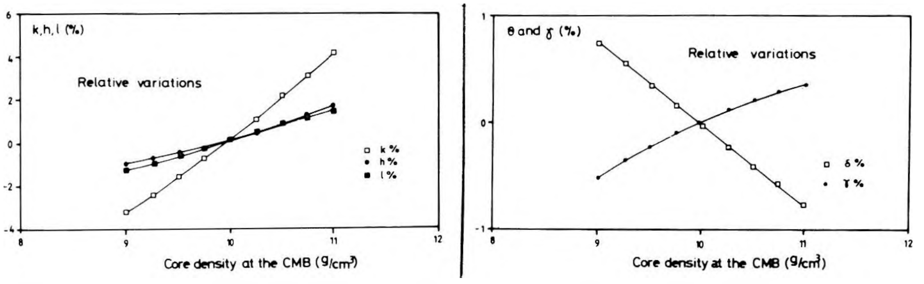


Fig. 2. Effect of the core density at the core–mantle boundary on Love numbers and their combinations in relation to the core density in the PREM (9.9037 g/cm^3)

2. ábra. A mag sűrűségének hatása a Love-számokra és kombinációikra a mag–köpeny határon a PREM mag sűrűségének értékéhez viszonyítva (9,9037 g/cm^3)

Рис. 2. Влияние изменений плотности земного ядра по сравнению с плотностью ядра в PREM (9,9037 г/см^3) на числа Лава и их комбинации на границе мантии с ядром

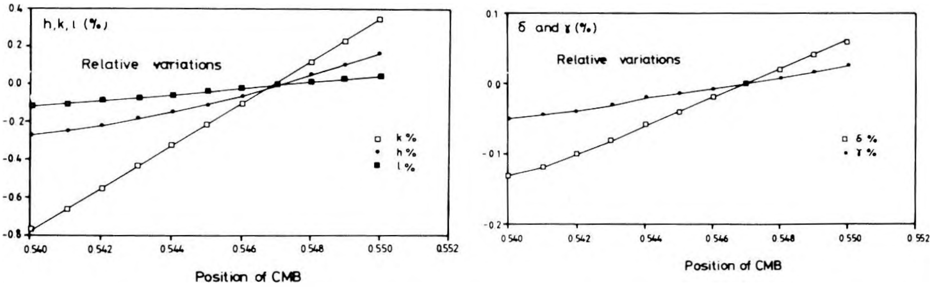


Fig. 3. Effect of the relative depth of the core–mantle boundary on Love numbers and their combinations in relation to the value (0.547) accepted in PREM

3. ábra. A mag–köpeny határ relatív mélységének hatása a Love-számokra és kombinációikra a PREM-ben elfogadott értékhez (0,547) viszonyítva

Рис. 3. Влияние изменений относительной глубины залегания границы мантии с ядром по сравнению с плотностью ядра в PREM (0,547) на числа Лава и их комбинации

The problem of the outer core being an ideal fluid or not has been an interesting question for geophysics for a long time. Therefore such PREM variations were calculated which assume a certain extent of rigidity in the outer core *(Fig. 4)*. Earlier it was assumed that the shear modulus of the outer can even be as high as 10^9 N/m^2 [Sato–Espinoza 1967, Ibrahim 1973]. Nowadays Kuo, Zhang and Chu [1986] showed that this value should be less than 10^8 N/m^2. If this is so, then the possible deviation of the outer core from the ideal fluid state does not influence Love numbers.

Summerizing what has been said concerning Love numbers and their combinations, one can say that:

- — the possible uncertainties of the core model do not influence the calculated values of Love numbers to any great extent;
- — the lateral variations of the 3D model in the upper mantle are considerable, whereas in the lower mantle they are not. If this is so, then the regional variations of the earth tide observations should reflect the lateral inhomogeneities of the upper mantle.

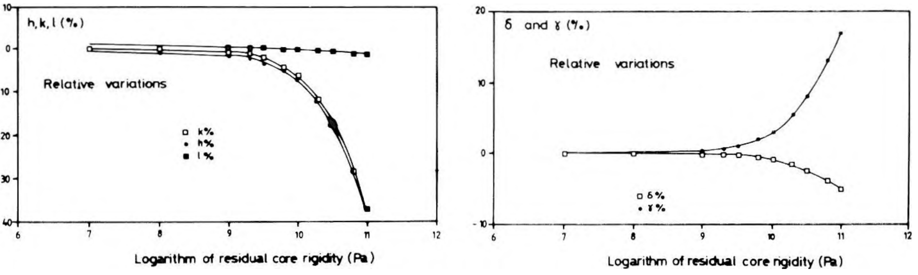


Fig. 4. Effect of the rigidity of the outer core on the Love numbers and their combinations compared to the ideal fluid state ($\mu = 0$)

4. ábra. A külső mag szilárdságának hatása a Love-számokra és kombinációikra. A változások viszonyítási alapja az ideális cseppfolyós állapot ($\mu = 0$)

Рис. 4. Влияние изменений твердости внешнего ядра по сравнению с идеально жидким состоянием ($\mu = 0$) на числа Лава и их комбинации

3. Possible variations of Love numbers and their combinations on the Earth's surface

The applicability of earth tide research for the investigation of lateral inhomogeneities primarily depends on whether or not these result in measurable variations of Love numbers and their combinations on the Earth surface. MOLODENSKII and KRAMER [1980] studied different lateral inhomogeneity models. The basic feature of their models is that the elastic wave velocities under the oceans are 5% lower than under the continents in the whole mantle or in certain depth ranges. The effects are determined in each case for the gravity tidal factor $\delta = 1+h-3/2k$.

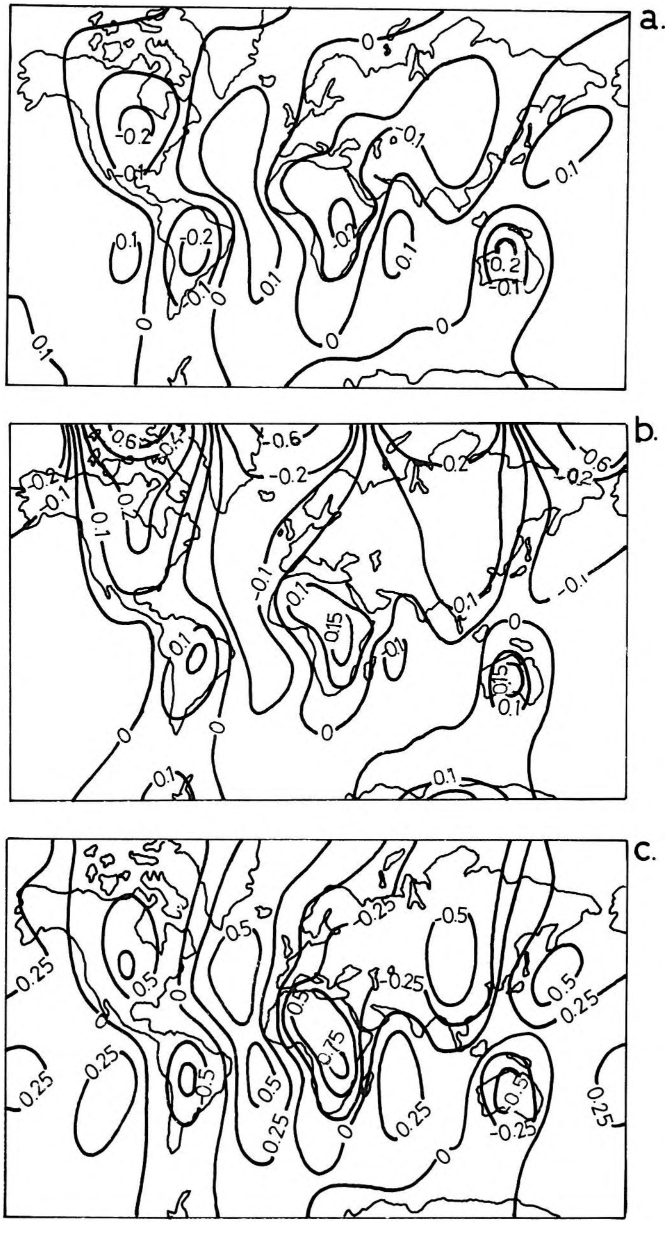
In the first model *(Fig. 5/a)* *P*-wave velocities (α) are 5% higher under the continents up to 300 km from the surface whereas in the second case this 5% increase in velocity characterises the *S*-waves (β) *(Fig. 5/b)*. In the third model the lateral inhomogeneity of the *P*-wave penetrates the whole mantle *(Fig. 5/c)*. For the first model the maximum change in δ is 0.3% whereas for the second it is 1.2% and for the third one it is 1.25%.

The common features of the results are as follows:

- the calculated δ anomalies follow the boundaries of the lateral inhomogeneities;
- the amplitudes of the anomalies do not depend on the size of the source (e.g. the amplitude of the anomaly for Australia equals that obtained for Eurasia).

From our investigations, in the course of which first the values of α and then that of β were changed by 5% in the upper-most 300 km in relation to the PREM, variations in δ of 0.2% and 0.3% were obtained. Changing α by 5% in the whole mantle resulted in a change of 1.0%. Comparing our results with MOLODENSKII–KRAMER's data for laterally inhomogeneous models, it can be stated that the magnitude of the regional Love-number variations caused by lateral inhomogeneities can be estimated by means of two radial models: one containing the modified value of the physical parameter as in a whole spherical layer, the other is the PREM. Naturally, in this way the regional anomalies cannot be described, however, the magnitudes of the possible anomalies will be estimated.

Starting from the above conclusions, the magnitude of δ variations expected on the basis of the 3D model prepared by DZIEWONSKI and WOODHOUSE can be determined. For this, however, it should be assumed that the Lamé constants are equal in the mantle, i.e. $\mu=\lambda$, and thus $\alpha = \sqrt{3}\,\beta$. As it is known, this assumption is true in the mantle to a rather good approximation. This assumption was necessary because the 3D model was based only on *S*-waves for the upper mantle whereas for the lower mantle only *P*-waves are used. Considering the above facts we tried to estimate the possible magnitudes of the regional variations of Love numbers and their combinations on the basis of the studies of WOODHOUSE and DZIEWONSKI [1984] as well as DZIEWONSKI [1984].


a.
0
-0.2
-0.1
0.1
-0.2
-0.1
0
0
0.1
-0.1
-0.2
0.1
0
0.1
0.2
-0.1
0
0.1
b.
0.6
0.4
-0.6
-0.2
0.2
0.6
0.2
-0.2
-0.1
0.1
0.2
-0.1
0.1
0.15
-0.1
0.1
0.1
0.15
0.1
0
0
0
0
0.1
0.1
-0.1
c.
0.5
-0.25
-0.5
-0.25
0.25
0.5
0.5
0.5
0.25
0.5
0.75
0.25
0.25
0.5
0.5
0.25
0.25
0
0
0.25
-0.5
-0.25
0.25

For this purpose a model was studied in which, in the upper mantle ($1.00 \geq r/a \geq 0.90$) α exceeds by 8% and β by 5% the PREM values *(Table IV)*, whereas in transition zone *C*, α and β exceed the PREM values by 3 and 2% respectively. In the lower mantle the seismic velocities are only 1% higher than those of the reference model, except in layer D'' at the core–mantle boundary. Here α exceeds by 3% and β by 2% the velocities accepted in the PREM. The deviation from the reference model (Table IV) is the least in the case of gravity tidal factor δ ($\Delta\delta = -0.72\%$) that can be best observed, whereas the greatest deviations were found in the extensometric components.

Definition of the model	Deviation of Love numbers and their combinations from PREM (in %)
Upper mantle	
$1.00 \geq r/a > 0.90$	$\Delta k = -1.80$
$\alpha = 1.08 \cdot \alpha_0$	$\Delta h = -2.67$
$\beta = 1.05 \cdot \beta_0$	$\Delta l = 0.82$
C layer	
$0.90 \geq r/a > 0.85$	$\Delta\delta = -0.72$
$\alpha = 1.03 \cdot \alpha_0$	$\Delta\gamma = 1.59$
$\beta = 1.02 \cdot \beta_0$	$\Delta\Theta = -4.99$
Lower mantle	
$0.85 \geq r/a > 0.60$	$\Delta\Sigma_H = -5.26$
$\alpha = 1.01 \cdot \alpha_0$	$\Delta\Sigma_V = -4.79$
$\beta = 1.01 \cdot \beta_0$	
D'' layer	
$0.60 \geq r/a > 0.55$	
$\alpha = 1.03 \cdot \alpha_0$	
$\beta = 1.02 \cdot \beta_0$	

Table IV. Variations of Love numbers and their combinations for the 3D model suggested by DZIEWONSKI and WOODHOUSE [1984] as well as DZIEWONSKI [1984]. (Reference basis: α_0 and β_0 velocities of PREM)

IV. táblázat. A Love-számok és kombinációik változásai a DZIEWONSKI és WOODHOUSE [1984] és DZIEWONSKI [1984] által javasolt 3D modellre. (Vonatkoztatási alap: PREM sebességek)

Таблица IV. Изменения чисел Лава и их комбинации в трехмерной модели, предложенной Дзевонским и Вудхаузом [DZIEWONSKI and WOODHOUSE 1984] и Дзевонским [DZIEWONSKI 1984] (основа для сравнения – скорости PREM)

Fig. 5. Effect of velocity inhomogeneity on gravity tidal factor δ
a) α is greater by 5% under the continents than under the oceans up to 300 km depth
⇦ b) β is greater by 5% under the continents than under the oceans up to 300 km depth
c) α is greater by 5% under the continents than under the oceans in the whole mantle

5. ábra. A sebesség inhomogenitásának hatása a földárapály paraméterre (δ)
a) α értéke 5%-kal nagyobb a kontinensek alatt, mint az óceánok alatt, 300 km mélységig
⇦ b) β értéke 5%-kal nagyobb a kontinensek alatt, mint a óceánok alatt, 300 km mélységig
c) α értéke 5%-kal nagyobb a kontinensek alatt, mint az óceán alatt, az egész köpenyben

Рис. 5. Влияние неоднородостей скоростей на параметр земных приливов (δ)
a) Значение α под континентами на 5% больше, нежели под океанами, до глубин 300 км
⇦ b) Значение β под континентами на 5% больше, нежели под океанами, до глубин 300 км
c) Значение α под континентами на 5% больше, нежели под океанами, на всю мантию

The gravity tidal factor obtained for the PREM model is $\delta = 1.1564$. This is in rather good agreement with the result of Dehant and Ducarme [1986] ($\delta = 1.1543$), which they obtained starting from Wahr's theory [1981]. Both results considerably differ, however, from the planetary mean value of the observation results $\delta = 1.161$ [Melchior 1977]. The reasons for this deviation need to be clarified both from the side of the theory and from that of observation techniques. This is one of the most important tasks of today's earth tide research.

Acknowledgement

P. Varga conducted his part of the research work as a Humboldt research fellow during his stay at the Geodetical Institute of Stuttgart University. Carlo Denis is grateful to the Fonds National de la Recherche Scientifique (Brussels, Belgium) for research funds which have enabled him to undertake this study.

REFERENCES

Alsop L. E., Kuo J. T. 1964: The characteristics of semi-diurnal earth tidal components for various Earth models. Ann. Geophys., **20**, 3, pp. 286–300

Alterman Z., Jarosch H., Pekeris C. L. 1959: Oscillation of the Earth. Proc., Roy. Soc. London, Ser. A, **252**, pp. 80–95

Creager K. C., Jordan T. H. 1986: Aspherical structure of the core–mantle boundary, from PKP travel times. Geophys. Res. Lett., **13**, 13, pp. 1497–1500

Dehant V., Ducarme B. 1987: Comparison between the theoretical and observed tidal gravimetric factors. Phys. Earth Plan. Int., **49**, 3–4, pp. 192–212

Denis C. 1974: Oscillations de configurations sphériques auto-gravitantes et applications á la Terre. Thése de doctorat, Université de Liége, 350 p.

Denis C. 1979: Static and dynamic effects in theoretical Love numbers. In: Bonatz M., Melchior P. (eds.) Proc. 8th Int. Symp. Earth Tides, Bonn, 1977. pp. 709–729

Dziewonski A. M., Gilbert F. 1972: Observations of normal modes from 84 recordings of the Alaskan earthquake of 1964 March 28. Geophys. J., **27**, 4, pp. 393–446

Dziewonski A. M., Anderson D. L. 1981: Preliminary reference Earth Model. Phys. Earth Plan. Int., **25**, 4, pp. 297–356

Dziewonski A. M. 1984: Mapping the lower mantle: determination of lateral heterogenity in *P* velocity up to degree and order 6. J. Geophys. Res., **89**, B 7, pp. 5929–5952

Dziewonski A. M., Woodhouse J. H. 1987: Global images of the Earth's interior. Science, **236**, pp. 761–797

Farrell W. E. 1972: Deformations of the Earth by surface loads. Rev. Geophys. Space Phys., **10**, 3, pp. 761–797

Farrell W. E. 1973: Earth tides, ocean tides and tidal loading. Phil. Trans. Roy. Soc., London, Ser. A, **274**, pp. 253–259

Hager B. H., Clayton R. W., Richards M. A., Comer R. P., Dziewonski A. M. 1985: Lower mantle heterogeneity, dynamic topography and the geoid. Nature, **313**, pp. 541–545

Ibrahim A. K. 1973: Evidences for a low velocity core–mantle transition zone. Phys. Earth Plan. Int., **7**, 2, pp. 187–198

Jordan T. H., Anderson D. L. 1974: Earth structure from free oscillations and travel times. Geophys. J., **36**, 2, pp. 411–459

Kuo J. T., Ewing M. 1966: Spatial variations of tidal gravity. In: Steinhart J. S., Smith T. J. (eds.) Earth beneath the continents. pp. 595–610

Kuo J. T., Zhang V., Chu Y. 1986: Time-domain total Earth tides. Proc. 10th Int. Symp. on Earth tides, Consejo Superior de Investigaciones Cientificas, Madrid, pp. 491–506

Longman I. M. 1962: A Green's function for determining the deformation of the Earth under surface mass loads. I. Theory. J. Geophys. Res., **67,** 2, pp. 845–850

Longman I. M. 1966: Computation of Love numbers and load deformation coefficients for a model Earth. Geophys. J. Roy. Astr. Soc. **11,** 1–2, pp. 133–137

Masters G., Jordan T. H., Silver P. G., Gilbert F. 1982: A spherical Earth structure from fundamental spheroidal mode data. Nature, **298,** pp. 609–611

Melchior P. 1977: Report on activities of the International Centre for Earth Tides. Proc. of the 8th Int. Symp. on the Earth Tides, Bonn, pp. 30–41

Molodenskii M. S. 1953: Elastic tides, free nutations, some questions concerning the internal structure of the Earth (in Russian). Trudy Geofis. Inst. Akad. Nauk S.S.S.R., Moscow, No. **19,** (146), pp. 3–42

Molodenskii M. S., Kramer M. V. 1961: Earth tides and nutations of the earth (in Russian). Izd. Akad. Nauk S.S.S.R., Moscow, 40 p.

Molodenskii M. S., Kramer M. V. 1980: Influence of big scale horizontal inhomogenities of the mantle and the tides of the earth. Fizika Zemli, **1,** pp. 3–20

Morelli A., Dziewonski A. M., Woodhouse J. H. 1986: Anisotropy of the inner core inferred from PKIKP travel times. Geophys. Res. Let., **13,** 13, pp. 1545–1548

Press F. 1970: Earth models consistent with geophysical data. Phys. Earth Plan. Int., **3,** pp. 3–22

Sato R., Espinosa A. F. 1967: Dissipation in the Earth's mantle and rigidity and viscosity in the Earth's core determined from waves multiply reflected from the mantle-core boundary. Bull., Seis. Soc. Am., **57,** 5, pp. 829–856

Stark M., Forsyth D. W. 1983: The geoid, small-scale convection, and differential travel time anomalies of shear waves in the central Indian ocean. J. Geophys. Res., **88,** Ser. B, 3, pp. 2273–2294

Stevenson D. J. 1987: Limits on lateral density and velocity variations in the Earth's outer core. Geophys. J. Roy. Astr. Soc., **88,** 1, pp. 311–319

Takeuchi M. 1950: On the earth tide of the compressible earth of variable density and elasticity. Trans. Am. Geophys. Un., **31,** 5, pp. 651–689

Varga P. 1974: Dependence of the Love numbers upon the inner structure of the Earth and comparison of theoretical models with results of measurements. PAGEOPH, **112,** pp. 777–785

Wahr J. M. 1981: Body tides on an elliptical, rotating, elastic and oceanless earth. Geophys. J. Roy. Astr. Soc., **64,** 3, pp. 677–703

Wilhelm H. 1978: Upper mantle structure and global earth tides. J. Geophys., **44,** 5, pp. 435–439

Woodhouse J. H., Dziewonski A. M. 1984. Mapping the upper mantle: three-dimensional modeling of Earth structure by inversion of seismic waveforms. J. Geophys. Res., **89,** Ser. B, 7, pp. 5953–5986

A FÖLDÁRAPÁLY PARAMÉTEREK FÖLDSZERKEZET OKOZTA LEHETSÉGES VÁLTOZÁSAI

VARGA Péter és Carlo DENIS

A Love számok meghatározásával foglalkozó korábbi munkák egyes konkrét földmodellekre alapozzák vizsgálataikat. Jelen dolgozat célja viszont: szisztematikusan vizsgálni, hogyan változnak a Love számok a Földmodell egyes elemeinek megváltoztatása függvényében. Megvizsgáljuk azt is, hogy a szeizmológiai alapokon nyugvó 3D földmodell esetében a Love számok és kombinációinak milyen regionális változásai lehetségesek.

ВОЗМОЖНЫЕ ИЗМЕНЕНИЯ ПАРАМЕТРОВ ЗЕМНЫХ ПРИЛИВОВ, СВЯЗАННЫЕ СО СТРУКТУРОЙ ЗЕМЛИ

Пётер ВАРГА и Карло ДЕНИС

В прежних работах по определению чисел Лава исследования базировались на каких-либо конкретных моделях Земли. Цель же настоящей статьи заключается в систематическом изучении изменений чисел Лава в зависимости от изменений тех или иных элементов модели Земли, а также региональных изменений чисел Лава и их комбинаций для трехмерной модели Земли, основывающейся на сейсмологических данных.