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DETERMINATION OF FILTRATION COEFFICIENT OF WATER-BEARING SAND LAYERS BY WELL LOGGING

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Darcy's law for fluid filtration and Kozeny's relationship for permeability are of a similar form. Utilizing this similarity a functional relationship can be derived for well-sorted water-bearing sands, connecting the coefficient of tortuosity and the specific grain surface (i.e. filtration path) with Kozeny's effective grain diameter and the average shape factor of the grains (grain geometry). The coefficient of tortuosity and Kozeny's effective grain diameter can be determined from resistivity- and porosity logs, the specific surface can also be found if the effective grain diameter is known. The kinematic viscosity of pore water can be computed from layer temperature, obtained from the temperature log. Substituting these data into Darcy's law, the filtration coefficient can be determined with a fair approximation. The formulae derived in the present paper are especially recommended for investigations of water-bearing layers within coal seams, with the aim of protection against water inrush.

Keywords: filtration coefficient, specific surface, tortuosity, permeability

1. Introduction

Between the hanging- and footwalls of coal-bearing formations one frequently finds such water-bearing clastic sediments, shaly or pure sands, sandstones, gravels or pyroclastics which must be opened up and drained by means of boreholes, either from the surface or from the mine galleries, for the sake of protections against flooding. The water yield of such formations can be determined from their so-called Darcy filtration coefficient, or *k*-factor. The *k*-factor of porous water-bearing layers is determined from the temporal changes of the water level created by means of drainage or absorption in a well (or several wells) drilled into the given layer, the amount of water yielded by pumping tests, as well as from data of the monitoring wells. This way of determining the *k*-factor, however, is time-consuming and, for several reasons, inaccurate.

The above-described way to determine the *k*-factor is rarely possible in mines because of technical difficulties. On the other hand, geophysical well logging in exploratory boreholes penetrating the given water-bearing layer provides a continuous record of the variation of certain geophysical quantities, which are directly related to the *k*-factor. For logging purposes sondes of small diameter are available, which can be used even in holes of 60 mm diameter. The movement of the probes in the drill holes and the recording of the measured data are performed in the mines by portable devices.

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The technique to be described is most advantageous for hydrogeologic and stratigraphic studies of water-bearing layers of coal measures with several coal seams, where the deeper coal beds are explored by drilling from the above-lying mines.

2. Relationship between filtration path and grain geometry

The filtration coefficient of sands and sandstones consisting of mixed-grains can be expressed in terms of the properties of pore-water and rock matrix, according to Darcy's law, as

$$k = \frac{1}{5}\frac{\gamma}{\eta}\frac{n^3}{(1-n)^2}\left(\frac{D_h}{\bar{\alpha}}\right)^2 \tag{1}$$

where γ is the specific weight of the pore water, η is its dynamic viscosity, n the void ratio (porosity) of the rock, D_h the effective grain diameter introduced by Kozeny, and $\bar{\alpha}$ the average shape factor of the grains [Kovács 1981]. In porous rocks the flow of water depends on four factors: (i) the first of these is connected with the geometrical characteristics of the grains (grain shape, grain size, grain size distribution); (ii) the second with such rock properties as compaction, cementation and tortuosity. These characteristics are in connection neither with the fluid properties affecting the flow (specific weight and viscosity) nor with the type of motion of the fluid (laminary and turbulent flows). The properties depending on the rock can be contracted into a single factor K, which is termed permeability, viz.

$$K = \frac{1}{5}\frac{n^3}{(1-n)^2}\frac{D_h}{\bar{\alpha}}; \tag{2}$$

(iii) the third factor affecting water flow is the ratio describing the fluid properties. For this we have

$$\frac{\gamma}{\eta} = \frac{g}{\nu} \tag{3}$$

where $g = 9.81\ \mathrm{ms}^{-2}$ is the gravitational acceleration, ν is the kinematic viscosity of water, related to layer temperature, T (°C) as

$$\nu = \frac{1.778 \cdot 10^{-2}}{1 + 0.0337T + 0.000221T^2} \quad [\mathrm{m}^2\mathrm{s}^{-1}]; \tag{4}$$

(iv) the fourth — generally neglectable — factor expresses the dependence on pressure and salinity. Summing up, the factor describing fluid properties becomes

$$\frac{g}{\nu} = \frac{9.81(1+3.37\cdot 10^{-2}T+2.21\cdot 10^{-4}T^2)}{1.778\cdot 10^{-2}} = 551.74\,C \quad [\mathrm{m^{-1}s^{-1}}] \tag{5}$$

where C denotes the temperature factor inside the brackets. Making use of Kozeny's equation, Eq. (2) can be rewritten so as to express the permeability of rock in terms of compaction, $n^3/(1-n)^2$, tortuosity coefficient, t and specific surface of mineral grain constituents S_v:

$$K = \frac{1}{5}\,\frac{n^3}{(1-n)^2}\left(\frac{1}{tS_v}\right)^2 \tag{6}$$

[Pirson 1963]. For a bundle of capillary tubes the numerical factor is 1/2, for real rocks, however, an empirical factor of 1/5 has been found [Carman 1956].

By Eqs. (2) and (6) we can relate the coefficient of tortuosity and the specific grain surface, that is the filtration path, to Kozeny's effective grain diameter and the average grain shape factor, that is, to the grain geometry:

$$\frac{1}{tS_v} = \frac{D_h}{\bar{\alpha}} \tag{7}$$

Since porosity, tortuosity and specific grain surface can be determined by geophysical well logging, the permeability as well as the filtration coefficient, k can be determined by means of Eqs. (6) and (1).

3. Determination of the factors affecting permeability

Determination of the porosity

The porosity n can be determined with the required accuracy either from individual porosity logs (density-, neutron-, or acoustic logs) or from their combinations.

If we have resistivity logs only, we cannot compute the real porosity by means of the formation factor, as Archie's formula or its modified forms do not suffice in themselves to solve this problem in the case of fresh-water-bearing porous formations.

Determination of the tortuosity coefficient

For a model of a bundle of capillary tubes the tortuosity coefficient is $t^2 = Fn$ [Pirson 1963]. Real rock however, has proved to be better approximated by the relation

$$t^{1.67} = Fn \tag{8}$$

[Ogbe–Bassiouni 1978], where

$$F = \frac{R_0}{R_w} \tag{9}$$

is the formation factor [Pirson 1963]. R_0 denotes the electric resistivity of the rock saturated by formation water, R_w that of the formation water, in situ. By Eqs. (8) and (9) the tortuosity coefficient can be expressed in terms of well-log data as

$$t = \left(\frac{R_0}{R_w} n\right)^{0.6} \tag{10}$$

Determination of the specific surface

The specific surface of a homogeneously dispersed system of spheres is

$$S_v = \frac{6}{D_h} \tag{11}$$

[cf. Pirson 1963]. In filtration calculations it is recommended that Kozeny's effective grain diameter be utilized since this takes into account grain size as well as its distribution [Kozeny 1953]. The effective grain diameter is defined as the diameter of spheres in a homogeneously dispersed system of spheres built up of identical diameter and density spheres in such a manner that the whole system has the same surface as the actual one.

Determination of Kozeny's effective grain-size diameter

In order to determine the effective grain-size diameter from well-log data, one should use several empirical formulae. In essence, these relationships serve as a substitute for the grain-size distribution histogram. If the grain-size distribution can be approximated by a mathematically easily describable curve then a relationship can be established between the characteristic grain diameters, which would also involve the "inequality factor" U [cf. Kovács 1981]. For the grain-size distribution of sand, the inequality factor U is defined as

$$U = \frac{D_{60}}{D_{10}} \tag{12}$$

where D_{60} denotes the grain diameter belonging to 60% on the grain-size distribution curve, while the value D_{10} corresponding to 10% is the so-called Hazen's standard grain diameter. For well-sorted water-bearing sands we usually have $2.0 \leqq U \leqq 2.5$ [Alger 1971].

For different kinds of sand the ratio of their characteristic grain-diameters is related to the logarithm of the inequality factor by

$$\frac{D_h}{D_{10}} = 1.919 \log U + 1.0 \tag{13}$$

in the above-mentioned range of U (see *Fig. 1.*), where D_h is the effective Kozeny grain size. If U changes between 2.0 and 2.5, the values of log U will range between 0.301 and 0.398, that is, on the average, the right-hand-side of Eq. (13) will be 1.671. Taking the mean value, the error is negligibly small, ± 0.093; that is, for fairly well-sorted fresh-water-bearing sands the effective Kozeny grain size is approximately related to the standard grain diameter of Hazen as

$$D_h = 1.671 \cdot D_{10} \tag{14}$$

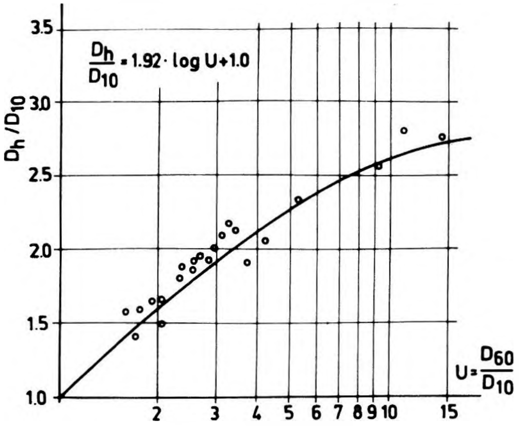

Fig. 1. Ratio of characteristic grain diameters for sand, as a function of the inequality factor [after KOVÁCS 1981]

1. ábra. Homokok jellemző szemcseátmérő hányadosa az egyenlőtlenségi együtthatójuk függvényében [KOVÁCS 1981 nyomán]

Рис. 1. Характерные отношения диаметров зерен в песках как функции коэффициента неравномерности [по KOVÁCS 1981]

Taking the mean value of the two empirical formulae proposed by ALGER [1971], in order to connect Hazen's standard grain diameter with the formation factor (cf. Eq. 9), we get

$$D_{10}[\mathrm{m}] = 5.22 \cdot 10^{-4} \log \frac{R_0}{R_w} \tag{15}$$

presented in *Fig. 2.* From Eqs. (14) and (15) the effective grain diameter becomes

$$D_h[\mathrm{m}] = 8.723 \cdot 10^{-4} \log \frac{R_0}{R_w} \tag{16}$$

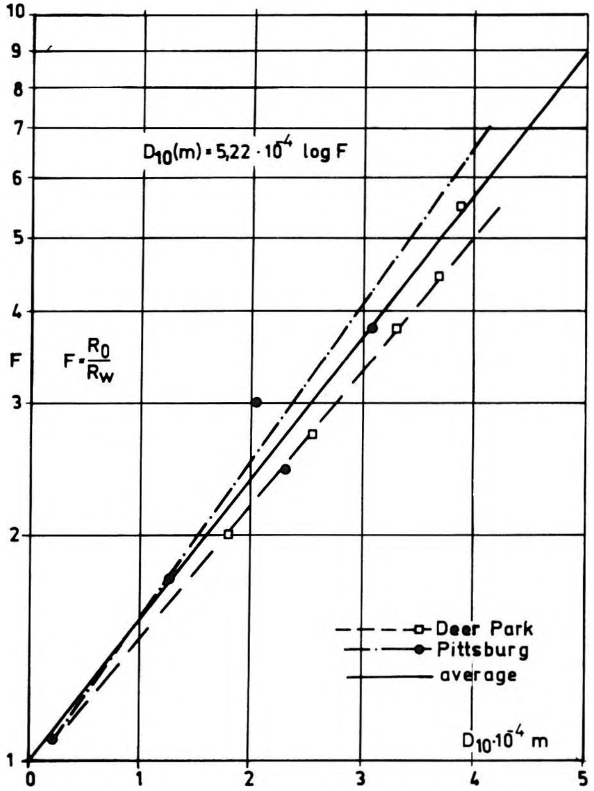

Fig. 2. Formation factor as a function of Hazen's standard grain diameter [after ALGER 1971]

2. ábra. A Hazen-féle mértékadó szemcseátmérő és a formációtényező összefüggése [ALGER 1971 nyomán]

Рис. 2. Зависимость формационного фактора от стандартного диаметра зерен по Хазену [по ALGER 1971]

That is, upon substituting Eq. (16) into (11), the specific surface becomes

$$S_v\left[\frac{m^2}{m^3}\right] = \frac{6}{8.723 \cdot 10^{-4} \log \dfrac{R_0}{R_w}} \tag{17}$$

If we substitute Eqs. (10) and (17) into (6) and carry out numerical operations, we obtain the following expression for permeability:

$$K[\mathrm{m}^2] = 4.227 \cdot 10^{-9} \frac{n^3}{(1-n)^2} \frac{\left(\log \dfrac{R_0}{R_w}\right)^2}{\left(\dfrac{R_0}{R_w} n\right)^{1.2}} \tag{18}$$

(Let us recall here that $10^{-9}\ \mathrm{m}^2 = 10^3$ darcy).

If the rock is poorly sorted ($U \gg 2.5$) and/or it contains a significant amount of flat grains ($\bar{\alpha} \gg 9$), the coefficient figuring in Eq. (18) should be modified empirically so as to give a more accurate value of the filtration coefficient.

4. Equation for the filtration coefficient

Upon multiplying Eq. (18), describing permeability, by the factor describing the temperature-dependence of the kinematic viscosity of the filtrating water (Eq. 5), an equation is obtained for the filtration coefficient, k:

$$k[\mathrm{ms}^{-1}] = 2.332 \cdot 10^{-6}\, C \frac{n^3}{(1-n)^2} \frac{\left(\log \dfrac{R_0}{R_w}\right)^2}{\left(\dfrac{R_0}{R_w} n\right)^{1.2}} \tag{19}$$

The coefficient C, as a function of temperature, can be read off the curve of *Fig. 3.*

If the assumptions made on grain-size distribution and grain shape are poorly met, the coefficient of Eq. (19) should be corrected on the basis of the grain-size distribution curve or by means of another log (gamma-ray) to achieve a better fit with permeability.

5. A case history

ALGER [1971], describes a study where the relations between grain-size, formation factor and permeability were investigated on nine sand samples taken from water wells. The standard grain diameters, D_{10} were determined from the grain-size distribution curves of the samples; the R_0 resistivities of the samples were measured for pore waters of three different resistivities; the porosities and the formation factors belonging to the three different pore-fluids were determined. These data, as well as the values of permeability and of the filtration coefficient are compiled in *Table I.*

On the basis of this table, the following conclusions can be made:

1. For all samples the formation factor, F, the filtration coefficient, k, and the permeability K decrease with increasing resistivity, R_w of the saturant.

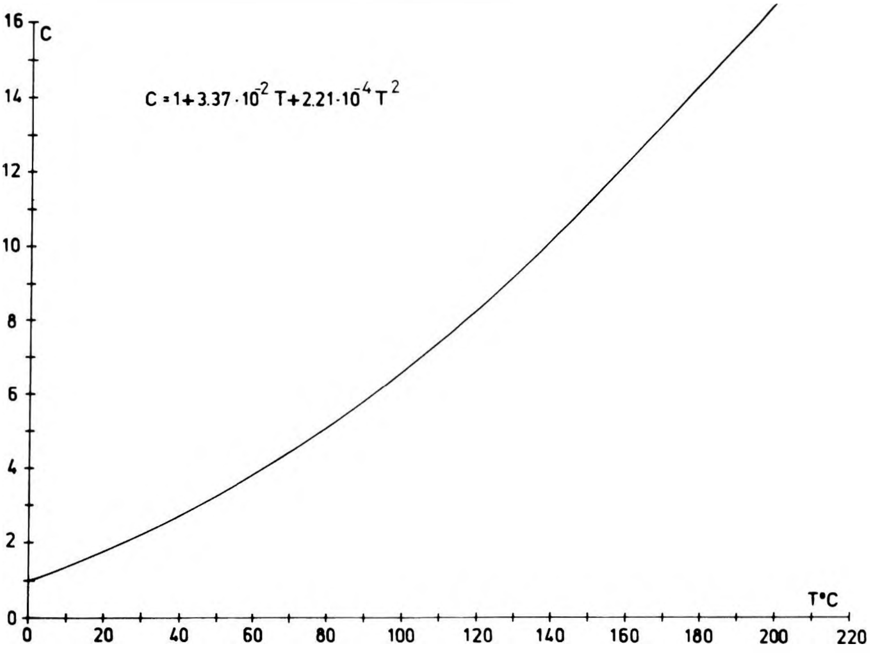

Fig. 3. Temperature factor of filtration coefficient

3. ábra. A szivárgási tényező hőmérsékleti együtthatója

Рис. 3. Температурный коэффициент фильтрационного фактора

2. When the samples were first saturated by brine of $R_w = 1.1$ Ωm resistivity ($5 \cdot 10^3$ ppm NaCl concentration at 25 °C), and subsequently by brine of 7.1 Ωm resistivity ($7 \cdot 10^2$ ppm), the filtration coefficient of the samples did not decrease more than a few per cent: $k(1.1) - k(7.1) = (2-6) \cdot 10^{-9}$ ms^{-1}.

3. The decrease of the filtration coefficient is by an order of magnitude larger if the samples are saturated by fresh water of 32.0 Ωm resistivity ($1.5 \cdot 10^2$ ppm):

$$k(1.1) - k(32.0) = (10-50) \cdot 10^{-9} \text{ ms}^{-1}$$
$$k(7.1) - k(32.0) = (\ 8-32) \cdot 10^{-9} \text{ ms}^{-1}.$$

4. The phenomenon is due to the fact that the resistivity, R_w of water was originally determined for the total mass of water. Rock resistivity R_0 was also accepted as a steady-state value, achieved after saturation and compaction. On the other hand, the resistivity, R_w belonging to the total mass of water has decreased because of ion exchange with the mineral grains, and because of dissociation, surface conductivity and adsorption. The less the original concentration of salt in the water saturating the samples, the larger would become this subsequent resistivity decrease.

No.	D_{10} [μm]	$n=\Phi$	R_w Ωm	$R_w^* =$ $= R_0/F$ $(R_w=1.1)$	R_0 Ωm	$F = \frac{R_0}{R_w}$	K darcy $[10^{-12}\ m^2]$	k$10^{-9}[ms^{-1}]$ $T=2{,}5\ °C,\ C=2.0$	k(1.1)–k(7.1)	k(7.1)–k(32)	k(1.1)–k(32)
1	76.2	0.375	1.1		4.30	3.90	126.3	139	—		
			7.1	—	—	—	—	—			
			32.0	17.6	68.50	2.14	81.1	89		—	50
2	76.2	0.372	1.1		—	—	—	—			
			7.1	—	23.40	3.30	116.0	128	—		
			32.0	22.3	73.60	2.30	87.0	96		32	—
3	76.2	0.375	1.1		4.30	3.90	126.3	139			
			7.1	—	—	—	—	—	—		
			32.0	20.4	79.40	2.48	96.9	107		—	32
4	127.0	0.341	1.1		4.73	4.30	97.8	108	2		
			7.1	6.5	27.75	3.91	95.8	106		8	
			32.0	21.7	93.50	2.92	88.4	98			10
5	177.8	0.358	1.1		4.84	4.40	112.9	125	3		
			7.1	6.3	27.82	3.92	110.3	122		13	
			32.0	22.0	96.60	3.02	98.7	109			16
6	177.8	0.345	1.1		4.92	4.47	101.7	112	3		
			7.1	6.2	27.82	3.92	99.1	109		15	
			32.0	20.4	91.30	2.85	85.4	94			18
7	190.5	0.332	1.1		5.16	4.70	91.8	105	6		
			7.1	6.2	29.10	4.10	89.9	99		—	
			32.0	—	—	—	—	—			—
8	203.2	0.336	1.1		5.16	4.70	94.9	105	2		
			7.1	6.5	30.40	4.27	93.7	103		11	
			32.0	20.9	98.30	3.07	83.1	92			13
9	304.8	0.340	1.1		5.50	5.00	98.6	109	2		
			7.1	6.0	29.80	4.20	96.6	107		—	
			32.0	—	—	—	—	—			—

Table I. Relation between grain-size, formation factor and permeability [after ALGER 1971]

I. táblázat. A szemcsenagyság, a formáció faktor és a permeabilitás közötti összefüggés [ALGER 1971 után]

Таблица I. Соотношение между размером зерен, формационным фактором и проницаемостью [по ALGER 1971]

5. The ratio of the resistivity, R_0 of the samples to the resistivity, R_w measured in the total mass of water is not equal to the real formation factor of the rock, as the resistivity of water changes inside the pores of the rock.

6. According to Table I, the decrease of the filtration coefficient of the rock samples in only 2–3% in the case of saturation by water of 1.1 Ωm and 7.1 Ωm resistivity, respectively, even though their porosity is 33.2–35.8%, and their standard grain-size D_{10} lies in the 127.0–203.2 μm range.

The resistivity of 32.0 Ωm of the fresh water has decreased – using the formation factor F belonging to $R_w = 1.1$ Ωm – to 22.3–17.6 Ωm inside the pores, that is by some 30–45%. On the other hand, the 7.1 Ωm dropped only to 6.5–6.0 Ωm, i.e. it has become only 8.5–15.5 per cent less. If we substitute the real formation factors – computed from the R_w specific resistivities valid inside the pores – into Eqs. (18) and (19), the permeabilities and filtration factors, respectively, of the different rock samples will be the same, independently of the specific resistivities of pore water and of rock, respectively.

The use of Eqs. (18) and (19) is primarily recommended for the delineation of producing layers penetrated by water-prospecting boreholes and for determining the expectable water yield. Additionally, these equations can be used for designing the protection system against water inrush in coal mines, by determining the filtration coefficients of water bearing sandstones and by locating running sand layers.

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VÍZTÁROLÓ HOMOKRÉTEGEK SZIVÁRGÁSI TÉNYEZŐJÉNEK MEGHATÁROZÁSA FÚRÓLYUKSZELVÉNYEZÉSSEL

CSÓKÁS János

A Darcy-féle szivárgási egyenlet és a Kozeny-féle permeabilitás formula hasonló. Ennek alapján jól osztályozott víztároló homokokra felírható a tortuozitási együttható és a fajlagos szemcsefelület (a szivárgási pálya), valamint a Kozeny-féle hatékony szemcseátmérő és a szemcsék átlagos alaki tényezője (a szemcse-geometria) közötti függvény. A tortuózitási tényezőt és a Kozeny-féle hatékony szemcseátmérőt a fajlagos ellenállás és a porozitás szelvényekből meg lehet határozni. A fajlagos felületet pedig a hatékony szemcseátmérő ismeretében lehet megadni. Hőmérséklet szelvényből kapott réteghőfok ismeretében a pórusvíz kinematikai viszkozitása számítható. A fent említett adatokat a Darcy-féle egyenletbe behelyettesítve a szivárgási tényezőt jó közelítéssel meg lehet kapni. A cikkben levezetett egyenletek főleg széntelepes összletek víztároló rétegeinek hidrológiai vizsgálatára javasolhatók a vízvédelem tervezéséhez.

ОПРЕДЕЛЕНИЕ КОЭФФИЦИЕНТА ФИЛЬТРАЦИИ ПЕСЧАНЫХ ВОДОНОСНЫХ КОМПЛЕКСОВ ПУТЕМ КАРОТАЖА БУРОВЫХ СКВАЖИН

Янош ЧОКАШ

Уравнение фильтрации Дарси сходна с формулой проницаемости Козени. На этом основании можно вывести формулу зависимости удельной поверхности зерен (траекторий фильтрации) с коэффициентом, характеризующим сложность поровых каналов, от эффективного диаметра зерен по Козени со средним параметром формы зерен (геометрии зерен). Коэффициент, характеризующий сложность поровых каналов, и эффективный диаметр зерен по Козени можно определить по кривым удельных сопротивлений и пористости, а удельную поверхность можно задать по известному эффективному диаметру. На основании известной температуры слоя, определенной по температурным кривым, можно рассчитать кинематическую вязкость поровых вод. Подставляя полученные данные в уравнение Дарси, можно с достаточной точностью определить коэффициент фильтрации. Уравнения, выведенные в настоящей работе, могут быть рекомендованы в первую очередь в гидрологических исследованиях водоносных горизонтов угленосных толщ.