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UPWARD CONTINUATION OF UNEVENLY SPACED POTENTIAL FIELD DATA USING EQUIVALENT SOURCES

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A complex gravity model is used to compare various upward continuation procedures based on solving integral equations. Such techniques that assume the processed field to be equal to the field of a certain distribution (equivalent source) located on the surface are useless in areas of very rugged topography. Good results are obtained by assuming the observed gravity field to be equal to the potential of a dipole distribution. The relation to the equivalent source concept is a formal one and the equations can be derived from the potential field theory. The accuracy of the results is discussed in relation to the edge effects, the values of the sampling interval, the number of iterations performed and the formula used for approximating the integrals.

Keywords: upward continuation, irregular surfaces, gravity field, integral equations, equivalent sources

1. Introduction

The equivalent source concept has been introduced in order to solve some inverse problems of applied geophysics [e.g. HALL 1958, CORBATÓ 1965, ZIDAROV 1965]. A density (magnetization) distribution is said to be equivalent to a gravity (magnetic) source body when the field of this distribution is equal (or very close in actual cases) to the observed field on the topographic surface. The goodness of fit is the only actual criterion on the reliability of this equivalent source if other geological or geophysical data are not used as constraints. The main problem of data processing by such procedures is to select the proper location for the equivalent sources. If the mass points (magnetic dipoles) are placed too far below (or too close to) the topographic surface, the equivalent source is unstable and so it cannot be used for data processing [DAMPNEY 1969, EMILIA 1973].

Despite the above, BHATTACHARYYA and CHAN [1977] proposed a technique for the upward continuation between irregular surfaces of potential field data (Dirichlet problem) based on equivalent sources located just on the surface. In the 2-D case, the gravity field of the source body is assumed to be equal to the field of a surface mass distribution at all the points situated above and on the surface, i.e.

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$$g(\bar{x}, H) = \int_{(T)} \frac{\sigma(x, h(x))\,(H - h(x))\,\mathrm{d}x}{(\bar{x} - x)^2 + (H - h(x))^2}, \tag{1}$$

where

$$\sigma = 2\mathrm{G}\varrho\sqrt{1 + (\mathrm{d}h/\mathrm{d}x)^2}, \tag{2}$$

ϱ = linear density distribution on the surface
and G = gravitational constant.
The topographic profile (T) is positive upward, defined as

$$z = h(x). \tag{3}$$

The distribution is obtained by solving iteratively the Fredholm integral equation

$$\sigma(\bar{x}, h(\bar{x})) = \frac{1 + (\mathrm{d}h/\mathrm{d}\bar{x})^2}{\pi}\left[g(\bar{x}, h(\bar{x})) - \int_{(T^*)} \frac{\sigma(x, h(x))\,(h(\bar{x}) - h(x))\,\mathrm{d}x}{(\bar{x} - x)^2 + (h(\bar{x}) - h(x))^2}\right], \tag{4}$$

where (T^*) represents the surface without the point of coordinates ($\bar{x}$, $h(\bar{x})$). The upward continued field is then obtained by using Eq. (1).

For magnetic data processing, a dipole (normal to the relief) distribution is similarly assumed such that its field F is equal to the observed field everywhere above and on the surface. The corresponding integral equation is

$$\mu(\bar{x}, h(\bar{x})) = \frac{1}{\pi}\left[F(\bar{x}, h(\bar{x})) - \int_{(T^*)} \mu(x, h(x))\,\frac{-(\bar{x} - x)\,\mathrm{d}h/\mathrm{d}x + h(\bar{x}) - h(x)}{(\bar{x} - x)^2 + (h(\bar{x}) - h(x))^2}\,\mathrm{d}x\right]. \tag{5}$$

Then

$$F(\bar{x}, H) = \int_{(T)} \mu(x, h(x))\,\frac{-(\bar{x} - x)\,\mathrm{d}h/\mathrm{d}x + H - h(x)}{(\bar{x} - x)^2 + (H - h(x))^2}\,\mathrm{d}x. \tag{6}$$

A significant difference appears between the above relationships. In Eq. (1), the gravity field of the source body is assumed to be equal to the field of a mass distribution (i.e. a true equivalent source). In Eq. (6), the anomalous field is equal not to the field but to the potential of a magnetic dipole distribution. The convergence properties of Eqs. (4) and (5) are consequently expected not to be the same. Kellog [1953] simply solves the Dirichlet problem without using the above equivalent source concept. By representing any potential field by Eq. (6), the unknown function μ results immediately from Eq. (5) which is just the limiting value of the dipole source potential when the observation point approaches the surface along the normal. At least in this case, the relation between the equivalent source concept and the upward continuation problem seems to be a formal one.

Actually, a finite number of observation points is available so that the equivalent distribution is a discrete one. Consequently, Eq. (1) and its magnetic equivalent are solvable as Fredholm integral equations of the first kind.

According to the uniqueness of the Dirichlet problem solution, no matter what kind of integral equation is solved and irrespective of the gravity or magnetic nature of the processed potential field, the same upward continued values are expected to be obtained. By assuming that the right side of Eq. (1) is the magnetic potential of a vertical dipole distribution, NAKATSUKA [1981] consequently processed a magnetic field by using Eq. (4). HANSEN and MIYAZAKI [1984] processed a magnetic field with Eq. (5) and outlined too that this relation can be used for the upward continuation of other potential fields.

When actual fields are processed, the finite number of measurements and the topographical irregularities have a substantially different impact upon the above techniques so that unreal results may often be obtained.

In this paper, a complex gravity model is used to compare various upward continuation procedures based on solving integral equations. The accuracy of the results is discussed in relation to the edge effects, the values of the sampling interval, the number of iterations performed and the formula used for approximating the integrals.

2. The model

The assumed topographic relief is given by

$$h(x) = 600/(p+(x/75)^2). \tag{7}$$

The parameter is set to $p=3$. The source body is represented by three horizontal infinite cylinders *(Fig. 1)*. Its gravity field is represented by

$$g(x, z) = \sum_{i=1}^{3} m_i(z-z_i)/(x^2+(z-z_i)^2), \tag{8}$$

where

$$m_1=50 \qquad m_2=112.5 \qquad m_3=200$$

and

$$z_1=125 \qquad z_2=0 \qquad z=-175.$$

This field has been sampled at the points having the x-coordinates equal to

$$x_k = \pm 25(k-1) \qquad k=1, \ldots, 25. \tag{9}$$

The lengths are given in meters and the field is in mGals.

In order to minimize the truncation effect, a horizontal infinite cylinder has been found such that its field is very close at the edges of the topographic profile to the field of Eq. (8) [IVAN in press]. Its field is

$$g_c(x, z) = 212.75(z+135.48)/(x^2+(z+135.48)^2). \tag{10}$$

This field is then subtracted from that of the source body and the result is continued upward to the desired level. The values of g_c at the same level are finally summed.

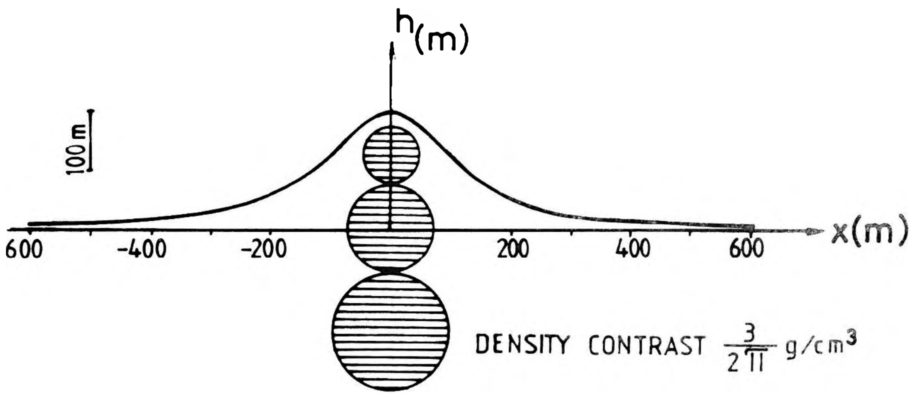

Fig. 1. Topographic profile with the source body

1. ábra. Topográfiai szelvény a hatóval

Рис. 1. Профиль с изображением рельефа и возмущающего тела

3. Data processing using Fredholm integral equations of the first kind

Infinite horizontal mass lines have been placed on the topographic profile at each point of the initial x-coordinates equal to

$$x_j = \pm(25j - 12.5) \qquad j = 1, \ldots, 24. \tag{11}$$

The field at point (x, z) due to the mass line placed at $(x_j, h(x_j))$ is

$$g_j(x, z) = S_j \Delta z / (\Delta x^2 + \Delta z^2), \tag{12}$$

where

$$\Delta x = x - x_j$$

and

$$\Delta z = z - z_j.$$

An iterative procedure was used in an endeavour to find the values of S_j and x_j so that the total field due to the mass lines distribution be as close as possible in the sense of least squares to the sampled field of Eq. (8). Different sets of values are obtained without a definite trend towards a certain distribution *(Table I/a)*. All of them give a good fit to the sampled field on the topographic profile *(Fig. 2)*. Their field above the relief has the same values for all these distributions but it is quite different from the exact values *(Fig. 3)*. Both the fit

(a)	$\pm x_j$	13	35	62	87	112	137	162	187	212	237	262	287	312	336	361	386	411	435	460	484	509	532	555	579
	S_j	0.4	3.1	6.1	7.1	7.5	8.3	9.3	10.7	12.5	14.7	17.4	20.6	24.4	28.7	33.7	39.2	45.6	52.5	60.7	68.2	81.1	81.9	71.2	78.6
(b)	$\pm x_j$	12.5	37.5	62.5	87.5	112.5	137.5	162.5	187.5	212.5	237.5	262.5	287.5	312.5	337.5	362.6	387.5	412.6	437.5	462.6	487.5	512.6	537.5	562.7	587.5
	D_j	103	245	312	227	166	102	73	49	39	30	24	21	16	16	11	13	8	12	5	11	3	10	1	10

Table I. Parameters of a simple- (a) and of a dipole- (b) source distribution. The edge effect is neglected

I. táblázat. Egyszerű- (a) és dipól- (b) hatóeloszlás paraméterei. A peremi hatás elhanyagolva

Таблица I. Распределение параметров над простым (a) и дипольным (b) возмущающим телом. Краевые влияния пренебрегаются

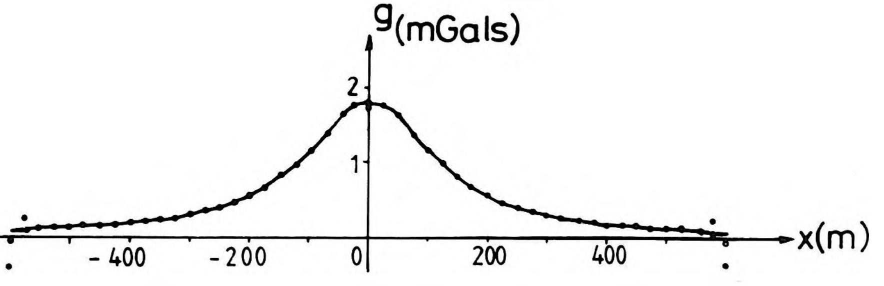

Fig. 2. Gravity fields along the topographic profile of Fig. 1. The solid line shows the field of the body; the filled circles are the values of the field due to the simple equivalent source from Table I/a; the empty circles are the same values when the edge effect is reduced

2. ábra. A gravitációs tér értékei az 1. ábra topográfiai szelvénye mentén. A folytonos vonal a ható tere, a pontok az I/a táblázatból vett egyszerű ekvivalens ható által létrehozott tér értékeit mutatják; a körök ugyanezen értékek, a peremi hatás csökkentésével

Рис. 2. Значение поля силы тяжести вдоль профиля, изображенного на рис. 1. Непрерывной линией показано поле от возмущающего тела, точками изображается поле силы тяжести от простого эквивалентного возмущающего тела, значения которого взяты из таблицы I/a; кружками обозначены те же значения, уменьшенные на краевой эффект

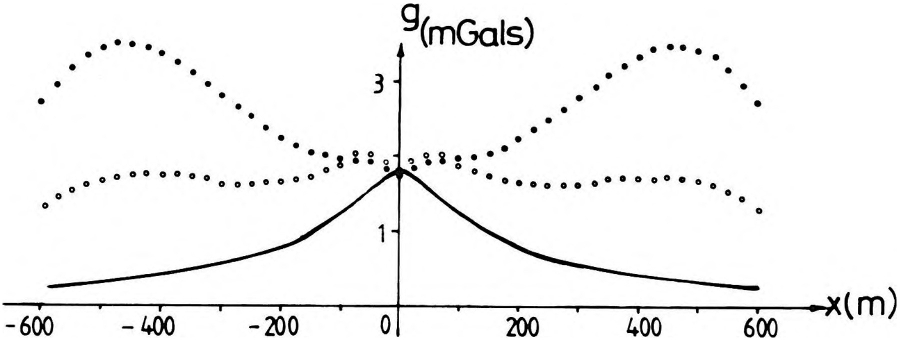

Fig. 3. Gravity fields on the plane at a height of 200 m. For legend, see Fig. 2

3. ábra. Térerősség értékek a 200 m magasságban levő síkon. Jelölések: mint a 2. ábrán

Рис. 3. Значения потенциального поля в плоскости на высоте 200 метров. Обозначения такие же, как на рис. 2.

and the upward continued values are improved when the edge effect is minimized but the differences with respect to the field of the body are significant even at great elevations *(Fig. 4)*.

In a second approach, the above mass lines have been replaced by dipoles having their axes normal to the relief. The field at point (x, z) due to the dipole placed at $(x_j, h(x_j))$ is

$$F_j(x, z) = D_j(\Delta x^2 - \Delta z^2 + 2\Delta x \Delta z \, dh/dx_j)/(\Delta x^2 + \Delta z^2)^2/\sqrt{1 + (dh/dx_j)^2}. \quad (13)$$

The above procedure now seems to be stable and the field of the obtained distribution *(Table I/b)* gives a remarkably good fit to the sampled field of Eq. (8) so that the maximum deviation is less than 0.01 mGal. But the field of this equivalent source almost vanishes above the relief so that a negative anomaly having its amplitude equal to -0.02 mGals is obtained on the plane at a height of 350 m. The minimization of the edge effect gives only a formal improvement.

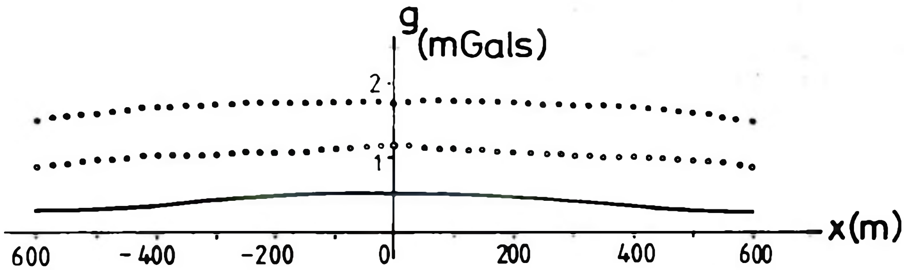

Fig. 4. Gravity fields on the plane at a height of 650 m. For legend, see Fig. 2

4. ábra. Térerősségértékek a 650 m magasságban levő síkon. Jelölések: mint a 2. ábrán

Рис. 4. Поле силы тяжести на высоте 650 метров. Обозначения такие же, как на рис. 2.

4. Data processing using Fredholm integral equations of the second kind

The sampled field of Eq. (8) has been processed by using Eqs. (4) and (1) in order to obtain the upward continued values at certain levels. The derivatives have been computed by differentiating Eq. (7). The integrals were evaluated by using a simple sum (the linear trapezoidal formula). *Figure 5* shows the failure of Eq. (4) to obtain convergence. No improvement appears even when the edge effect is minimized and the observed field is sampled at an interval of 5 m.

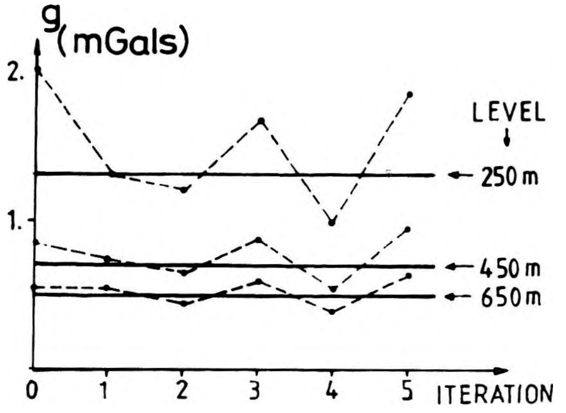

Fig. 5. Central values of upward continued field obtained by using Eqs. (4) and (1) at different levels and iterations (dashed lines). Exact values are represented by solid lines

5. ábra. A (4) és (1) egyenlettel számitott fölfelé folytatott tér középponti értékei, különböző szintekre, és iterációkkal (szaggatott vonal). A pontos értékeket a folytonos vonalak jelölik

Рис. 5. Средние центральные значения поля, продолженного вверх на различные уровни, рассчитанные по уравнениям (1) и (4) с итерациями (прерывистая линия). Точные значения указаны сплошной линией

Convergent results are always obtained by using Eqs. (5) and (6) so that only 3 or 4 iterations are needed. By setting $p=2$ in Eq. (7) a more rugged relief results *(Fig. 6)*. The observed field is now clearly disturbed *(Fig. 7)* and an upward continuation becomes really necessary. The gravity values sampled at various intervals have been continued to planes located at different heights above the relief. The results are unreal when these planes are placed near the top of the relief so that the minimum height seems to be around twice the sampling interval.

A more careful evaluation of the integrals has been used [Hansen and Miyazaki 1984] with good results. In this case, the computer time necessary is essentially increased with respect to the simple summing formula. The value of the sampling interval has an important impact on the accuracy of the upward continued field and the errors are reduced to almost half their value when the edge effect is minimized *(Fig. 8)*.

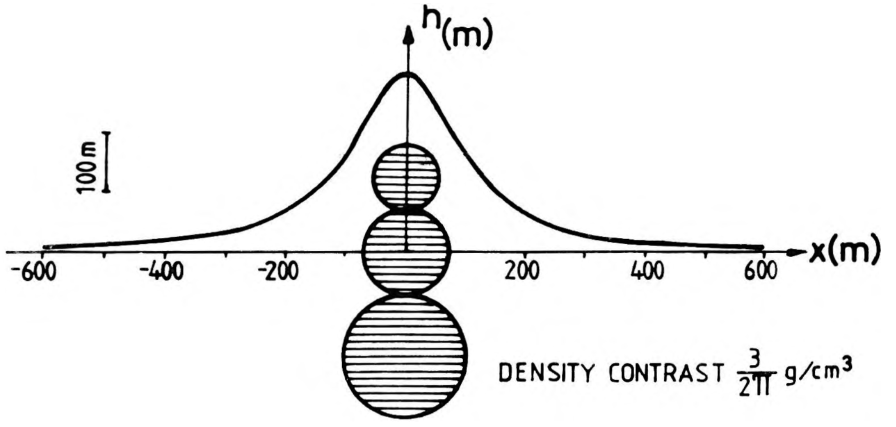

Fig. 6. A more rugged topographic relief with the source body

6. ábra. Erősebben tagolt topográfiai szelvény a hatóval

Рис. 6. Профиль с сильно изрезанным рельефом, показано положение возмущающего тела

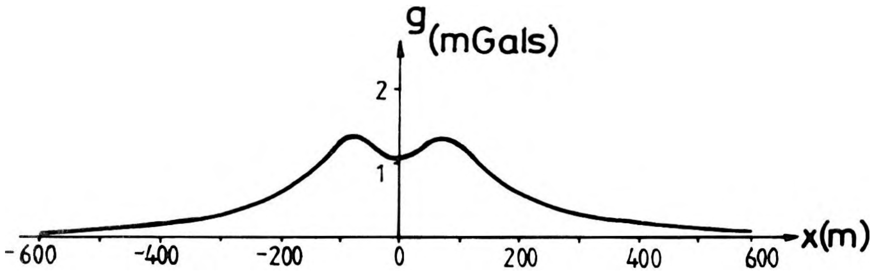

Fig. 7. Gravity field along the topographic profile of Fig. 6

7. ábra. Gravitációs tér a 6. ábrán bemutatott topográfiai szelvény mentén

Рис. 7. Поле силы тяжести вдоль профиля, приведенного на рис. 6.

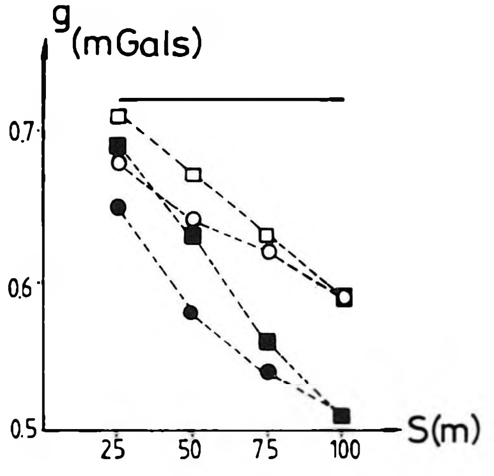

Fig. 8. Central values of upward continued field on the plane at a height of 450 m versus the value of the sampling interval. The integrals are evaluated by using the HANSEN and MIYAZAKI [1984] formula (squares) and by simple summing (circles). The filled circles and squares show the values obtained when the edge effect is not reduced, the empty circles and squares show the same values obtained when the edge effect is minimized. The solid line represents the exact value

8. ábra. A felfelé folytatott tér középponti értékei a 450 m magasságban levő síkon, a mintavételi köz függvényében. A függvényértékeket kiszámítottuk HANSEN és MIYAZAKI [1984] képletével (négyzetek) és egyszerű összegezéssel (körök). A tele körök és négyzetek jelölik a peremhatás figyelembevétele nélküli értékeket, az üres körök és négyzetek pedig ugyanezen értékeket, a peremhatás minimalizálásával. A folytonos vonal a pontos értéket adja

Рис. 8. Средние центральные значения поля, продолженного вверх на высоту 450 метров, в зависимости от расстояния между точками наблюдения. Значения рассчитаны по уровнениям ХАНСЕНА и МИЯЗАКИ [1984] (по квадратам) и простым суммированием (по кругам). Зачерненные круги и квадраты обозначают значения, полученные без учета краевых влияний, а пустые круги и квадраты при их учете. Сплошной линией показаны точные значения

5. Conclusions

All procedures assuming the processed field to be equal to the field of a certain source distribution placed on a rugged topography are useless for the upward continuation. Good results are obtained by using the potential of a dipole distribution but the relation to the equivalent source concept is a formal one and the equations are immediately derived from potential field theory. Due both to the value of the sampling interval and the edge effects, only qualitative results are expected to be obtained in many actual cases.

One of the main problems of upward continuation procedures is the great amount of computations required especially when surface data are processed.

In view of this, the simplest evaluation of the integrals is desirable. Numerical tests have indicated that the zero-order approximation of Eq. (5) always restores the correct shape of the gravity field by erasing the disturbing effects of the topography. This formula also represents an approximate generalization of the plane Dirichlet integral [Grant and West 1965] valid for the upward continuation between irregular surfaces of any potential field data [Ivan in press]. Bearing in mind that upward continuation is only an intermediate stage towards source modelling, it sometimes seems desirable to limit ourselves to performing this procedure only.

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SZABÁLYTALAN MINTAVÉTELEZÉSŰ POTENCIÁLTÉR FELFELÉ FOLYTATÁSA EKVIVALENS HATÓK ALKALMAZÁSÁVAL

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Több hatóból álló gravitációs modellt alkalmaztunk az integrál egyenleteket felhasználó különféle fölfelé folytató eljárások összehasonlítására. Ezek az eljárások feltételezik, hogy a fölfelé folytatott tér megegyezik egy alkalmasan választott felszíni eloszlás (az ún. ekvivalens ható) terével. Erősen tagolt topográfiájú területeken ez az eljárás nem válik be. Jó eredményeket kaphatunk annak feltételezésével, hogy az észlelt gravitációs tér megegyezik egy dipól eloszlás potenciáljával. Az egyenletek levezethetők a potenciáltér elméletéből, ezért az ekvivalens ható fogalmának használata pusztán formális. A cikk részletezi, hogy az eredmények pontossága miként függ a peremhatásoktól, a mintavételi távolságtól, az elvégzett iterációk számától és az integrálok megközelítésének módjától.

АНАЛИТИЧЕСКОЕ ПРОДОЛЖЕНИЕ НЕРАВНОМЕРНО ИЗМЕРЕННОГО ПОЛЯ СИЛЫ ТЯЖЕСТИ В ВЕРХНЕЕ ПОЛУПРОСТРАНСТВО С ПРИМЕНЕНИЕМ ЭКВИВАЛЕНТНЫХ ВОЗМУЩАЮЩИХ ТЕЛ

Мариан ИВАН

В целях сравнения различных способов продолжения поля потенциалов силы тяжести в верхнее полупространство использовалась модель, состоящая из нескольких возмущающих тел. Обычно, при рассчетах этими способами, предполагается, что поле, продолженное в верхнее полупространство, совпадает с наблюдаемым на поверхности от выбранного (так наз. эквивалентного) возмущающего тела. В случае сильно изрезанного рельефа этот метод дает большие ошибки. Хорошие результаты получаются, если предположить, что наблюдаемая аномалия гравитационного поля совпадает с распределением потенциального поля от диполя. Уравнения выводятся из теории поля, поэтому применение термина эквивалентного возмущающего тела пустая формальность. В статье подробно рассматривается каким образом на точность результатов влияют краевые условия, расстояния между пунктами наблюдений, зависимость результатов от числа итераций и от метода приближения интегралов.