GEOPHYSICAL TRANSACTIONS 1988
Vol. 34. No. 1. pp. 59—81

STRESS-INDUCED ANISOTROPY IN ELASTIC MEDIA

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The linear Hooke's law for elastic media represents in general the first order approximation of a nonlinear stress–strain relation for small stresses and strains. If also second order expressions are considered, the influence of static stresses on the propagation of elastic waves will be included in the wave equation. These effects are discussed in view of applications to seismic work. The microscopic origin of stress-induced anisotropy is reviewed. The stress-field due to the overburden pressure is responsible for a stress-induced transverse isotropy, while horizontal tectonic stresses additionally generate azimuthal anisotropy, leading to a splitting (birefringence, double refraction) of vertically propagating shear waves. Inherent and stress-induced anisotropy can be distinguished from their different symmetry properties.

Keywords: stress-induced anisotropy, shear-wave splitting, birefringence, nonlinear elasticity, Murnaghan constants

1. Introduction

Stress-induced anisotropy in solid materials is well known in optics. This anisotropy leads to a splitting of the electromagnetic (transverse) waves into two components that travel at different velocities in the medium: This is termed birefringence or double refraction. Both wave components show a time delay against each other after they have passed through the medium, leading to distinct interference phenomena which can be used for stress analysis in engineering modelling [see e.g. MEUTH 1973, BLÜML et al. 1982]. Similarily, during the past decade, the application of stress-induced elastic anisotropy, appearing as birefringence of elastic transverse waves ("shear wave splitting") has found growing interest for testing materials with regard to internal stresses by ultrasonic shear waves [e.g.: HSU 1974, BLINKA and SACHSE 1976, KINO et al. 1979, SCHNEIDER and GOEBBELS 1982].

In addition, increasing consideration has been given to the study of stress-induced elastic anisotropy by seismic waves both for understanding the origins of anisotropy in general and for the intention of deriving the tectonic the origins of anisotropy in general and for the intention of deriving the tectonic stress field from seismic measurements. A first attempt to formulate a rigorous and general theory on wave propagation in an elastic medium under stress was made by BIOT [1940], but it was HUGHES and KELLY [1953] who derived expressions in closed form from the stringent framework given by MURNAGHAN

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Manuscript received: 14 September, 1987

[1951] for the theory of elasticity in the case of finite deformations. On this basis many other authors treated their study of waves in prestressed solids, e.g. WALTON [1974], BONAFEDE et al. [1978], BACH and ASKEGAARD [1979]. Reference is also made to SEEGER and MANN [1959] and SEEGER and BUCK [1960] and, for collateral reading to the fundamental article of HUGHES and KELLY [1953]. TRUESDELL and NOLL [1965] present expressions for the velocities in the medium under stress in a different notation, but these can be transformed into those of HUGHES and KELLY [1953] in terms of the Murnaghan constants (see Appendix B). Corresponding considerations on stress-induced anisotropy of elastic wave propagation were made by TOLSTOY [1982] and NORRIS [1983]; they regard implicitly the (finite) strain caused by the static stress in terms of a generally nonlinear elasticity and superimpose the small strains of the elastic wave in terms of *linear* elasticity.

The application of stress-induced anisotropy for stress evaluation is still in its initial phase. AGGSON [1978] proposed a sonic tool for borehole measurements of the tectonic stress by observation of the interference between the *SV*- and *SH*(tangential)-wave, caused by shear-wave splitting. The analysis of shear wave *(SH)* vertical seismic profiles by consideration of the change of the state of polarization as well as by cepstral analysis for tectonic stress estimation has been studied by TÖNNIES [1986]. ZOBACK [1985] proposed that the horizontal polarization of tube-waves recorded on vertical seismic profiles may be used to measure the tectonic stress.

2. Phenomenological structure of stress-induced anisotropy

In this contribution, let us neglect any inherent (intrinsic) anisotropy of the medium. Thus, we consider a homogeneous medium which is isotropic if there are no stresses applied. Since we wish to consider the influence of high static stresses (overburden pressure, tectonic stress) on the propagation of elastic waves, we can no longer use the linear Hooke's law as a stress–strain relation. Instead, the strength of the static stress field makes it necessary to take into account the nonlinear elasticity of the medium. Up to the quadratic order, we have to deal with a stress–strain relation of the following form, in symbolic notation:

$$\sigma = (\lambda, \mu)\varepsilon + (l, m, n)\varepsilon^2 \quad (1)$$

where σ indicates the tensor (of second rank) of stress, ε the tensor (of second rank) of strain, (λ, μ) represents the tensor (of fourth rank) of the elastic moduli for linear elasticity, which, for a homogeneous and isotropic medium, consists of only two independent parameters, the Lamé moduli λ and μ, in this notation. Correspondingly, (l, m, n) indicates a tensor of elastic moduli of the sixth rank, describing the quadratic component in the stress–strain relation; for a medium which is homogeneous and isotropic in the stress-free state, it contains three independent parameters, which are the Murnaghan constants l, m, n in this notation [MURNAGHAN 1951]. A different choice of notation is presented in

Appendices A and B. Thus Hooke's law, which is the linear approximation of a generally nonlinear elastic stress–strain relation, is extended to the quadratic order and three further elastic constants – the Murnaghan constants – are involved for a homogeneous and initially isotropic medium. We will use the term *elastic* for a medium described by a nonlinear stress–strain relation in order to express that we assume no elastic hysteresis to be present, the latter property usually being ascribed as *anelastic*. Occasionally the term *hyperelastic* is used for the nonlinear medium without hysteresis [TRUESDELL 1961] if the stress–strain relation can be derived from an energy function — as is the case in our problem (see Appendix B).

Furthermore, in the frame of a nonlinear theory of elasticity, we must take into account that the strain tensor ε is also nonlinearily related to the displacement vector $\mathbf{u}$, which is neglected in linear elasticity:

$$\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) + \frac{1}{2}\frac{\partial u_j}{\partial x_i}\cdot\frac{\partial u_j}{\partial x_k} \tag{2}$$

where u_n represents the component n of the displacement vector $\mathbf{u}$ and x_m is the mth coordinate. In equation (2) and in the following, we make use of a summation convention requiring that all expressions must be summed up from one to three over that index which appears twicefold within the expression. As a consequence of the nonlinearity in equation (2), the Lamé moduli of linear elasticity will also appear in addition to the Murnaghan constants in nonlinear terms in the wave equation for the displacement $\mathbf{u}$. The wave equation for the displacement $\mathbf{u}$ is derived in the usual way, starting from Newton's law (summation convention on index k!):

$$\varrho\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k} \tag{3}$$

where ϱ is the mass density.

Inserting the nonlinear stress–strain relation (equation (1)) as well as the nonlinear relation of strain and displacement (equation (2)) we get the following wave equation in symbolic notation:

$$\varrho_o\frac{\partial^2 \mathbf{u}}{\partial t^2} = (\lambda, \mu)\frac{\partial^2 \mathbf{u}}{\partial x^2} + (\lambda, \mu, l, m, n)\frac{\partial \mathbf{u}}{\partial x}\cdot\frac{\partial^2 \mathbf{u}}{\partial x^2} + \frac{\partial \sigma_{ik}^{static}}{\partial x_k} \tag{4}$$

In this *symbolic* formulation, the derivatives $\frac{\partial \mathbf{u}}{\partial x}$ and $\frac{\partial^2 \mathbf{u}}{\partial x^2}$ represent any of the vectorial derivatives curl, div, grad div and curl curl, as they occur in the equation, ϱ_o indicates the density of the medium in the undeformed state. $\frac{\partial \sigma_{ik}^{static}}{\partial x_k}$ represents the divergence of the static portion of the total stress field acting on the medium. In geophysics, this includes tectonic and gravity forces.

If now the total displacement $\mathbf{u}$ is regarded as consisting of a static part $\mathbf{u}_{static}$ and of a wave part $\mathbf{u}_{wave}$

$$\mathbf{u} = \mathbf{u}_{static} + \mathbf{u}_{wave} \tag{5}$$

we see immediately that the wave equation can no longer be split — as in the case of linear elasticity — into separate equations for only the static displacement and only the wave displacement. The reason is the existence of nonlinear expressions in the wave equation (4), which also comprises mixed terms of the form

$$\frac{\partial \mathbf{u}_{static}}{\partial x} \cdot \frac{\partial^2 \mathbf{u}_{wave}}{\partial x^2} .$$

Such mixed terms express a coupling of the wave propagation with the static stress field.

The static displacement is time independent by definition, viz:

$$\frac{\partial \mathbf{u}_{static}}{\partial t} = 0 \tag{6}$$

and hence, using the partition of the displacement in equation (5), we derive from equation (4) the following form of the wave equation in the lowest order of nonlinearity:

$$\begin{aligned} \varrho_0 \frac{\partial^2 \mathbf{u}_{wave}}{\partial t^2} = & \left[(\lambda, \mu) + (\lambda, \mu, l, m, n) \frac{\partial \mathbf{u}_{static}}{\partial x} \right) \right] \cdot \frac{\partial^2 \mathbf{u}_{static}}{\partial x^2} + \frac{\partial \sigma_{ik}^{static}}{\partial x_k} + \\ & + \left[(\lambda, \mu) + (\lambda, \mu, l, m, n) \frac{\partial \mathbf{u}_{static}}{\partial x} \right] \cdot \frac{\partial^2 \mathbf{u}_{wave}}{\partial x^2} + \\ & + \left[\text{terms of the form } \frac{\partial \mathbf{u}_{wave}}{\partial x} \cdot \frac{\partial^2 \mathbf{u}_{wave}}{\partial x^2} \text{ that are neglected} \right] \end{aligned} \tag{7}$$

Although this equation is nonlinear in the total displacement (and also in the static displacement), it is, in this order of approximation, linear in that portion of the displacement which is caused by the wave. I will therefore term this approximation "quasilinear".

In this quasilinear approximation, the wave equation, equation (7), is formally separable into an equation for the static displacement and an equation for the wave displacement, if we formally introduce a new, stress dependent tensor of elastic moduli by

$$(\lambda^*, \mu^*) = \left[(\lambda, \mu) + (\lambda, \mu, l, m, n) \frac{\partial \mathbf{u}_{static}}{\partial x} \right]$$

This tensor of stress dependent moduli (λ^*, μ^*) contains in general more than two independent parameters and it formulates the anisotropy of the medium as a function of the static stress. The wave propagation then is *formally* described in terms of linear elasticity. This is the reason why linear elasticity works so well for the elastic wave propagation in the Earth, if only pressure dependent wave velocities and occasionally anisotropy are allowed. In simplified words, we may regard a general nonlinear stress–strain relation (see *Fig. 1*) and introduce "new" stress dependent moduli, thus expressing the curvature of the stress–strain curve. The wave propagation is then governed in terms of linear elasticity by the slope of the tangent, $(\lambda^*, \mu^*) = \frac{d\sigma}{d\varepsilon}$, to the stress–strain curve at the value of the static load. In this treatise, however, we will prefer the explicit formulation of nonlinear elasticity in terms of stress-independent elastic moduli. The inhomogeneous wave equation (7) can be solved in two steps:

a) In the absence of a seismic wave

$$\mathbf{u}_{wave} = 0$$

a nonlinear equation holds for the static displacement alone, depending on the static stress field σ^{static};

b) Once the static displacement $\mathbf{u}_{static}$ is derived as a function of static stress, this solution can be inserted into the entire wave equation (7) and a linear wave equation for $\mathbf{u}_{wave}$ will be left.

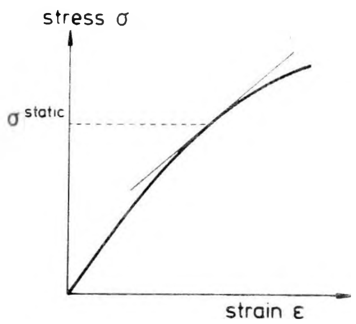

Fig. 1. Nonlinear stress–strain curve with tangent modulus $d\sigma/d\varepsilon$ at a load σ^{static}

1. ábra. Nem lineáris feszültség–alakváltozás függvény a $d\sigma/d\varepsilon$ tangens modulussal, σ^{static} terhelésnél

Рис. 1. Нелинейная функция напряжение–деформация с тангенсовым модулем $d\sigma/d\varepsilon$ при нагрузке σ^{static}

The solution of this problem was given by HUGHES and KELLY [1953] for three typical conditions:

1) For pure hydrostatic (lithostatic) pressure p (isotropic stress)

$$\varrho_0 V_P^2 = (\lambda + 2\mu) - \frac{p}{3K}[10\mu + 7\lambda + 6l + 4m] \tag{8a}$$

$$\varrho_0 V_S^2 = \mu - \frac{p}{3K}\left[3(\lambda + 2\mu) + 3m - \frac{1}{2}n\right] \tag{8b}$$

where V_P, V_S are the compressional and shear-wave velocities, respectively, and K is the bulk modulus

$$K = \lambda + \frac{2}{3}\mu \tag{9}$$

Equations (8a) and (8b) represent a description of the pressure dependence of the wave velocities in terms of the pressure independent Lamé and Murnaghan constants;

2) For axial compressive stress σ parallel to the direction of wave propagation

$$\varrho_0 V_P^2 = (\lambda + 2\mu) - \frac{\sigma}{3K}\left[\frac{\mu+\lambda}{\mu}(10\mu + 4\lambda + 4m) + \lambda + 2l\right] \tag{10a}$$

$$\varrho_0 V_S^2 = \mu - \frac{\sigma}{3K}\left[4(\mu+\lambda) + \frac{\lambda}{4\mu}\cdot n + m\right] \tag{10b}$$

3) For axial compressive stress perpendicular to the direction of wave propagation

$$\varrho_0 V_P^2 = (\lambda + 2\mu) - \frac{\sigma}{3K}\left[2l - \frac{2\lambda}{\mu}(2\mu + \lambda + m)\right] \tag{11a}$$

$$\varrho_0 V_{S\perp}^2 = \mu - \frac{\sigma}{3K}\left[m - 2\lambda - \frac{\mu+\lambda}{2\mu}\cdot n\right] \tag{11b}$$

$$\varrho_0 V_{S\parallel}^2 = \mu - \frac{\sigma}{3K}\left[(\lambda + 2\mu) + m + \frac{\lambda}{4\mu}\cdot n\right] \tag{11c}$$

where the symbols $\perp$ and $\parallel$ indicate if the polarization of the shear wave is perpendicular or parallel, respectively, to the stress.

Hence, the medium becomes anisotropic, as soon as the stress field is not isotropic. In this order of approximation, the corrections describing the influence of the stress field on the wave propagation are linear in the stress. Therefore, the solutions given in equations (8), (10), and (11) can be additively superposed in order to apply to the general case.

3. Geophysical relevance of stress-induced anisotropy

We will not treat in this contribution the geophysical consequences of anisotropy in general, but present instead discussions on special cases of stress-induced anisotropy. For general reading on wave propagation in anisotropic media, irrespectively of its origin, the reader is referred to the other contributions in this volume and e.g. to HELBIG [1981], CRAMPIN [1981], HELBIG [1983], CRAMPIN [1984a, b, c], CRAMPIN et al. [1984], HELBIG [1984], CRAMPIN [1985] and for the more practical aspects see e.g. TODD et al. [1973], WINTERSTEIN [1986], HAKE [1986] and THOMSEN [1986].

3.1 Transverse stress-induced isotropy

Suppose there are no horizontal tectonic stresses, then, in a homogeneous and originally isotropic formation, the overburden pressure generates a non-lithostatic (anisotropic) stress field at depth (see e.g. p. 108 of TURCOTTE and SCHUBERT [1982], or JAEGER and COOK [1976], p. 113 and p. 369):

$$\sigma_{zz} = \varrho g z = S_V \text{ (overburden pressure)}$$
$$\sigma_{xx} = \sigma_{yy} = \frac{\lambda}{\lambda + 2\mu} \varrho g z = \frac{\nu}{1-\nu} \varrho g z = S_H \tag{12}$$

where z is the depth, σ_{zz} and σ_{xx}, σ_{yy} are the principal stresses in the vertical and horizontal directions, respectively, for which also the notation S_V for the vertical and S_H for the horizontal stress may be used. The stresses are defined as positive in this treatise if they are compressive. ν is Poisson's ratio. The ratio

$$\frac{S_H}{S_V} = \frac{\nu}{1-\nu} \tag{13}$$

is independent of depth. For a Poisson ratio of $\nu = 1/4$ the ratio S_H/S_V becomes 1/3. The pressure p, being the isotropic part of the stress field, is given by

$$p = \frac{1}{3}(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) = \frac{1}{3}(2S_H + S_V) = \frac{1}{3}\varrho g z \frac{1+\nu}{1-\nu} \tag{14}$$

while the deviatoric stresses S'_V and S'_H are

$$S'_V = S_V - p = \frac{2}{3}\varrho g z \frac{1-2\nu}{1-\nu}$$
$$S'_H = S_H - p = -\frac{1}{3}\varrho g z \frac{1-2\nu}{1-\nu} \tag{15}$$

and hence

$$\frac{S'_H}{S'_V} = -\frac{1}{2}$$
$$S'_V - S'_H = \varrho g z \frac{1-2\nu}{1-\nu} \tag{16}$$

While pressure p leads, according to equations (8) to an isotropic change of the velocities, depending on the magnitude of the Lamé and Murnaghan elastic constants as well as on the sign of the Murnaghan constants, the deviatoric stress components lead to transverse isotropy according to equations (10) and (11). For the P-wave, the vertical velocity follows from equation (10a) for σ being S'_V, plus the pressure dependent part of equation (8a) for p. Correspondingly, the horizontal P-wave velocity follows from S'_V being inserted in equation

(11a) plus p being inserted in equation (8a). Similarily, but in a more complicated way, the stress-induced transverse (azimuthal) isotropy affects the shear wave. For the vertical shear-wave velocity, equation (10b) – with σ being S'_V plus equation (8b) with p according to equation (14) – must be taken. For the horizontal shear-wave velocity, equation (11b) or (11c) – depending on whether it is an *SH*- or *SV*-polarized shear wave, with σ being S'_V plus the pressure dependence from equation (8b) – is valid. Thus, a general polarization direction leads to a splitting of the horizontally propagating shear wave into two components (*SH* and *SV*) that travel at different velocities (double refraction, birefringence).

For a general direction of wave propagation, care must be taken that the stress-dependent expressions in equations (10) and (11) transform as components of a tensor of second rank. Shear-wave splitting will occur whenever the shear wave is not purely *SH* polarized and propagates in a non-vertical direction. Thus, for a homogeneous layer, anisotropy in the form of transverse isotropy will always be present from stress-induced anisotropy. This contributes to other sources of transverse isotropy like lithological and stratigraphic (fine layering) anisotropy. Winterstein [1986] demonstrates impressively how seriously transverse isotropy influences the stacking velocities. His conclusion that these effects contain information on lithology can be specified for the stress-induced part of transverse isotropy in the remark that the Murnaghan constants may be regarded as lithological parameters.

3.2 Azimuthal anisotropy

We assume now, in addition to the stress field which is generated by the overburden pressure, a horizontal tectonic (axial, compressive deviatoric) stress S'_t. If the tectonic axial stress alone would be present, not in addition to the vertical deviatoric stress, it would induce a transverse isotropy with a horizontal axis of symmetry. In the general case, for the *P*-wave, the anisotropy must be described for the vertical direction of propagation by equation (11a), with S'_t being inserted for σ, plus the stress-dependent part of equation (10a), with σ being S'_V, and with the pressure influence of equation (8a). The horizontal *P*-wave velocity follows correspondingly from equation (11a), with the perpendicular tensorial component of the horizontal tectonic stress S'_t plus S'_V, inserted for σ, plus the parallel tensorial component of S'_t for σ in equation (10a), and with the pressure influence of equation (8a).

The situation becomes even more complicated for shear waves. Therefore, for simplicity, let us regard the case of a vertically travelling shear wave *(SH)*, as it occurs in practice for shear-wave vertical seismic profiles *(VSP)* or for nearly vertical rays in shear-wave reflection seismics. The tectonic stress S'_t is then perpendicular to the propagation of the wave. In addition to the pressure influence in equation (8b) and the influence of the vertical deviatoric stress S'_V

in equation (10b), the tectonic stress in equations (11b) and (11c), respectively, gives different velocities, depending on whether the tectonic stress lies in the plane of polarization or perpendicular to it. Thus, a shear wave of an arbitrary orientation of its polarization direction will split into a parallel and a perpendicular component, which travel at different velocities.

As a result, from the time delay between both components, an originally linearly polarized shear wave becomes elliptically polarized after the passage of a finite path length; the general relation between time delay of orthogonally polarized waves and the state of polarization may be found, for example in monographs on optics, like BORN and WOLF [1980]. For a transient shear-wave signal, as seismic wavelets are, with a beginning and an end, the faster travelling component determines the polarization of the beginning of the recorded signal, while its end is determined by the slow wave component. In the main phase of the recorded signal, the composition of both components generally leads to elliptical polarization. Thus, we need only to know which is the parallel and which is the perpendicular component, respectively, the slow or the fast shear wave, in order to derive the orientation of the tectonic stress from polarization studies; in general (see below) it is the component polarized parallel to the stress that travels faster. In *Figure 2* such a change of the state of polarization over the duration of the recorded wavelet is demonstrated in a sequence of synthetically generated hodographs for the stress assumed to act with an angle of 30° with respect to the x-axis in a mathematically positive sense [after TÖNNIES 1986]. The plane of polarization of the source is rotated in steps of 10 degrees. The source signal is a Ricker wavelet of 25 Hz dominant frequency, and a time delay of 40 ms is assumed between both components. Furthermore, the spectrum of the composed wavelet will periodically be modulated by an interference structure caused by the interference of the two components of the wave from their time delay (cf. Appendix C). AGGSON [1978] has proposed that the frequency interval between these maxima and minima be used to estimate the strength of the tectonic stress field from shear-wave borehole logging and TÖNNIES [1986] has given synthetic examples for such evaluation by cepstral analysis.

Quantitatively, we derive for the time delay Δt between the parallel and perpendicular to the stress polarized shear wave:

$$\Delta t = \frac{\Delta z}{V_{\parallel}} - \frac{\Delta z}{V_{\perp}} = \frac{\Delta z}{V_{\parallel} \cdot V_{\perp}} \frac{V_{\perp}^2 - V_{\parallel}^2}{V_{\perp} + V_{\parallel}} \tag{17}$$

where Δz is the length of the travel path. By use of equations (11b) and (11c):

$$\Delta t = \frac{\Delta z}{V_{\parallel} \cdot V_{\perp}} \frac{\mu}{\varrho_0} \frac{1}{V_{\perp} + V_{\parallel}} \frac{4\mu + n}{4\mu^2} \sigma \tag{18}$$

σ being the axial stress perpendicular to the propagation direction (equal to S_t' for the tectonic origin of this stress). Furthermore,

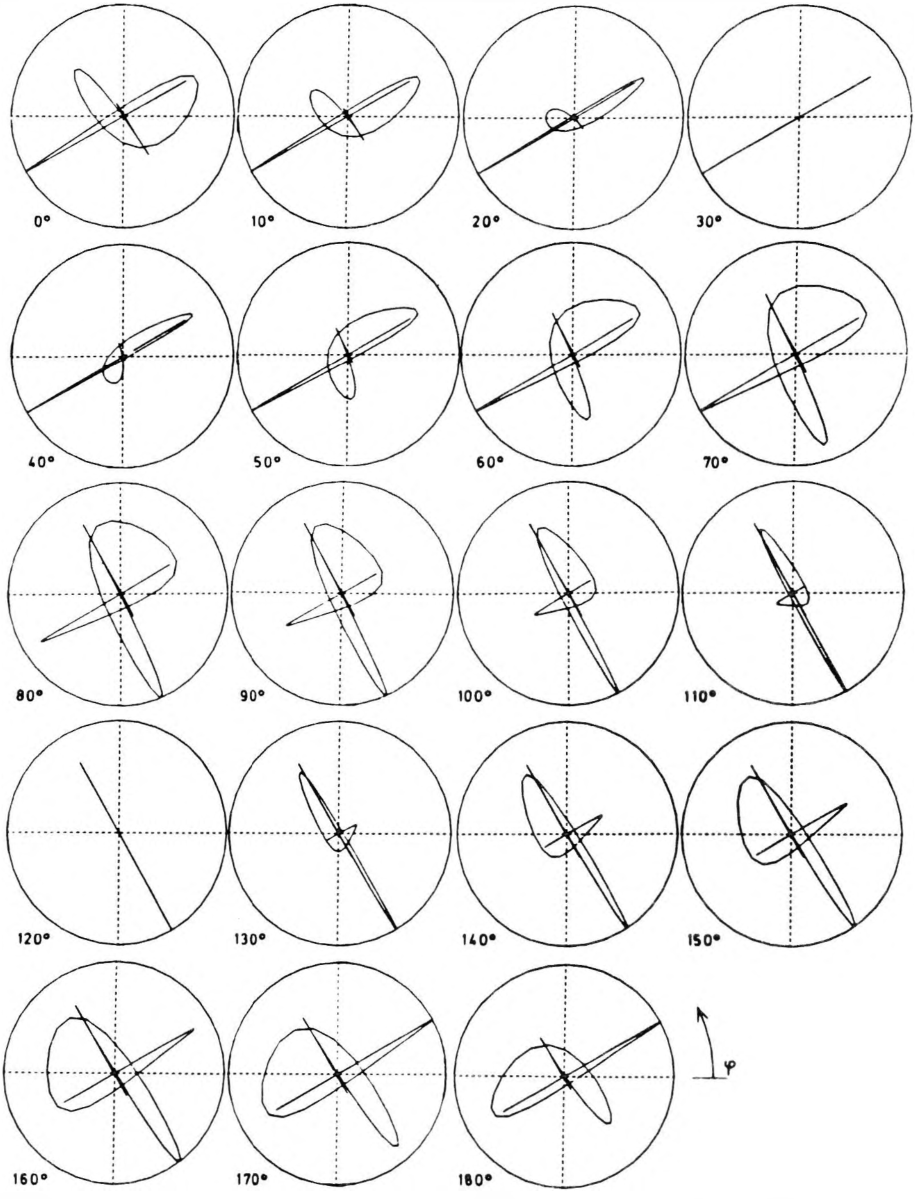

Fig. 2. Variation of the form of the hodograph as a function of the azimuth of the polarization of the source. The axis of the tectonic stress is assumed to be oriented at 30° with respect to the x-axis [after Tönnies 1986]

2. ábra. A hodográf alakváltozásai a hullámforrás polarizációs azimutja függvényében. Feltételezzük, hogy a tektonikai feszültség tengelye az x tengellyel 30°-os szöget zár be [Tönnies 1986 nyomán]

Рис. 2. Изменения годографа в зависимости от азимута поляризации источника волн. Предполагается, что ось тектонических напряжений составляет угол 30° с осью x [по Tönnies 1986].

$$\frac{\mu}{\varrho_0} = V_{SO}^2 \tag{19}$$

where V_{SO} is the (isotropic) shear-wave velocity in the medium without stress. In this order of approximation, which includes only expressions that are linear in stress, we estimate in equation (18)

$$\begin{aligned} V_\perp + V_\parallel &\approx 2V_{SO} \\ V_\parallel \cdot V_\perp &\approx V_{SO}^2 \end{aligned} \tag{20}$$

We then have for the time delay Δt:

$$\Delta t = \alpha_s \frac{\Delta z}{V_{SO}} \sigma \tag{21}$$

where α_s is termed the "constant of stress induced birefringence".

$$\alpha_s = \frac{4\mu + n}{8\mu^2} \tag{22}$$

V_{SO} in equation (21) may be taken in practice as the geometric or arithmetic mean velocity, according to equations (20). Since

$$t_0 = \frac{\Delta z}{V_{SO}} \tag{23}$$

is the mean travel time of the shear waves, we can rearrange equation (21) in the form

$$\frac{\Delta t}{t_0} = \alpha_s \sigma \tag{24}$$

3.3 Order of magnitude of stress-induced anisotropy

The magnitude of the stress-induced anisotropy is determined by the strength of the axial stresses and by the magnitude of the Lamé and Murnaghan elastic constants. While the Murnaghan constants are known for a variety of "laboratory materials", like pure metals or synthetic materials, only very few measurements of these constants for minerals and rocks are reported in the literature. In *Figure 3*, the measurements on granite, presented by Aggson [1978], show the stress-induced shear-wave splitting to be of remarkable order of magnitude. This diagram allows one to estimate the constant of stress-induced birefringence α_s for this sample of granite; the difference between the velocities at zero stress indicates the presence of inherent anisotropy in this rock sample. Walton [1974] presented a set of Lamé and Murnaghan constants for Barre granite, and Tönnies [1986] derived another set for Barre granite from the data of Nur and Simmons [1969]. These data, together with corresponding sets of elastic moduli of some other materials for comparison, are presented in *Table I*. While the Murnaghan constants for metals are in general of the order

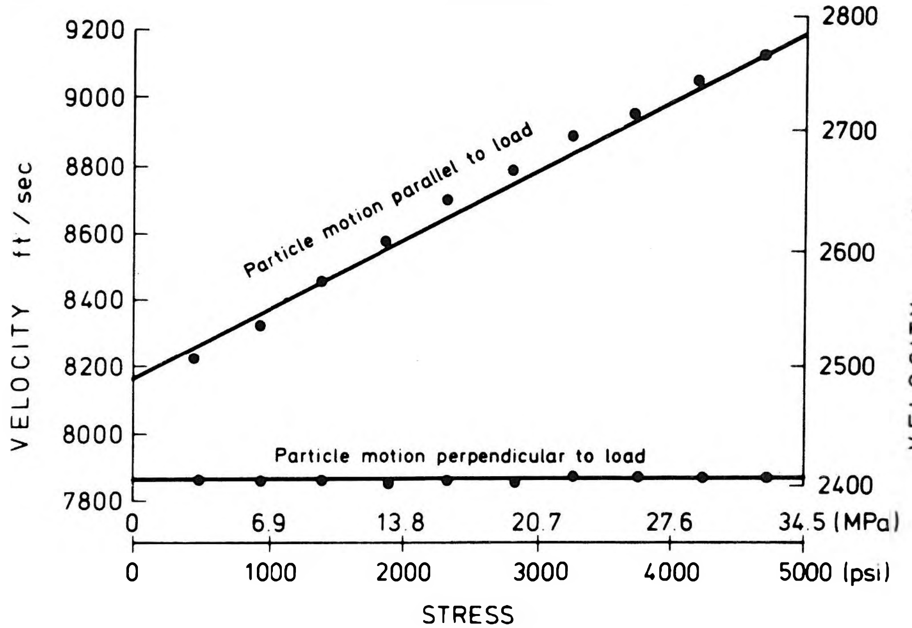

Fig. 3. Measurement of stress-induced shear-wave splitting in granite [after AGGSON 1978]

3. ábra. A feszültség által kiváltott nyíróhullám kettős-törés mérése gránitban [AGGSON 1978 nyomán]

Рис. 3. Измерение двупреломления поперечных волн, вызванных напряжениями, в гранитах [по AGGSON 1978].

of five to ten times the magnitude of the Lamé constants, they seem to be far greater in rocks. This may be caused by the micro-heterogeneity in the interior of rocks (see section 4). The negative sign of α_s for most of the materials indicates that the shear wave polarized parallel to the stress travels faster.

Thus, for granite, we estimate the order of magnitude of shear-wave splitting induced by stress to be

$$\alpha_s = -2.3 \text{ (dry)} \ldots -4.8 \text{ (wet) GPa}^{-1}$$

The axial tectonic stress may typically be of the order of some ten MPa, hence from Eq. (24)

$$\frac{\Delta t}{t_0} \approx -0.05$$

This means that the time delay between both shear-wave components will be of the order of 20 ms for a mean shear-wave travel time of $t_0 = 400$ ms, which may correspond to a travel path length of about 1000m in granite. The time delay expressed in terms of the phase delay $\Delta\varphi$ between both shear-wave components turns out to be

Material		λ in GPa	μ in GPa	l in GPa	m in GPa	n in GPa	ϱ_0 in g/cm³	α_S in GPa⁻¹
Barre-granite (dry)	1)	1.16	18.38	−3600	−6540	−6300	2.650	$-2303 \cdot 10^{-3}$
Barre-granite (wet)	2)	29.7	25.3	−4800	−8400	−25000	2.66	$-4862 \cdot 10^{-3}$
Granite	3)							$-4495 \cdot 10^{-3}$
Stone—Mountain-granite	4)	4.0	15.4				2.614	
Polystyrene	5)	2.89	1.38	−19	−13	−10	1.056	$-293 \cdot 10^{-3}$
Pyrex-glass	5)	13.5	27.5	+14	+92	+420		$+88 \cdot 10^{-3}$
Armco-iron	5)	110	82	−348	−1030	+1100		$+27 \cdot 10^{-3}$
Iron	6)	113	81	−167	−755	−1490		$-22 \cdot 10^{-3}$
Copper	6)	105	47	−157	−608	−1560		$-78 \cdot 10^{-3}$
Steel	7)	115.8	79.8	−248	−623	−714		$-8 \cdot 10^{-3}$

1) TÖNNIES [1986], based on NUR and SIMMONS [1969]
2) WALTON [1974]
3) AGGSON [1978]
4) LANDOLT—BÖRNSTEIN [1982], p. 42 (included in this compilation to demonstrate the large variations that the same type of rock may exhibit in its elastic properties)
5) HUGHES and KELLY [1953]
6) SEEGER and BUCK [1960]
7) EGLE and BRAY [1976]

Table I. Lamé and Murnaghan constants and the constant of stress-induced shear-wave birefringence for some materials

I. táblázat. A Lamé és Murnaghan állandók, valamint a feszültség által keltett nyíróhullám kettős-törés konstansa néhány anyagra

Таблица I. Константы Ламэ и Мурнагана, а также константа двупреломления поперечной волны, вызванной напряжением, для некоторых веществ.

$$\Delta\varphi = \frac{\Delta t}{T} 2\pi = \frac{t_0}{T} 2\pi\, \alpha_s\, \sigma \tag{25}$$

where T is the period of the shear wave. Hence, the phase delay $\Delta\varphi$ becomes 2π if t_0/T equals about 20 (for the values that have been assumed above). Thus, for a wavelet of a dominant frequency of 20 Hz, a phase delay of 2π occurs if t_0 is about 1000 ms. Remember that a linearly polarized wave becomes circularly polarized for a phase delay of $\Delta\varphi = \pi/2$ if both orthogonal components are of equal magnitude [for details, see BORN and WOLF 1980]. All the other effects of stress-induced anisotropy, as discussed above, may be of the same order of magnitude. Note that the deviatoric vertical stress S'_V, according to equation (15), is of the order of 10 MPa at a depth of 1000 m for $\varrho = 2.6 \cdot 10^3$ kg/m³ (granite) and a Poisson number of $\nu = 1/4$, and increases linearly with depth. The data presented in Table I also confirm that it was necessary, for the derivation of a theory of stress-induced anisotropy, to deal not only with the nonlinear relation of the strain and displacement (equation (2)), but also with the non-linear stress–strain relation (equation (1)). In fact, in the stress-dependent expressions of equations (8), (10) and (11) the terms of the Murnaghan constants become the overwhelming parts compared with those of the Lamé moduli.

4. Further remarks on the nature of stress-induced anisotropy

Nothing is said in the frame of this theory about the petrophysical origin of stress-induced anisotropy on a microscopic scale ("microscopic" with respect to the wavelength), because our description is purely phenomenological. Several causes may contribute, whereas in any given formation one of the phenomena will be expected to be dominant. In preference, the oriented closure and/or oriented generation of microcracks by stress is mainly discussed in the literature [NUR and SIMMONS 1969, TODD et al. 1973, CRAMPIN et al. 1980, CRAMPIN and MCGONIGLE 1981, CRAMPIN et al. 1984, CRAMPIN and ATKINSON 1985, ROBERTS and CRAMPIN 1986, BROOKS et al. 1987]. But we also think of such sources of stress-induced anisotropy like the change of the shape of pores (e.g. spherical pores become ellipsoidal under stress), the change of the contact between the individual grains in a preferred direction, the elastic differential rotation of nonspherical grains if they are embedded in a matrix of different elastic properties, and the change of structure of the crystal lattice in the grains. The last phenomenon dominates in the studies of stress-induced anisotropy in solid-state physics, e.g. BIRCH [1947], SEEGER and MANN [1959], SEEGER and BUCK [1960], BATEMAN et al. [1961], THURSTON [1965] and, is a main part, in acoustoelastic imaging of internal stress fields by ultrasonic shear-wave birefringence. All the other phenomena mentioned above account for the material heterogeneity. These dominate in natural rocks and thus the excess of Murnaghan's constants for rocks in relation to homogeneous materials may be explained. Obviously, the Murnaghan elastic constants express the readiness of the material to close or generate cracks, to change the shape of the pores, to change the contact of the grains, the ability of elongated grains to rotate and the facility to change the structure of the crystal lattice.

In view of the prospect of deriving information on the stress field in the Earth from the observation of stress-induced anisotropy, the distinction between inherent and stress-induced anisotropy is of major importance. In ultrasonic experiments MAHADEVAN [1966] found the inherent shear-wave birefringence to depend on the frequency whereas the stress-induced part proved to be frequency independent. SCHNEIDER et al. [1985], who have analysed this effect in more detail, successfully applied a procedure on this basis to separate both portions of anisotropy. It seems questionable whether similar effects occur in seismics, but careful studies should be executed in the future. In general, the separation of stress-induced and inherent anisotropy is possible by taking advantage of the difference in symmetry of the medium, expressed in different symmetry properties of the modulus tensor c_{ijkl} [THURSTON 1974, p. 227, KING and FORTUNKO 1983, THOMPSON et al. 1984, NIKITIN and CHESNOKOV 1984]. This tensor of elastic moduli itself depends on the stress to account for the nonlinearity of the stress–strain relation (this is, in the first order expansion of c_{ijkl} with respect to the stress, fully equivalent to the formulation of equation (1), see KING and FORTUNKO [1983]). It turns out, that

$$\varrho(V_{ij}^2 - V_{ji}^2) = \begin{cases} \sigma_{ii} - \sigma_{jj} & \text{for stress-induced anisotropy} \\ 0 & \text{for inherent anisotropy} \end{cases} \tag{26}$$

where V_{ij} is the velocity of a shear-wave propagating in the i direction and polarized in the j direction. Equation (26), indicates the lower symmetry of stress-induced anisotropy compared with inherent anisotropy. Furthermore it implies as a consequence that the abbreviated Voigt notation for the modulus tensor, as used by many authors to describe anisotropy [e.g. THOMSEN 1986], cannot be used for stress-induced anisotropy [THURSTON 1974]. Hence, it follows from equation (26) that the stresses can also be derived from shear-wave anisotropy observations in the presence of inherent anisotropy if the material is traversed by rays in orthogonal directions and correspondingly mutually exchanged polarization directions; this is easier to achieve in the laboratory with ultrasonic experiments than in seismics. In seismic work, one may observe shear-wave anisotropy from a steep ray and from oblique rays under different azimuths and polarization directions and use equations (10) and (11) in tensorial rotated form, corresponding to the directions of the ray and of the polarization.

Anisotropy depending on the stress field has also been observed in the subcrustal lithosphere and in the upper mantle [e.g. CRAMPIN 1977, ANDO et al. 1980, ANDO and ISHIKAWA 1982, ANDO et al. 1983, FUCHS 1983, FUKAO 1984, SHEARER and ORCUTT 1986]. Two models for the origin of this anisotropy have been subject to discussion, the crack alignment model and the olivine alignment model [ANDO et al. 1983]. While in the crack alignment model magma filled cracks in a preferred direction are assumed, in the olivine alignment model, which is now widely favoured [CRAMPIN et al. 1984], the orientation of olivine crystallites—which themselves show a crystalline anisotropy—by flow processes is supposed [FUCHS 1983, CHRISTENSEN 1984, ARTYUSHKOV 1984, SHEARER and ORCUTT 1986, SAYERS 1987]. Although this type of anisotropy is *stress*-influenced, hence allowing for stress analysis in these depths, it is due to rheology, that is not based on elasticity as described by equation (1) but that accounts also for the flow [see e.g. FUCHS 1983]. In view of this it does not strictly belong to the class of stress-induced anisotropy, whereas flow-induced anisotropy seems to characterize this type of anisotropy more precisely.

5. Conclusions

The study of stress-induced anisotropy from seismic observations is not only of importance for the analysis of the Earth's stress field but also for lithological information in the sense of WINTERSTEIN [1986], from the estimation of third-order (Murnaghan) elastic moduli. Laboratory measurements of the Murnaghan elastic moduli for a variety of sedimentary and crystalline rocks are now urgently required to estimate the order of magnitude of the stress-induced anisotropy in more detail than is done in this treatise in section 3.3, and to get an idea of its lithological span of variability. On the other hand, more observational data under controlled conditions, like shear-wave vertical seismic profiles (VSP) using 3-component geophones in the borehole, are needed. Care must be taken that the polarization direction of the shear-wave source is neither parallel nor perpendicular to the tectonic stress otherwise no shear-wave splitting would occur; thus, in general, at least two polarization orientations, forming an angle of 45° to each other, should be chosen.

In shear-wave polarization studies, furthermore, attention must be paid to changes of the state of polarization due to other origins, such as from transmission and reflection at dipping interfaces [Douma and Helbig 1987] or from the effect of the free surface [Evans 1984].

The linear increase with depth of the vertical deviatoric stress S'_V (cf. equation (15)) suggests that stress-induced transverse isotropy may play a significant role in the lithosphere. This assumption is consistent with the model of Dziewonski and Anderson [1981] and Anderson and Dziewonski [1982] for a general transverse isotropy in the lithosphere.

At last, the author wishes to draw attention to higher order elastic effects, which influence the frequency of a seismic wave. In equation (7), we have neglected terms that describe an interaction of the propagating wave with itself. These terms involve nonlinear spectral mixing leading to the generation of harmonic frequencies. A theory in closed form like that for the quasilinear approximation has not yet been published. If such effects would become noticeable in seismic work, depending once more on the magnitude of the Murnaghan elastic moduli, they would influence spectral studies as, for example, seismic attenuation determinations. Observational indications, on the other hand, have been reported by Agnew [1981] and by Beresnev et al. [1986].

APPENDIX A
Murnaghan's elastic moduli in Eulerian and Lagrangian coordinates

In a theory of finite elastic deformations, attention must be paid to a strict and persistent definition of the variables, since initial coordinates and final coordinates are no longer interchangeable. The choice of the initial coordinates as independent variables is called "Lagrangian formulation", while the choice of final coordinates as independent variables is termed "Eulerian formulation" [Hughes and Kelly 1953]. Care must be taken if use is made of the literature, whether Eulerian or Lagrangian coordinates are used since the set of third order elastic moduli is different in both formulations. In general, however, the Lagrangian description is preferably used in the literature—as it is in this contribution. Seeger and Mann [1959] presented the relations that allow the conversion between both representations. If l, m, n are Murnaghan's moduli in Lagrangian formulation and l', m', n' are those as defined in Eulerian formulation, then the following interrelations hold:

$$\begin{aligned} l &= -2(3\lambda+4\mu)+3l'+m' \\ m &= -2\mu-\frac{1}{2}m' \\ n &= -12\mu+n' \end{aligned} \qquad \text{(A-1)}$$

$$\begin{aligned} l' &= 2(\lambda+2\mu)+\frac{1}{3}(l+2m) \\ m' &= -4\mu-2m \\ n' &= 12\mu+n \end{aligned} \qquad \text{(A-2)}$$

APPENDIX B
Other than Murnaghan notation of elastic third order moduli

The elastic energy density in Lagrangian representation $\varphi(\varepsilon_{ik})$, where ε_{ik} are the components of the strain tensor, reads in the formulation according to MURNAGHAN [1951]:

$$\varphi = -p_0 I_1 + \frac{\lambda+2\mu}{2} I_1^2 - 2\mu I_2 + \frac{l+2m}{3} I_1^3 - 2mI_1I_2 + nI_3 \qquad \text{(B-1)}$$

p_0 is the initial (hydrostatic/lithostatic) pressure, λ and μ are the Lamé-, and l, m, n the Murnaghan elastic moduli, and I_1, I_2, I_3 the first, second, and third tensorial invariants:

$$\begin{aligned} I_1 &= \varepsilon_{ii} = \mathrm{Tr}\,(\varepsilon_{ik}) \\ I_2 &= \det\begin{pmatrix}\varepsilon_{11} & \varepsilon_{12}\\ \varepsilon_{21} & \varepsilon_{22}\end{pmatrix} + \det\begin{pmatrix}\varepsilon_{22} & \varepsilon_{23}\\ \varepsilon_{32} & \varepsilon_{33}\end{pmatrix} + \det\begin{pmatrix}\varepsilon_{33} & \varepsilon_{31}\\ \varepsilon_{13} & \varepsilon_{11}\end{pmatrix} \\ I_3 &= \det\,(\varepsilon_{ik}) \end{aligned} \qquad \text{(B-2)}$$

Recall the summation convention as defined in Section 2, following equation (2), also for the following. The stress–strain relation in initial coordinates is given by (cf. SEEGER and MANN [1959]):

$$\sigma_{ik} = \mathbf{J}_{ij}\frac{\partial\varphi}{\partial\varepsilon_{jk}} \qquad \text{(B-3)}$$

where the Jacobian matrix $\mathbf{J}_{ij}$, connecting the final coordinates x_i' with the initial coordinates x_j is

$$\mathbf{J}_{ij} = \frac{\partial x_i'}{\partial x_j} = \delta_{ij} + \frac{\partial u_i}{\partial x_j} \qquad \text{(B-4)}$$

where the u_i are the components of the displacement vector.

TRUESDELL and NOLL [1965] use in their treatise the following formulation of the elastic energy density for vanishing initial pressure p_0 (their equation (93.1)):

$$\varphi = \frac{\lambda+2\mu}{2} I_1^2 - 2\mu I_2 + \mu\beta_1 I_1^3 + \mu\beta_2 I_1 I_2 + \mu\beta_3 I_3 \tag{B–5}$$

Therefore, from comparison with equation (B–1), this notation can be converted into Murnaghan's expression by

$$\begin{aligned} \mu\beta_1 &= l+2m \\ \mu\beta_2 &= -2m \\ \mu\beta_3 &= n \end{aligned} \tag{B–6}$$

On the other hand, these authors compare the stress–strain relation, following from (B–5), with an expression which they define as the second-order stress–strain relation for general elastic materials (their equation (66.3)), and they derive that

$$\begin{aligned} \alpha_3 &= -\alpha_1+3\beta_1+\beta_2 \\ \alpha_4 &= \beta_2+\beta_3 \\ \alpha_5 &= 2\alpha_1-2-\beta_2-\beta_3 \\ \alpha_6 &= 4+\beta_3 \end{aligned} \tag{B–7}$$

where α_i are the elastic constants as introduced in a second-order expansion of the stress with respect to the strain. Substituting Murnaghan's constants from (B–6) into (B–7), and α_1 being λ/μ, we derive:

$$\begin{aligned} \mu\alpha_3 &= -\lambda+l \\ \mu\alpha_4 &= -2m+n \\ \mu\alpha_5 &= 2\lambda-2\mu+2m-n \\ \mu\alpha_6 &= 4\mu+n \end{aligned} \tag{B–8}$$

This result, however, is inconsistent with the conversion they themselves give on p. 230 of their work.

If, on the other hand, these conversions (B–8) are inserted into their own expressions for the velocities in their notation, then using α_i, one gets exactly the velocities as given by Hughes and Kelly [1953], see equations (8), (10), and (11). Hence, we conclude that the conversion in the form (B–8) and not the conversion of Truesdell and Noll [1965], p. 230, for α_i into Murnaghan constants is valid [Tönnies 1986].

APPENDIX C

Structure of the spectral interference pattern for a double-refracted shear wave

Consider, for simplicity, the case of a vertical seismic profile (VSP), where a horizontally polarized shear-wave travels in the vertical direction. Since the orientation of the horizontal tectonic stress will not be known in general, we assume the angle of the polarization direction with respect to the direction of the tectonic stress to be α. By the double refraction in the medium the shear wave splits into components polarized parallel and perpendicular with respect to the stress:

$$\begin{aligned} s_{\parallel}(t) &= s_0(t) \cdot \cos\alpha \\ s_{\perp}(t) &= s_0(t) \cdot \sin\alpha \end{aligned} \tag{C–1}$$

where s_0 is the source wavelet in the time domain.

The orientation of a horizontal geophone at depth will generally not be under control, although it may be known, for example, from a compass signal. Let β be the angle of the geophone orientation against the stress direction, then the signal s_G, recorded by the geophone, will be

$$s_G(t) = s_{\parallel}(t)\cos\beta + s_{\perp}(t+\delta)\sin\beta \tag{C–2}$$

where δ is the time delay (positive or negative) between both shear-wave components, caused by the birefringence. Together with equation (C–1) follows the equation

$$s_G(t) = a s_0(t) + b s_0(t+\delta) \tag{C–3}$$

where

$$\begin{aligned} a &= \cos\alpha\cos\beta \\ b &= \sin\alpha\sin\beta. \end{aligned} \tag{C–4}$$

By Fourier transformation, we derive for the spectrum of the geophone signal:

$$S_G(\omega) = S_0(\omega)\,[a + b e^{i\omega\delta}] \tag{C–5}$$

where $S_0(\omega)$ is the source spectrum. The expression within the brackets represents the spectral interference pattern, superposed upon the source spectrum. For the power spectrum of the geophone signal we get

$$|S_G(\omega)|^2 = |S_0(\omega)|^2\,[a^2 + b^2 + 2ab\cos(\omega\delta)]. \tag{C–6}$$

The power spectrum of the interference pattern is schematically depicted in *Fig. 4*. Its amplitude, and thus its sensitivity for detection from experimental data, obviously depends on the product ab, which is maximal for $\alpha = \beta = \pi/4$: $(ab)_{max} = 1/4$. Depending on the signs of a and b, respectively, the interference can also begin with a minimum at zero frequency. Since the spectral modulation by the interference is periodic, a cepstral analysis is appropriate for analysing the modulation quefrency, i.e. to derive the time delay δ.

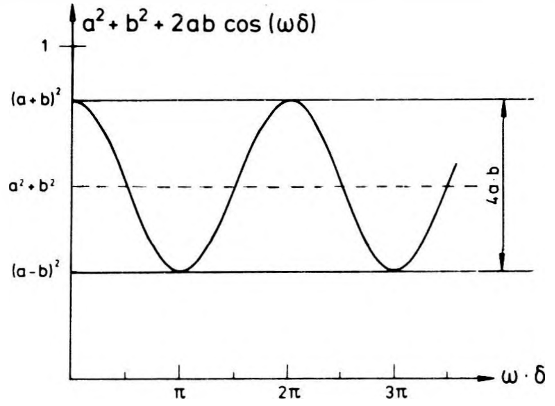

Fig. 4. Power spectrum of the interference pattern

4. ábra. Az interferencia kép teljesítményspektruma

Рис. 4. Спектр мощности интерференционной картины.

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FESZÜLTSÉG ÁLTAL KIVÁLTOTT ANIZOTRÓPIA RUGALMAS KÖZEGBEN

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A rugalmas közegekre vonatkozó lineáris Hook törvény általánosságban a nemlineáris terhelés–alakváltozás összefüggés első rendű közelítésének tekinthető, kis feszültségekre és alakváltozásokra. Ha másodrendű tagokat is figyelembe veszünk, akkor a hullámegyenlet tartalmazza a statikus feszültségek hatását a rugalmas hullámok terjedésére. Ezeket a hatásokat tárgyalja a cikk, szeizmikus szemszögből. A feszültség által okozott anizotrópia mikroszkopikus eredetéről áttekintést ad. A fedő terhelés okozta feszültségtér indukált transzverzális izotrópiát hoz létre, a horizontális tektonikai feszültségek pedig iránytól függő anizotrópiát idéznek elő, amely a vertikálisan terjedő nyíróhullámok kettős töréséhez vezet. Az eredendő és a terhelés által indukált anizotrópia megkülönböztethető az eltérő szimmetria tulajdonságok alapján.

АНИЗОТРОПИЯ, ВЫЗВАННАЯ НАПРЯЖЕНИЯМИ В УПРУГОЙ СРЕДЕ

Людвиг ЭНГЕЛЬХАРД

Линейный закон Гука, описывающий упругие среды, в общем может рассматриваться в качестве первого приближения зависимости деформации от нелинейных нагрузок, действительного при малых напряжениях и деформациях. Если учесть и члены второго порядка, то в полученном волновом уравнении будет содержаться и влияние статических нагрузок на распространение упругих волн. В статье рассматриваются именно эти соотношения с упором на сейсморазведку. Дается обзор микроскопического происхождения анизотропии, вызванной напряжениями. Полем напряжений, возникающим из-за нагрузок со стороны кровли, обуславливается поперечная изотропность, в то время как горизонтальными тектоническими напряжениями вызывается азимутальная анизотропия, приводящая к двойному преломлению поперечных волн распространяющихся в вертикальном направлении. Первичная и вызванная нагрузками анизотропия может различаться по неодинаковым особенностям симметрии.