

AUTOMATIC RELATIVE DEPTH MATCHING OF BOREHOLE INFORMATION I. THEORETICAL REVIEW

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One of the prerequisites for interpreting borehole information is that data of the given well should be correct according to depth. In order to obtain a common depth point it is assumed that the $A(X)$ relative depth deviations changing from point to point can be approximated by a polynomial. Developing in a series the $Y(X)$ log or logs to be matched according to depth, the $A(X)$ depth deviations in the Taylor's series agree with the substitution values of the polynomial. Minimizing the error function which can be formed from the data to be matched, the coefficients of the polynomial can be calculated and the corrected data obtained. If the process is repeated several times the calculated values converge. The method is suitable not only for correcting linear slips but, depending on the degree of the polynomial, also for eliminating deviations of varying sign. If the order of the polynomial describing the depth deviation is zero, i.e. it is a constant slip, the result obtained by the method is as good as that of the conventional cross correlation method. It is, however, substantially faster than the conventional one because of calculating the slip. The method is suitable for correcting depth deviations between well logs, between core data and well logs, and between the lithological column and well logs.

Keywords: well logging, depth deviation, borehole information, computer programs, algorithm, matching

1. Introduction

Similarly to every measurement, borehole information has its uncertainties characterizing the method, viz. the conditions, the instrument and the physical parameters of measurement. Both the method and the measured quantities may considerably differ from each other, but when determining their characteristics common features can be found as well. In the case of borehole information this common feature is their being recorded as a function of depth. Since measurements generally follow each other, measured values of geophysical and geological parameters corresponding to the same depth will not appear at the same place on the records, depth differences may occur. The causes of depth deviations will be discussed later.

In order to decrease the depth differences either the methodology should be modified or the logs should be corrected afterwards. In the first case the application of sonde trains would be necessary but even then there would not

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Manuscript received: 4 November, 1986

be any possibility to perform all the measurements simultaneously because of the great number of geophysical parameters. In the second case, in the conventional manual evaluation, the characteristic points of the curves (maximum, minimum, inflexion point, etc.) are taken into consideration when fitting the logs, i.e. matching relative depth. Using this method the experience of the expert and visual examination of the curves yield good results, but in the field of computer aided processing there is limited reference in the literature to depth matching.

In recent years attempts have been made to shift the curves to an extent determined by the operator, after reproducing the logs on a graphic display connected to a computer. It seems that this interactive method is suitable only for correcting very great deviations. The method mostly applied automatically corrects the constant slip of the logs. A reference log is selected, it is recorded for each run together with the geophysical parameters to be measured. The repeatedly measured log is considered as the base log. For depth matching cross correlation is computed between the base logs. Maxima of the correlation coefficient mark out the corresponding values. If this method is employed for the complete log, it only eliminates the constant deviation, although not only the extent but even the direction of the depth differences may vary from point to point. If cross correlation is performed for short intervals, then the problem arises in smoothing the differences at the boundaries of the intervals.

In this paper the mathematical phrasing of the possibilities of depth matching is presented, and a computer aided method is described which eliminates the above mentioned difficulty. A further advantage of the method to be described is that it is not necessary to measure the base log for each run.

2. Mathematical phrasing of depth differences

In order to describe mathematically the relative depth differences of well logs one has to start by examining the measurements. For well logging the sonde is lowered into the borehole by a cable. The signals emitted by the sonde are transmitted to galvanometers or to the magnetic tape recorder through cable-conductors. The camera is controlled by the movement of the cable through a transmission system whereas when using magnetic tape recording two independent depth determinations are used, viz. magnetic depth marks on the cable and the sampling interval controlled by the logging speed. Deviations from the correct depth values may originate from the following causes: differences in the reference points of the sondes; stretching of the cable caused by the interaction of the cable, the sonde and the borehole; inaccuracy of the transmission system between the camera and the cable; deviations from the set logging speed when recording on magnetic tape.

Dealing with the causes of the $\Delta(X)$ depth discrepancies in increasing order of the powers of the recorded X depth of the sonde, leads to the following grouping:

- a) The constant term (of zero order) comes from the difference between the reference points of the sondes:

$$\Delta(X)_0 = C \quad (2.1)$$

- b) The linear term is obtained from cable stretching caused by:

- cable stretching due to sonde weight. This can be calculated on the basis of Hooke's and Archimedes' laws supposing elastic deformation and including the buoyant force of the mud:

$$\Delta(X)_1 = k \cdot Q \left(1 - \frac{\gamma_m}{\gamma_s} \right) \cdot X \quad (2.2)$$

where k is the elastic module of the cable,

Q is the weight of the sonde in air,

γ_s is the specific weight of the sonde,

γ_m is the specific weight of the mud;

- cable stretching due to the friction of the sonde on the wall of the borehole and/or to the pressing of it against the wall:

$$\Delta(X)_2 = k\mu N \cdot X \quad (2.3)$$

where μ is the friction coefficient between the sonde and the sidewall,

N is the pressure force against the sidewall;

- the changing of the actual size of the film or paper, when digitizing analog logs:

$$\Delta(X)_3 = k_1 X \quad (2.4)$$

- c) The second order term is obtained by means of the following:

- the weight of the cable lowered in the borehole is in linear ratio with the length of the cable thus if cable stretching obtained by Hooke's and Archimedes' laws is integrated according to depth, a relation is obtained which is a depth function of second order:

$$\Delta(X)_4 = \frac{1}{2} kq \left(1 - \frac{\gamma_m}{\gamma_c} \right) \cdot X^2 \quad (2.5)$$

where q is the weight of the cable for unit length,

γ_c is the specific weight of the cable;

- the hydrostatic compression on the surface of the cable in the mud is proportional to its length. Thus the relation obtained by integrating the frictional force—proportional to the hydrostatic compression—is also a depth function of second order:

$$\Delta(X)_5 = \frac{1}{2} k\gamma_m (W + j) X^2 \quad (2.6)$$

where W is the friction of rest between cable and mud,

$j = j(v)$ is a quantity depending on logging speed;

- the force due to the friction of the cable on the wall of deviated boreholes is proportional to the component, perpendicular to the wall of the borehole, of the weight proportional to the length of the cable. Integrating this effect according to depth gives a quadratic relationship:

$$\Delta(X)_6 = \frac{1}{2} kq\mu_1 \left(1 - \frac{\gamma_m}{\gamma_k} \right) \sin \varphi \cdot X^2 \quad (2.7)$$

where μ_1 is the friction coefficient between cable and sidewall.

φ is the angle between the axis of the borehole and the vertical:

- integrating the effect of the temperature increasing quasi-linearly in depth, again a quadratic relation is obtained:

$$\Delta(X)_7 = \frac{1}{2} \alpha g_i X^2 \quad (2.8)$$

where α is the linear thermal expansion coefficient of the cable,

g_i is the geothermic gradient.

- d) Added to the former terms, the following can be approximated with those of higher order:

- the effect of sticking and restarting of the sonde.
- the effect of harmonic vibration of the sonde during the run.
- the "depth correction" of the operator or, with digital recording, that of the special electronic unit.

If logs of different runs are matched then the relative depth deviations are obtained as the difference of the two polynomials — which is also a polynomial. The coefficients of the terms describing the relative depth difference are obtained from the changing of the parameters in relations (2.1)—(2.8) between two runs.

In the case of sidewall coring the same reasoning can be applied since the depth difference is caused by cable stretching here too. For conventional coring, deformation of the drill pipe should be taken into consideration instead of that of the cable. During well logging, tensile load affects the cable whereas in coring compressive forces are acting on the drill pipe, thus the depth differences owing to elastic strain are supposed to sum up.

3. Relative depth matching of well logs

For the mathematical phrasing let us consider *Fig. 1*. As a first approach let us suppose that curves $Y_2(X)$, $Y_3(X)$, ..., $Y_N(X)$ are—related to each other—correct in depth and we should like to match function $Y_1(X)$ to them. At depth point X_i , ($i = 1, 2, \dots, L$), to function values $Y_2(X_i)$, $Y_3(X_i)$, ..., $Y_N(X_i)$ belongs the value $Y_1[X_i + \Delta(X_i)]$ of the function to be matched. For the sake of clarity the function values belonging to each other are marked in the figure and the deviation function $\Delta(X)$ is plotted at the bottom.

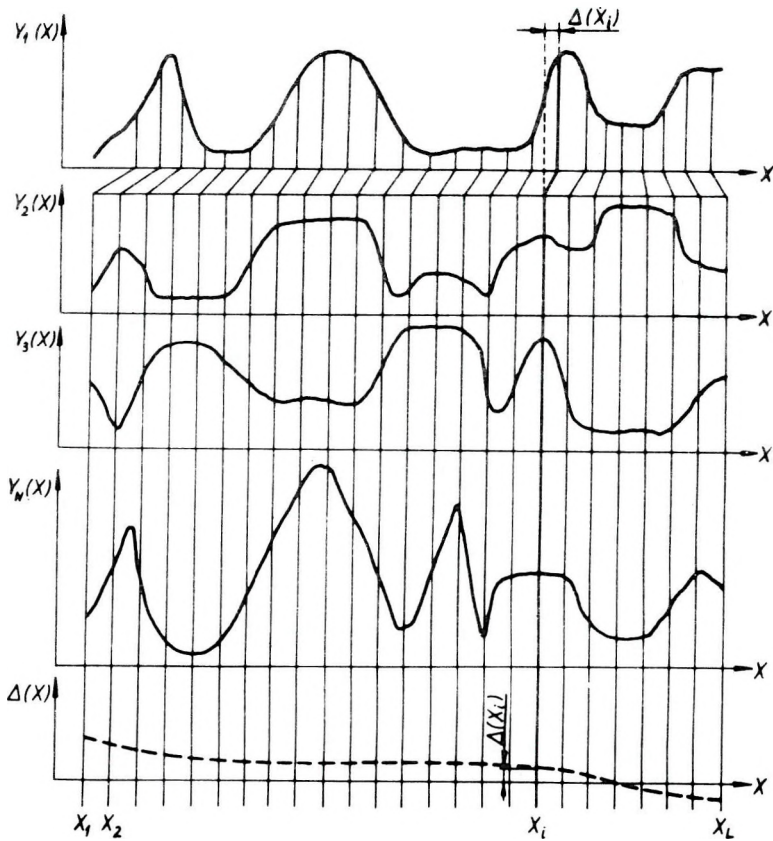


Fig. 1. Illustration of depth deviations of well logging curves

$Y_1(X)$ — curve to be matched; $Y_2(X)$, $Y_3(X)$, ..., $Y_N(X)$ — curves considered to be correct in depth; $\Delta(X)$ — function of depth deviation

1. ábra. Karotázgörcbék mélységtérésének szemléltetése

$Y_1(X)$ — egyeztetni kívánt görbe; $Y_2(X)$, $Y_3(X)$, ..., $Y_N(X)$ — mélységre helyesnek tekintett görbék; $\Delta(X)$ — a mélységtérést leíró függvény

Рис. 1. Демонстрация расхождений между каротажными кривыми по глубине

$Y_1(X)$ — кривая, подлежащая согласованию по глубине; $Y_2(X)$, $Y_3(X)$, ..., $Y_N(X)$ — кривые, считающиеся правильными по глубине; $\Delta(X)$ — функция, описывающая расхождения по глубине

Our aim is to determine $\Delta(X)$ since in the knowledge of this, depth correction means substituting the respective $Y_1[X_i + \Delta(X_i)]$ function value into $Y_1(X_i)$ at the $i = 1, 2, \dots, L$ sampling points.

To perform the calculation we assume that the values belonging to each other are related; this relationship can be defined by an operator F since each log provides certain geophysical information about the same place:

$$Y_1^{corr}(X_i) = Y_1[X_i + \Delta(X_i)] = F[Y_2(X_i), Y_3(X_i), \dots, Y_N(X_i)]$$

$$i = 1, 2, \dots, L \quad (3.1)$$

If the operator were precisely known, then the system of equations (3.1) would theoretically be solvable because it contains L unknowns in the values at the $i = 1, 2, \dots, L$ sampling points of the $\Delta(X_i)$ function of depth deviation and it consists of altogether L equations. Knowing the F operator, however, would give the impression of a contradiction since it would mean that the $Y_1(X)$ curve could be obtained from the other curves and so it would not be necessary to measure it. Naturally from geophysical aspects it cannot be true since the individual logging methods yield additional characteristic information related to the other measurements. Since our aim is, besides keeping the characteristic features of the curve $Y_1(X)$, to match its characteristic places with those of the other curves—and not produce it from the other curves—exact knowledge of operator F is not required. Between certain logs there is evident correlation, e.g. the resistivity logs correlate with each other and with the SP log. In practice, in the course of processing, the SP and the gamma-ray logs are replaced by each other many times because of their similar characteristics. The porosity indicator logs, the neutron-gamma, the neutron-neutron and the acoustic logs are necessarily correlated with each other.

With regard to quasi symmetrical logs it can be assumed that operator F can be approximated by their linear combination. The more curves there are in it, mathematically the more probable it is that with one of them the correlation is close. If the theoretical function-connection is not linear, it results in a decrease of the correlation coefficient; this, however, does not considerably influence the result of the subsequent calculations. (If a gradient curve is correlated then in the F operator the derivatives of the symmetrical curves should be used.)

From the above it can be assumed that operator F can be approximated by the linear combination of the $Y_2(X)$, $Y_3(X)$, ..., $Y_N(X)$ logs:

$$F[Y_2(X_i), Y_3(X_i), \dots, Y_N(X_i)] \approx b_1 + b_2 Y_2(X_i) + b_3 Y_3(X_i) + \dots + b_N Y_N(X_i)$$

$$i = 1, 2, \dots, L \quad (3.2)$$

Here parameters b_1, \dots, b_N are further unknowns characterizing the correlation of the functions.

To determine the depth deviation varying from point to point, let us develop in a series the left side of relation (3.1) and to preserve the linearity in $\Delta(X_i)$, i.e. approximating it up to the first term, the following can be written:

$$Y_1[X_i + \Delta(X_i)] \approx Y_1(X_i) + \Delta(X_i) \cdot Y_1'(X_i) \quad i = 1, 2, \dots, L \quad (3.3)$$

Here the following notation was used:

$$Y_1'(X_i) \equiv \left. \frac{dY_1(X)}{dx} \right|_{x=X_i} \quad (3.4)$$

The derivative (3.4) exists for all analog logs since, due to the continuous recording of a finite speed, the curve is continuous and always has a definite tangent not perpendicular to the abscissa. Thereby it satisfies the criterion concerning the existence of the derivative.

If the $Y_1(X_i)$ curve is known from sampling points, then using the function values in the two-two neighbouring digitization points, the derivative (3.4) can be approximated by the formula [OBÁDOVICS 1977]:

$$Y_1'(X_i) \approx \frac{1}{12h} [Y_1(X_{i+2}) - 8Y_1(X_{i+1}) + 8Y_1(X_{i-1}) - Y_1(X_{i-2})] \quad (3.5)$$

$$i = 3, 4, \dots, L - 2$$

where h is the sampling interval. There are other approximations using fewer or more neighbouring function values than Eq. (3.5), but this was chosen because with fewer points, statistical noise would be increased whereas relations with more sampling points result in an increase in machine time. Naturally at both ends of the curves where there are no neighbouring points one has to be content with an approximation with the left or right derivatives [OBÁDOVICS 1977]:

$$Y_1'(X_i) \approx \frac{1}{h} [Y_1(X_{i+1}) - Y_1(X_i)] \quad i = 1, 2$$

$$Y_1'(X_i) \approx \frac{1}{h} [Y_1(X_i) - Y_1(X_{i-1})] \quad i = (L - 1), L$$
(3.6)

As we have seen in Section 2 relative depth deviations can be approximated by a polynomial:

$$\Delta(X_i) \approx a_0 + a_1X_i + a_2X_i^2 + \dots + a_pX_i^p \quad i = 1, 2, \dots, L \quad (3.7)$$

It should be noted that relation (3.7)—disregarding the physical meaning—is mathematically according to Weierstrass' theorem [OBÁDOVICS 1977] in the case of a continuous function, since—choosing a suitably great number of power P —any accuracy of the approximation can be achieved.

Substituting approximation (3.3)—using relation (3.7)—into the left side and, approximation (3.2) into the right side of equation system (3.1) the following is obtained:

$$Y_1(X_i) + Y_1'(X_i) \cdot [a_0 + a_1X_i + a_2X_i^2 + \dots + a_pX_i^p] \approx$$

$$\approx b_1 + b_2Y_2(X_i) + b_3Y_3(X_i) + \dots + b_NY_N(X_i) \quad i = 1, 2, \dots, L \quad (3.8)$$

One can see that instead of the $\Delta(X_i)$, $i = 1, 2, \dots, L$ unknowns of equation system (3.1), in relation (3.8) considerably fewer, only the a_p , $p = 0, 1, \dots, P$ coefficients in the polynomial of the depth deviation and the b_n , $n = 1, 2, \dots, N$ parameters in the linear combinations of the functions, should be determined.

Since in this way the number of unknowns in (3.1) could be made much smaller than the number of equations, relation (3.8) becomes overdetermined

and the unknowns can be determined, e.g. by the method of least squares [JÁNOSY 1965]. Forming the difference of the left and right side of relation (3.8) and then the quadratic sum the following can be written:

$$\Theta = \sum_{i=1}^L \{ Y_1(X_i) + Y_1'(X_i) \cdot [a_0 + a_1 X_i + \dots + a_p X_i^p] - b_1 - b_2 Y_2(X_i) - \dots - b_N Y_N(X_i) \}^2 \quad (3.9)$$

The unknowns are determined so that Θ should be minimal, i.e. the derivatives according to the wanted parameters should be zero:

$$\begin{aligned} \frac{\partial \Theta}{\partial a_p} &= 0, \quad p = 0, 1, \dots, P \\ \frac{\partial \Theta}{\partial b_n} &= 0, \quad n = 1, 2, \dots, N \end{aligned} \quad (3.10)$$

The normal equation system obtained after performing the derivations (3.10) includes $(N + P + 1)$ linear equations and as many unknowns. It should be noted that coefficients b_n , $n = 1, 2, \dots, N$ will not be necessary further on: these are the so called surplus parameters needed only to establish the system of equations.

By introducing matrices the solution of (3.8) will be clearer using the method of least squares. Let:

$$\mathbf{M} = \begin{bmatrix} 1 & Y_2(X_1) & Y_3(X_1) & \dots & Y_N(X_1) & -Y_1'(X_1) & -Y_1'(X_1)X_1 & \dots & -Y_1'(X_1)X_1^p \\ 1 & Y_2(X_2) & Y_3(X_2) & \dots & Y_N(X_2) & -Y_1'(X_2) & -Y_1'(X_2)X_2 & \dots & -Y_1'(X_2)X_2^p \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & Y_2(X_L) & Y_3(X_L) & \dots & Y_N(X_L) & -Y_1'(X_L) & -Y_1'(X_L)X_L & \dots & -Y_1'(X_L)X_L^p \end{bmatrix}$$

and

$$\mathbf{Y}_1 = \begin{bmatrix} Y_1(X_1) \\ Y_1(X_2) \\ \vdots \\ Y_1(X_L) \end{bmatrix} \quad \mathbf{I} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \\ a_0 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} \quad (3.11)$$

In matrix form (3.8) is:

$$\mathbf{M} \cdot \mathbf{I} \approx \mathbf{Y}_1 \quad (3.12)$$

The matrix equation obtained from (3.9) and (3.10) using the method of least squares is:

$$(\mathbf{M}^T \cdot \mathbf{M}) \cdot \mathbf{I} = \mathbf{M}^T \cdot \mathbf{Y}_1 \quad (3.13)$$

where \mathbf{M}^T is the transposed matrix of \mathbf{M} . The solution of (3.13) is:

$$\mathbf{I} = (\mathbf{M}^T \cdot \mathbf{M})^{-1} \cdot (\mathbf{M}^T \cdot \mathbf{Y}_1) \quad (3.14)$$

One can see that the overdetermined linear system of equations (3.8) can be solved relatively easily in a suitably ordered form by means of transposition, multiplication and inversion of the matrix.

Knowing the polynomial coefficients a_0, a_1, \dots, a_p determined by (3.14) the depth deviation curve $\Delta(X_i)$ varying from point to point can numerically be given for every $i = 1, 2, \dots, L$ sampling point by relation (3.7). After calculating deviations $\Delta(X_i)$ the corrected curve values are determined as follows: knowing the values of $\Delta(X_i)$ and the sampling interval h , sampling point X_k nearest to the corrected depth value can be determined. The distance $d(X_k)$ from this can be written as follows:

$$d(X_k) = \Delta(X_i) - (X_k - X_i) \quad (3.15)$$

where:

$$|d(X_k)| < h$$

Knowing X_k and $d(X_k)$ the interpolated value of the corrected function value is:

$$Y_1^{corr}(X_i) = Y_1[X_i + \Delta(X_i)] \approx Y_1(X_k) + d(X_k) \cdot Y_1'(X_k) \quad (3.16)$$

$$i = 1, 2, \dots, L$$

where $Y_1'(X_k)$ can be calculated from (3.5).

In order to preserve the linearity in $\Delta(X_i)$ the series development of (3.3) went up to the first derivative only, (3.16) can be regarded as a first approximation only. Considering the corrected curve always as an initial value the iteration can be continued until the value of the quadratic deviation (3.9) no longer decreases to any great extent, i.e. the form of the corrected curve does not vary any more. It should be noted that because of assumption (3.2) the value of Θ given by (3.9) will not compulsorily approach zero with increasing number of iterations. This does not matter since, according to what was said at the beginning of the section, operator F in (3.1) cannot accurately be given. Moreover parameters b_0, b_1, \dots, b_N in the linear combination of Eq. (3.2) are not directly included in the values of the polynomial calculated from Eq. (3.7); thus, presumably the polynomial is not too sensitive to these parameters. As the results discussed later will also prove, the stipulation that from the good or less good Θ values the parameters belonging to the lowest possible Θ should be chosen seems acceptable even if this Θ is relatively still too high. (We do not intend to determine the function $Y_1(X)$ from the other curves, we only want to match it to them and, at the same time, retain its characteristic features.)

If there are K logs to be corrected and $(N-K)$ logs considered to be correct in depth, where K can be one of the $1, 2, \dots, N$ values, then every iteration phase consists of K cyclically inverted iterations. Taking one of the logs to be corrected for Y_1 , it is corrected in the way described above using the other $(N-1)$ curves.

Then the second, third, K th log will be taken for Y_1 so that the $(N-1)$ curves to be used will include those corrected before. For depth matching, apart from giving the number of iterations, only the order number of the polynomial describing the relative depth deviation in (3.7) should be prescribed.

4. Relative depth matching of well logs and the quantities derived from coring

In the course of geological exploration, for the integrated interpretation of all information depth matching of data of different origin is required. Depth errors may lead to apparent contradiction between well logging and core data. Since the latter represent a small volume of rock, a small depth shift may cause great difference. The depth correction method described in Section 3 cannot directly be applied to this case since cores are not known at equidistant intervals, and — as generally the yield is not complete — the missing neighbouring points make derivation impossible even by approximation.

For phrasing the problem let us consider Fig. 2. The computed porosity logs $Y_1(X_i), Y_2(X_i), \dots, Y_N(X_i), i = 1, 2, \dots, L$, are assumed to be correct in depth in relation to each other. (This can be obtained by the method described in Section 3.) Our aim is to match the quantities derived from the $\Phi(X_m)$, $m = 1, 2, \dots, M$ core samples known at not equidistant sites to these logs. At depth point X_m ($m = 1, 2, \dots, M$), the $Y_1[X_m + \Delta(X_m)], \dots, Y_N[X_m + \Delta(X_m)]$ curve values, taken at the real depth point $(X_m + \Delta(X_m))$, belong to the $\Phi(X_m)$ quantity to be matched. The function describing the depth deviation is also illustrated in the figure. The task is to define the function $\Delta(X)$ since in the knowledge of this, depth correction means the transfer of the corresponding $\Phi(X_m)$ quantity from the X_m depth point to the $[X_m + \Delta(X_m)]$ point.

One can see that as opposed to the depth correction of the well logs, here not a new function value will be calculated in every sampling point but the corresponding quantity will be transferred to a new depth. The steps of the solution are similar to those in Section 3. We assume that the values belonging to each other are related; this relationship can be described by an operator F :

$$\Phi(X_m) = F[Y_1(X_m + \Delta(X_m)), Y_2(X_m + \Delta(X_m)), \dots, Y_N(X_m + \Delta(X_m))] \\ m = 1, 2, \dots, M \quad (4.1)$$

Operator F is approximated by a linear combination of the curves:

$$F[Y_1[X_m + \Delta(X_m)], \dots, Y_N[X_m + \Delta(X_m)]] = \\ = b_0 + b_1 Y_1[X_m + \Delta(X_m)] + \dots + b_N Y_N[X_m + \Delta(X_m)] \quad (4.2) \\ m = 1, 2, \dots, M$$

Developing in a series the right side of Eq. (4.1), and — for the linearity in $\Delta(X_m)$ — approximating it up to the first term, we can write:

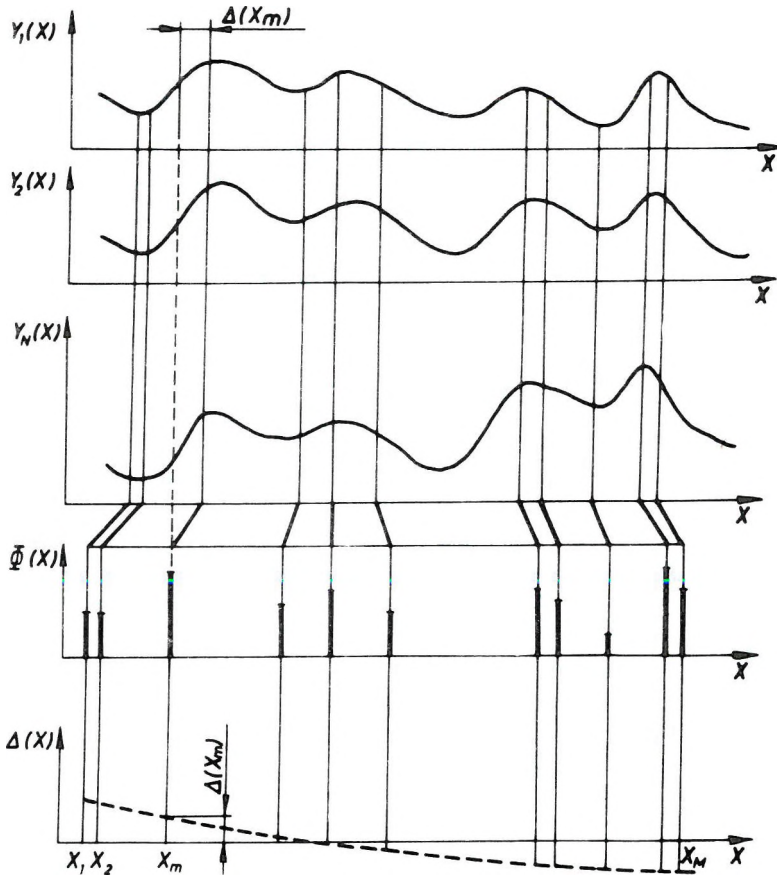


Fig. 2. Matching of core data to porosity–depth functions derived from well logging curves $Y_1(X)$, $Y_2(X)$, ..., $Y_N(X)$ — porosity logs considered to be correct in depth; $\Phi(X)$ — porosity values of core samples; $\Delta(X)$ — function of depth deviation

2. ábra. Magadatok mélységének illesztése karotázs mérésekből származtatott porozitás görbékhöz

$Y_1(X)$, $Y_2(X)$, ..., $Y_N(X)$ — mélységileg helyesnek tekintett porozitás szelvények;
 Φ — a magadatokból számított, nem azonos közökként ismert porozitás értékek;
 $\Delta(X)$ — a mélységtérést leíró függvény

Рис. 2. Согласование глубины ядра с кривой пористости, выведенной по скважинным измерениям

$Y_1(X)$, $Y_2(X)$, ..., $Y_N(X)$ — диаграммы пористости, считающиеся правильными по глубине;
 Φ — вычисленные по ядру значения пористости, полученные за неравные интервалы;
 $\Delta(X)$ — функция, описывающая расхождения по глубине

$$Y_n[X_m + \Delta(X_m)] \approx Y_n(X_m) + \Delta(X_m) \cdot Y'_n(X_m) \quad (4.3)$$

$$n = 1, 2, \dots, N \quad m = 1, 2, \dots, M$$

In Eq. (4.3) the notations of Eq. (3.4) were used and the derivative can numerically be approximated by formulae (3.5) and (3.6).

The $\Delta(X_m)$ depth deviation is approximated by a polynomial the same way as in Eq. (3.7):

$$\Delta(X_m) \approx a_0 + a_1 X_m + a_2 X_m^2 + \dots + a_p X_m^p \quad m = 1, 2, \dots, M \quad (4.4)$$

Substituting the approximations (4.2), (4.3) and (4.4) into the system of equations (4.1) one gets:

$$\Phi(X_m) \approx b_0 + \sum_{n=1}^N b_n \left[Y_n(X_m) + Y'_n(X_m) \cdot \sum_{p=0}^P a_p X_m^p \right] \quad (4.5)$$

$$m = 1, 2, \dots, M$$

This system of equations consists of M equations corresponding to the number of core samples and includes $(N + P + 2)$ unknowns from which $(P + 1)$ are the coefficients of the polynomial describing the depth deviation and $(N + 1)$ are the parameters in the linear combination of the well logs. Since Eq. (4.5) contains the products of the parameters b_n $n = 0, 1, \dots, N$ and a_p $p = 0, 1, \dots, P$, the system of equations is not linear. If $M \geq N + P + 2$ then the overdetermined system of equations of this type can be solved by iteration using the method of least squares. Because of series expansion (4.3) even the result obtained by the iteration can be considered only as a first approximation; thus, in order to avoid double iteration it is expedient to look for a perhaps less accurate but clearer and faster method for the solution of Eq. (4.5).

Let us write Eq. (4.5) in the form:

$$\Phi(X_m) \approx \left\{ b_0 + \sum_{n=1}^N b_n Y_n(X_m) \right\} + \left\{ \left[\sum_{n=1}^N b_n Y'_n(X_m) \right] \cdot \left[\sum_{p=0}^P a_p X_m^p \right] \right\} \quad (4.6)$$

$$m = 1, 2, \dots, M$$

For the two terms in braces in equation system (4.6) let us introduce the notations

$$A(X_m) = b_0 + \sum_{n=1}^N b_n Y_n(X_m) \quad (4.7)$$

$$B(X_m) = \left[\sum_{n=1}^N b_n Y'_n(X_m) \right] \left[\sum_{p=0}^P a_p X_m^p \right] \quad m = 1, 2, \dots, M \quad (4.8)$$

Using (4.7) and (4.8), (4.6) can be written as follows:

$$\Phi(X_m) \approx A(X_m) + B(X_m) \quad m = 1, 2, \dots, M \quad (4.9)$$

Relation (4.9) expresses in an illustrative way that the quantity $\Phi(X_m)$ from the core sample can be composed of two terms. The first is a linear combination of the quantities obtained from the well log measurements and the second is the perturbation due to the depth deviation.

Since the $B(X_m)$ part describing the perturbation is probably much smaller than the $A(X_m)$ term, the system of equations (4.6) can be solved in two steps. First let us take the following quadratic sum:

$$\Theta_1 = \sum_{m=1}^M [\Phi(X_m) - A(X_m)]^2 \tag{4.10}$$

As one can see from (4.7), in (4.10) only the parameters $b_n, n = 0, 1, 2, \dots, N$ are included. Their determination can be carried out by minimizing Θ_1 :

$$\frac{\partial \Theta_1}{\partial b_n} = 0, \quad n = 0, 1, 2, \dots, N \tag{4.11}$$

The normal equation system obtained by derivation is linear thus the determination of the unknowns presents no problem.

After calculating the parameters $b_n, n = 0, 1, \dots, N$ the following difference can be formed:

$$\Delta\Phi(X_m) = \Phi(X_m) - A(X_m) \quad m = 1, 2, \dots, M \tag{4.12}$$

This can be approximated by the perturbation term of Eq. (4.9):

$$\Delta\Phi(X_m) \approx B(X_m) \quad m = 1, 2, \dots, M \tag{4.13}$$

In the expression of $B(X_m)$ the polynomial coefficients $a_p, p = 0, 1, \dots, P$ are the only unknowns (see Eq. 4.8) since coefficients $b_n, n = 0, 1, \dots, N$ were calculated before. For the computation of a_p , let us produce the following quadratic sum:

$$\Theta_2 = \sum_{m=1}^M [\Delta\Phi(X_m) - B(X_m)]^2 \tag{4.14}$$

The value of Θ_2 is required to be minimal i.e. the derivatives according to $a_p, p = 0, 1, 2, \dots, P$ should become zero:

$$\frac{\partial \Theta_2}{\partial a_p} = 0; \quad p = 0, 1, \dots, P \tag{4.15}$$

The normal equation system which is obtained after performing the derivation is also linear in the variables, thus the unknowns can easily be determined.

By introducing matrices, the algorithm of the solution is the following. Let:

$$\mathbf{M}\mathbf{I} = \begin{bmatrix} 1 & Y_1(X_1) & Y_2(X_1) & \dots & Y_N(X_1) \\ 1 & Y_1(X_2) & Y_2(X_2) & \dots & Y_N(X_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & Y_1(X_M) & Y_2(X_M) & \dots & Y_N(X_M) \end{bmatrix} \tag{4.16a}$$

$$\Phi = \begin{bmatrix} \Phi(X_1) \\ \Phi(X_2) \\ \vdots \\ \Phi(X_M) \end{bmatrix} \quad \mathbf{I1} = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_N \end{bmatrix} \quad (4.16b)$$

The solution obtained by minimizing the quadratic sum (4.10) is

$$\mathbf{I1} = (\mathbf{M1}^T \cdot \mathbf{M1})^{-1} \cdot (\mathbf{M1}^T \cdot \Phi) \quad (4.17)$$

The difference (4.12) in vector form is

$$\Delta\Phi = \Phi - \mathbf{M1} \cdot \mathbf{I1} \quad (4.18)$$

Knowing the coefficients $b_n, n = 1, 2, \dots, N$ let us take

$$C(X_m) = \sum_{n=1}^N b_n Y_n'(X_m) \quad m = 1, 2, \dots, M$$

$$\mathbf{M2} = \begin{bmatrix} C(X_1) & X_1 C(X_1) & \dots & X_1^p C(X_1) \\ C(X_2) & X_2 C(X_2) & \dots & X_2^p C(X_2) \\ \vdots & \vdots & \dots & \vdots \\ C(X_M) & X_M C(X_M) & \dots & X_M^p C(X_M) \end{bmatrix} \quad \mathbf{I2} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_p \end{bmatrix} \quad (4.19)$$

The solution obtained from the minimization of the quadratic sum (4.14) is:

$$\mathbf{I2} = (\mathbf{M2}^T \cdot \mathbf{M2})^{-1} \cdot (\mathbf{M2}^T \cdot \Delta\Phi) \quad (4.20)$$

The non-linear, overdetermined equation system of (4.5) has been reduced to two linear systems of equations, to be solved one after the other, by the matrix algorithm of (4.16)–(4.20).

By means of solution (4.20) obtained for the coefficients of the polynomial describing the depth deviation the numerical value of the depth deviation ΔX_m can be computed for every core sample from relation (4.4). Depth correction means the transferring of the quantities $\Phi(X_m), m = 1, 2, \dots, M$ consecutively from depth X_m to depth $[X_m + \Delta(X_m)]$. Since the series expansion (4.3) was performed only up to the first derivative, the depth correction can be considered only as a first approximation. Considering the new depth values always as initial data, the method can be repeated till the values of the quadratic differences, (4.10) and (4.14), begin to decrease substantially. Note that core sampling point X_m should not necessarily coincide with one of the sampling points of well log curves since interpolated curves can be obtained, for example, by formula (4.3) as well.

In the algorithm it was assumed that the accuracy of the depth data of the well log measurements is much greater than that of coring because of the continuous measurement, thus the depth correction was performed only for the depth values of the core samples. The point of interest in the method is that the correlation coefficients often needed in practice are obtained together with the depth correction.

5. Relative depth matching of well logs and geological columns

In the course of industrial application it is often the case that the geological column obtained from coring or approximately known from neighbouring wells should be made accurate by means of the well logs of the given borehole. The $Y_1(X_j), Y_2(X_j), \dots, Y_N(X_j), j = 1, 2, \dots, L$ well logs sampled and already corrected according to depth by the method described in Section 3 are illustrated in Fig. 3. The approximate knowledge of the geological column means that the lithological code representing the rock type cannot unconditionally be given even in a first approximation at every sampling point. (This may, for instance, be due to the insufficient core yield.) Moreover, where it is known, at that sampling point the indices $k = 1, 2, \dots, K$ are introduced in order to differentiate the rock types. Thus $Y_n^{(k)}(X_j)$ means that the X_j sampling value of the n th well log can be assigned to the rock type denoted by the index k . Let the number of sampling points belonging to the rock types denoted by the indices $k = 1, 2, \dots, K$ consecutively be $J_k = J_1, J_2, \dots, J_K$. As in the previous Section, we consider that the well logs are correct in depth, thus the X_j place of the j th lithological code will be corrected at the sampling points for every j . We can assume that on the well logging curves some $[X_j + \Delta(X_j)]$ real depth value belongs to the X_j place to be corrected.

Approximating the function values taken at the real depth points the same way as was done for (3.3), we can write:

$$Y_n^{(k)}[X_j + \Delta(X_j)] \approx Y_n^{(k)}(X_j) + \Delta(X_j) \cdot Y_n'(X_j) \tag{5.1}$$

$$j = 1, 2, \dots, L \quad n = 1, 2, \dots, N$$

The depth deviation function $\Delta(X_j)$ is approximated by a polynomial, as in (3.7):

$$\Delta(X_j) \approx a_0 + a_1 X_j + \dots + a_p X_j^p = \sum_{p=0}^P a_p X_j^p \tag{5.2}$$

$$j = 1, 2, \dots, L$$

Substituting approximation (5.2) into relation (5.1) we get:

$$Y_n^{(k)}[X_j + \Delta(X_j)] \approx Y_n^{(k)}(X_j) + \left(\sum_{p=0}^P a_p X_j^p \right) Y_n'^{(k)}(X_j) \tag{5.3}$$

The average of the function values corrected by Eq. (5.3) can be calculated by rock types for every well logging curve:

$$A'_{nk} = \frac{1}{J_k} \sum_{j=1}^L Y_n^{(l)}[X_j + \Delta(X_j)] \cdot \delta_{kl}(X_j) =$$

$$= \frac{1}{J_k} \sum_{j=1}^L \left[Y_n^{(l)}(X_j) + Y_n'^{(l)}(X_j) \cdot \sum_{p=0}^P a_p X_j^p \right] \cdot \delta_{kl}(X_j) \tag{5.4}$$

$$n = 1, 2, \dots, N \quad k = 1, 2, \dots, K$$

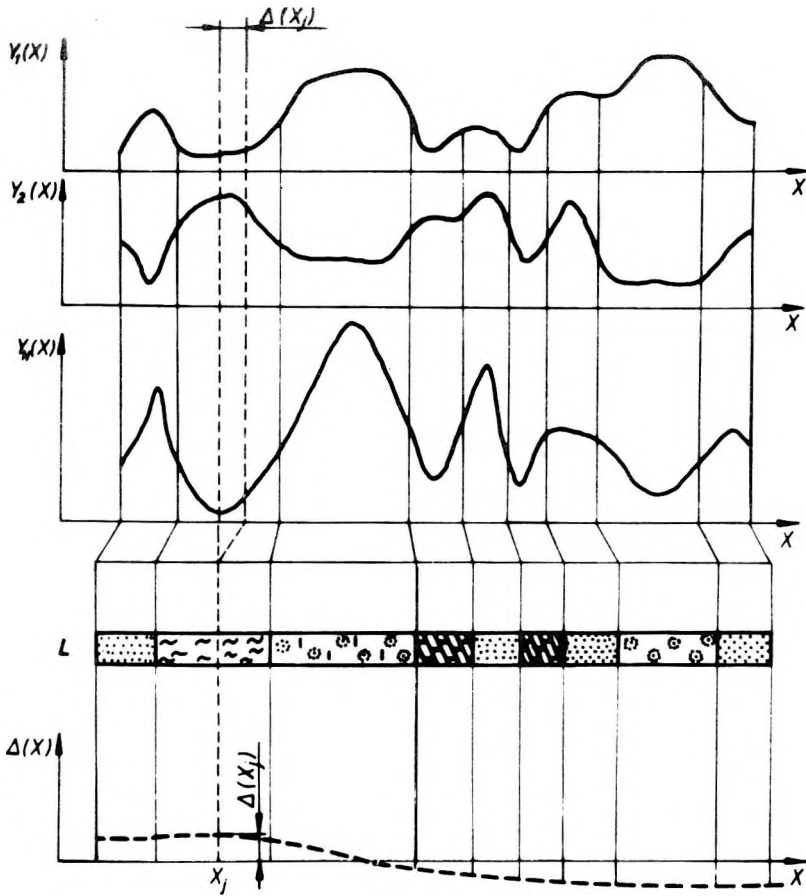


Fig. 3. Matching of geological column to well logging curves
 $Y_1(X), Y_2(X), \dots, Y_N(X)$ — well logs considered to be correct in depth; L — lithological column;
 $\Delta(X)$ — function of depth deviation

3. ábra. Geológiai rétegsor karotázs görbékhez történő igazítása
 $Y_1(X), Y_2(X), \dots, Y_N(X)$ — mélységleg helyesnek tekintett karotázs szelvények; L — litológiai
 rétegsor; $\Delta(X)$ — a mélységtérést leíró függvény

Рис. 3. Согласование литологической колонки с каротажными кривыми
 $Y_1(X), Y_2(X), \dots, Y_N(X)$ — каротажные диаграммы, считающиеся правильными по глубине;
 L — литологическая колонка; $\Delta(X)$ — функция, описывающая расхождения по глубине

where $\delta_{kl}(X_j)$ is the Kronecker-delta:

$$\delta_{kl}(X_j) = \begin{cases} 1, & \text{if } l=k \\ 0, & \text{if } l \neq k \end{cases}$$

In formula (5.4) the summation should be performed log by log at places of the same k lithological code and the sum should be divided by the number of points in the sum. If there are N curves and K different lithological codes then a total of $N \cdot k$ different values of A'_{nk} will be obtained.

Let us introduce the following quantities that can numerically be calculated from the well logging curves:

$$A_{nk} \equiv \frac{1}{J_k} \sum_{j=1}^L Y_n^{(l)}(X_j) \cdot \delta_{kl}(X_j) \quad n = 1, 2, \dots, N \quad k = 1, 2, \dots, K \quad (5.5)$$

$$A_{nk}^{(p)} \equiv \frac{1}{J_k} \sum_{j=1}^L X_j^p \cdot Y_n^{(l)}(X_j) \delta_{kl}(X_j) \quad (5.6)$$

$$n = 1, 2, \dots, N, \quad p = 0, 1, 2, \dots, P, \quad k = 1, 2, \dots, K$$

Expression (5.5) means the average function value of the n th curve belonging to the k th lithological code. Expression (5.6) represents the mean of the derivative of the n th curve weighted by the corresponding power of the depth value belonging to the k th rock type.

Using expressions (5.5) and (5.6) the corrected mean value defined by relation (5.4) can be written — after some rearrangement — as follows:

$$A'_{nk} = A_{nk} + \sum_{p=0}^P a_p \cdot A_{nk}^{(p)} \quad n = 1, 2, \dots, N \quad k = 1, 2, \dots, K \quad (5.7)$$

The quadratic sum of the deviations from the average can be produced for every rock type and for every type of well logging:

$$\Theta_{nk} = \sum_{j=1}^L \{Y_n^{(l)}[X_j + \Delta(X_j)] - A'_{nk}\}^2 \cdot \delta_{kl}(X_j) \quad (5.8)$$

$$n = 1, 2, \dots, N \quad k = 1, 2, \dots, K$$

where the summation for j relates to places of the same rock type.

Substituting approximation (5.3) and relation (5.7) into the quadratic sum of (5.8), we get:

$$\Theta'_{nk} = \sum_{j=1}^L \left\{ Y_n^{(l)}(X_j) + \left(\sum_{p=0}^P a_p X_j^p \right) \cdot Y_n^{(l)}(X_j) - A_{nk} - \sum_{p=0}^P a_p A_{nk}^{(p)} \right\}^2 \cdot \delta_{kl}(X_j) \quad (5.9)$$

$$n = 1, 2, \dots, N \quad k = 1, 2, \dots, K$$

After rearranging expression (5.9) and summing by curves and by rock types we can write:

$$\Theta = \sum_{n=1}^N \sum_{k=1}^K \sum_{j=1}^L \left\{ [Y_n^{(l)} X_j - A_{nk}] + \left[\sum_{p=0}^P a_p (X_j^p \cdot Y_n^{(l)}(X_j) - A_{nk}^{(p)}) \right] \right\}^2 \delta_{kl}(X_j) \quad (5.10)$$

The meaning of the terms in the first square brackets of the quadratic sum (5.10) is clear: the mean value belonging to the given rock and to the given curve and calculable by (5.5) should be subtracted from the respective curve value for every curve and every sampling point. The second square brackets contain the coefficients $a_p, p = 0, 1, \dots, P$ of the polynomial describing the deviation which is to be determined. Furthermore, the derivatives of the logs weighted with the powers of the depth value and the respective mean derivatives weighted with the powers of the depth as defined by formula (5.6) can also be calculated numerically.

The coefficients of the polynomial describing depth deviations are determined so that (5.10) should be minimal, i.e. the derivatives according to the variables should be zero:

$$\frac{\partial \Theta}{\partial a_p} = 0, \quad p = 0, 1, 2, \dots, P \tag{5.11}$$

By performing the derivations of (5.11), a linear system of equations is obtained for the coefficient of the polynomial consisting of $(P + 1)$ equations and including $(P + 1)$ unknowns. The solution ensures that the quadratic sum of the deviations from the means characterizing the rock types will be minimal for each log. The solution using matrix formalism is the following. Let:

$$\mathbf{M} = \begin{bmatrix} \sum_{n=1}^N (Y_n^{(k)}(X_1) - A_{nk}^{(0)}) & \sum_{n=1}^N (X_1 Y_n^{(k)}(X_1) - A_{nk}^{(1)}) & \dots & \sum_{n=1}^N (X_1^P Y_n^{(k)}(X_1) - A_{nk}^{(P)}) \\ \sum_{n=1}^N (Y_n^{(k)}(X_2) - A_{nk}^{(0)}) & \sum_{n=1}^N (X_2 Y_n^{(k)}(X_2) - A_{nk}^{(1)}) & \dots & \sum_{n=1}^N (X_2^P Y_n^{(k)}(X_2) - A_{nk}^{(P)}) \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{n=1}^N (Y_n^{(k)}(X_L) - A_{nk}^{(0)}) & \sum_{n=1}^N (X_L Y_n^{(k)}(X_L) - A_{nk}^{(1)}) & \dots & \sum_{n=1}^N (X_L^P Y_n^{(k)}(X_L) - A_{nk}^{(P)}) \end{bmatrix} \tag{5.12}$$

Matrix \mathbf{M} consists of as many lines in as many sampling points the lithological code is known. The $A_{nk}^{(p)}$ weighted mean values of the derivatives previously calculated by formula (5.6) for the k th lithological code should be subtracted from the derivatives of each log multiplied by the powers of the depth. Further notations are:

$$\mathbf{Y} = \begin{bmatrix} \sum_{n=1}^N (A_{nk} - Y_n^{(k)}(X_1)) \\ \sum_{n=1}^N (A_{nk} - Y_n^{(k)}(X_2)) \\ \vdots \\ \sum_{n=1}^N (A_{nk} - Y_n^{(k)}(X_L)) \end{bmatrix} \quad \mathbf{I} = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_P \end{bmatrix} \tag{5.13}$$

The number of the lines of vector \mathbf{Y} corresponds to the number of the columns of matrix \mathbf{M} . Here, the curve values taken at the sampling points should be subtracted from the mean function value defined by formula (5.5) for the k th lithological code. The solution obtainable by minimizing (5.10) using matrices (5.12) and (5.13) is:

$$\mathbf{I} = (\mathbf{M}^T \cdot \mathbf{M})^{-1} \cdot (\mathbf{M}^T \cdot \mathbf{Y}) \quad (5.14)$$

Knowing the polynomial coefficients a_0, a_1, \dots, a_p determined by solution (5.14) the $\Delta(X_1)$ curve defining the depth deviation can be obtained by means of relation (5.2) for the sampling points $i = 1, 2, \dots, L$.

The correction of the lithological column consists of transferring the places of lithological code jumps i.e. those of the layer boundaries from place X_j to the $[X_j + \Delta(X_j)]$ point by correction (5.2), and thus a new, corrected lithological column is obtained. Since the series expansion (5.1) was performed only up to the first term in order to preserve the linearity in $\Delta(X)$, the corrected geological column can be regarded as a first approximation. If averages (5.5), (5.6) and matrices (5.12), (5.13) are determined according to the new lithology, the iteration can be continued by solution (5.14) till the value of Θ defined by (5.10) substantially decreases.

To sum up, lithological columns are corrected by the above mathematical statistical method using the constraint describing depth deviations by a polynomial so that the quadratic sum of the differences between the measured values and the respective means should be minimal for the entirety of rock types with different mean values on different logs.

6. Conclusions

Relative depth matching of the information obtained from boreholes is an essential condition for interpretation purposes. The elaborated mathematical statistical method makes it possible for a computer to be used for the intermediate step between measurement and interpretation, i.e. for relative depth matching. It was illustrated that the measuring features enable the value of the depth deviation varying from point to point to be approximated by a polynomial. In this way the accordion-like depth correction is given a mathematical phrasing.

The method enables the simultaneous correction of all the given logs but it is possible that supposing certain curves to be correct in depth the others may be matched to them. It follows from the mathematics of the method that at the boundaries there are no missing values left thus the number of depth points will not change during correction. The method is suitable for dealing with the problems of matching core data and lithological columns to well logs as well. Depth deviations are determined by calculation — instead of trials — thus being

substantially faster even than the cross correlation method in spite of the fact that — depending on the order of the polynomial — higher order deviations are also considered.

The method induces hope that in production drillings the parameters of reservoir geology may be determined with sufficient accuracy without coring, merely from well logging, using the correlation coefficients from certain exploration drillings. To apply the method in practice a computer program was written [SZENDRŐ 1978, 1980]. Its description and the experience gained with its application as well as the results are due to be dealt with in a further paper.

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A MÉLYFŰRÁSBÓL SZÁRMAZÓ INFORMÁCIÓK AUTOMATIKUS RELATÍV MÉLYSÉGEGYEZTETÉSE I. ELMÉLETI ÁTTEKINTÉS

SZENDRŐ Dénes

A mélyfúrásból származó információk alapján történő értelmezés előfeltétele, hogy az adott kútban levő adatok mélységhelyesek legyenek. A közös mélységpontra hozás céljából feltesszük, hogy a pontról pontra változó nagyságú $A(X)$ relatív mélységeltérések polinommal közelíthetők. Sorba fejtve a mélységegyeztetésben részt vevő $Y(X)$ szelvényt a mélység szerint, a Taylor-sorban levő $A(X)$ mélységeltérés éppen a polinom helyettesítési értékével egyezik meg. Minimalizálva az egyeztetésben részt vevő adatokból képezhető hibafüggvényt, a polinom együtthatói kiszámíthatók, s a korrigált adatok a sorfejtés alapján megkaphatók. Az eljárást néhányszor az összes mennyiségre megismételve, a számított értékek a mélységkorrigált adatokhoz konvergálnak. A módszer nem csak a lineáris elcsúszások korrigálására alkalmas, hanem a polinom fokszámától függően „harmonikázó” eltolódások kiküszöbölésére is. Ha a mélységeltérést leíró polinom fokszáma nulla, azaz konstans elcsúszásról van szó, akkor a módszer a hagyományos keresztkorrelációs eljárással megegyező eredményt szolgáltat. Mivel azonban az elcsúszást kiszámolja, a hagyományos eljárásnál lényegesen gyorsabb. Az eljárás alkalmas a karotázs szelvények közötti, a magadatok és a karotázs szelvények közötti, s a geológiai rétegsor és a karotázs szelvények közötti mélységeltérések korrigálására.

**АВТОМАТИЧЕСКОЕ СОГЛАСОВАНИЕ ДАННЫХ СКВАЖИННОЙ ГЕОФИЗИКИ
ПО ОТНОСИТЕЛЬНЫМ ГЛУБИНАМ
I. ТЕОРЕТИЧЕСКОЕ ОБОСНОВАНИЕ**

Денеш СЕНДРЁ

Предпосылкой интерпретации данных скважинной геофизики является правильное определение глубины, к которой относятся те или иные данные. Для приведения данных к общей глубинной точке предполагается, что относительные расхождения по глубине $A(X)$, величина которых меняется от точки к точке, аппроксимируются полиномом. После разложения подвергнутой согласованию по глубине кривой $Y(X)$ в ряд, отклонение по глубине $A(X)$ в ряду Тэйлора точно совпадает со значением подстановки полинома. После приведения к минимуму функции ошибок, образуемой из участвующих в согласовании данных, можно вычислить коэффициенты полинома и получить исправленные данные на основе разложения в ряд. Если такая процедура повторяется несколько раз для всех величин, вычисленные значения приблизятся к исправленным за расхождение по глубине данным. Метод пригоден не только для исправления линейных смещений, но также и для устранения отклонения с переменными знаками в зависимости от степени полинома. Если степень смещение, метод дает результат, совпадающий с традиционным методом взаимной корреляции. Поскольку, однако, при этом вычисляется смещение, данный метод значительно быстрее традиционного, он также позволяет ввести поправки за расхождения по глубине между каротажными диаграммами, между керном и каротажными диаграммами, а также между литологической колонкой и каротажными диаграммами.

