GEOPHYSICAL TRANSACTIONS
Vol. 30. No. 1. pp. 63–72

TWO- STEP DECONVOLUTION OF GAMMA-RAY LOGS

Jiři ŠAMŠULA*, Jarmila ZEMČÍKOVÁ*

The natural radioactivity of rocks in a borehole is measured by gamma well-logging. The measured curve, however, is affected by the measurement itself, the logging equipment and by the interaction, dispersion and mutual shielding of layers. The possibilities of reducing these effects are shown using deconvolution operators supposing knowledge of certain measurement conditions. The corrected values better correspond with the true geological situation.

d: gamma-ray logging, computer evaluation, deconvolution, filter technique

1. Introduction

The application of computer techniques in well-logging measurements has enabled the use of certain transformations in the processing of field data which, even if useful, have not been utilized up to now because of the demands on numerical operations, though these are elementary ones. Many authors have been inspired by this problem and considerations and attempts similar to those proposed here have appeared in the literature [George et al. 1962, 1964, Rice 1962, Conaway 1980].

The natural radioactivity of rocks in a borehole is characterized by the intensity of gamma-rays measured by a moving scintillation probe with appropriate integration equipment. However, the measured curve is affected both by the parameters of the measuring equipment, and by the interaction, dispersion and mutual shielding of layers. Consequently, the data from the measurements may only partially correspond to the true geological situation. The borehole profile can be differentiated by the measured curve with partial resolution only. In this paper it is proposed that there may be the possibility to improve the situation by mathematical transformation of the measured curve—by convolution with a suitable operator.

* Geofyzika n.p., Brno, POB 62, 612 46 Brno 12, Czechoslovakia
Paper presented at the 28th International Geophysical Symposium, 28 September–1 October, 1983, Balatonszemes, Hungary

2. Mathematical base

Let us consider the following: for an idealized model course of a physical parameter along a borehole profile the response of the measuring equipment is either known or is to be derived. The relationship between these course is to be defined either with high precision or with a suitable approach by the convolution integral

$$v(h) = \int_{-\infty}^{\infty} g(h-\tau)\, s(h)\, d\tau = g(h) * s(h) \qquad (1)$$

where $s(h)$ represents the true course of a physical variable depending on a depth h, $v(h)$ the response of the measuring system, and $g(h)$ the impulse response of a linear filter which may represent the measuring system. The convolution integral may be expressed as a weight integration with a shifting weight function $g(h-\tau)$. The analysis of a filtering function has often been carried out in a frequency domain. If the term of frequency has been introduced, then it has to be considered as a variable reciprocal whether to time or length, according to the nature of the solution.

If equation (1) holds, a reciprocal solution is proposed

$$s(h) = v(h) * \frac{1}{g(h)} \qquad (2)$$

which implies a theoretical possibility to determine by the convolution operation the true course of a measured physical variable. The possibility, which has long been known, is based on the knowledge of the inversion of the impulse response $1/g(h)$ of a linear filter. In practical applications, however, the above mentioned computation difficulties need to be overcome. The executing of the convolution operation, e.g. for two long numerical series, requires a great number of elementary arithmetical operations.

3. Application to gamma-logging measurements

Let us propose a borehole profile, as outlined in *Fig. 1*, a radioactively barren material with a thin active layer (a). In static measurements along the borehole profile by a sensor we obtain a curve (b). In measurements by a moving sensor the measured curve has been deformed as a function of velocity and direction of the motion—curves (c) and (d)—and partly due to the time constant of the impulse counter. Curve (b) was termed by Conaway (1980) as the Geologic Impulse Response (GIR). This curve represents the physical variable measured along the borehole profile and the input into the measuring

* is the symbol designating the convolution

apparatus, which causes further deformations. If the original geological situation is to be achieved from the measured curve, we proceed in two steps. Firstly, the deforming influence of the measuring channel is to be suppressed, whereby the GIR curve is obtained. Then, by the inversion of the GIR curve the true geological situation is achieved.

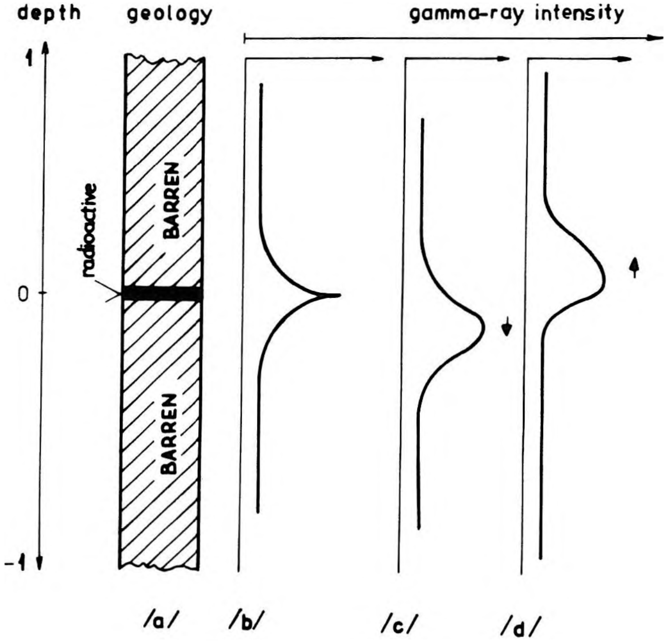


Fig. 1. a) Geologic column showing a thin zone of radioactive material embedded in barren rock
b) Ideal simulated gamma-ray log run past the thin radioactive zone with a digital system having a point detector
c) Simulated log run past the thin radioactive zone with an analog logging system having an exponential ratemeter; log run in downhole direction
d) Same as c) except log run in uphole direction [after CONAWAY 1980]

1. ábra. a) Meddő kőzetbe ágyazott vékony radioaktív zónát szemléltető rétegsor
b) Ideálisan szimulált gamma-sugár szelvény pontszerű detektoros digitális rendszerrel
c) Szimulált szelvény exponenciális rateméterrel működő analóg szelvényező rendszerrel; lefelé való szelvényezéskor
d) Mint c) de felfelé való szelvényezéssel [CONAWAY 1980 után]

Рис. 1. а) Геологический разрез с тонкой прослойкой радиоактивного вещества, вмещенной в пустой породе
b) Идеально симулированная диаграмма ГК при использовании сифровой системы с точечным детектором
c) Симулированная диаграмма при использовании аналоговой измерительнй установки с экспоненциальным счетчиком импульсов; замер был проведен при пуске зонда
d) г. ж. как c) при подъеме зонда
[по КОНЭВЕЙ, 1980 г.]

The reaction of most commercially manufactured counters with integrators can be expressed in a simple exponential function

$$f(h) = 0, \quad h<0$$
$$f(h) = \frac{1}{vT} \cdot e^{-h/vT}, \quad h \geq 0 \tag{3}$$

where h is the depth, T the time constant of the sensor, and v the velocity of the sensor.

This function can be called the impulse response of the sensing equipment. To eliminate the influence of the sensing equipment one must know the inverse of the $f(h)$ function. This function can be calculated by two methods—either in the frequency or time domain. Using the first method, Conaway [1980] calculated a three-element operator of the following type

$$\frac{1}{f(h)} = \left(\frac{vT}{2\Delta h}, \quad 1, -\frac{vT}{2\Delta h}\right) \tag{4}$$

where Δh represents the sampling interval. Essentially, this operator solves the first deconvolution step.

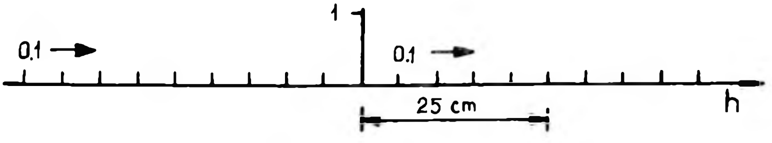
In practical applications of these operators problems may arise. If we take a sampling interval from 5 cm to 50 cm in deep drilling, and consider that the measurement is suppressed by some sort of fluctuations. then the application of the two- or more element operator is undesirable. In practical applications the following procedure and solution have been introduced. Let us calculate with the parameters:

$$v = 0.09 \text{ m/s}, \quad T = 5 \text{ s}, \quad \Delta h = 5 \text{ cm}$$

To calculate the operator elements let us take $\Delta h = 25$ cm

$$\frac{vT}{2\Delta h} = 0.9$$

This value enables the operator to be constructed, its marginal elements will be distributed into nine samples:



This procedure is practically a combination of the calculated operator and a smoothing one.

By the above demonstrated transformation we obtain the curve which corresponds to that designated by CONAWAY [1980] as GIR. In theory, the next step—which implies additional inversion—gives a possibility to obtain a model of the original geological situation. The GIR curve in *Fig. 2* has the form

$$(h) = e^{-\alpha(h)} \tag{5}$$

where α is the constant including the influence of the activities of a thin layer, the probe sensitivity, the borehole influence, the dispersing effect of a barren media, etc. While in a preceding procedure for determining the inversion to eliminate the influence of the sensor the parameters v and T remained constant during the measurement, this cannot be stated definitely. In view of this we have carried out the analysis of the relation with respect to the changing of α. Accounting for the unchangeable borehole diameter, the probe diameter approaching the borehole diameter and, further, providing for the non-existence of materials along the borehole profile with the shielding effects for gamma-ray approaching that of lead and similar materials, we can consider the α constant as the invariable along the borehole profile. Nevertheless, the calculation of the concrete value of α still remains difficult. Similarly, the construction of a model in 4π geometry is not simple, nor is its calculation. In the first approach we proceeded as follows: a GIR curve with a double exponential course was proposed and we could estimate the α value directly from the measured curve. If α equals 1, then $\varphi(h)$ is equal to 1 for $h=0$. For $h=1$ the $\varphi(h)$ value decreases to 0.36. If the GK log exhibits an expressive active layer, then the depth interval (distance) in which the anomaly decreases from its 100% to 36% can be considered as characteristic for the α value of the given probe and the given situation in a borehole. Assuming all this, we can derive the inverse for the $\varphi(h)$ function, which represents the deconvolution operator of the second step. We can derive the inverse for the $\varphi(h)$ function by numerous methods. Problems of deriving the inversion model have been analysed and solved by the fundamental approach of ROBINSON [1967], KANASEWICH [1973], KULHÁNEK [1976], and others. In our applications we have advantageously utilized the procedure and algorithms developed by MEREU [1978]. The above quoted author has published a complete description of a computer program to obtain the weights of a time-domain wave-shaping filter program. Although this was developed for seismic applications, its applications may be even wider.

The following table shows the output from the program in which the values

$$\varphi(h)^* = (0.05; \quad 0.13; \quad 0.36; \quad 1; \quad 0.36; \quad 0.13; \quad 0.05)$$

have been introduced in the calculation as the given function (W column). The desired function has been given as the value 1 in column D. The desired operator is calculated in column F. In column H a checking calculation has been entered

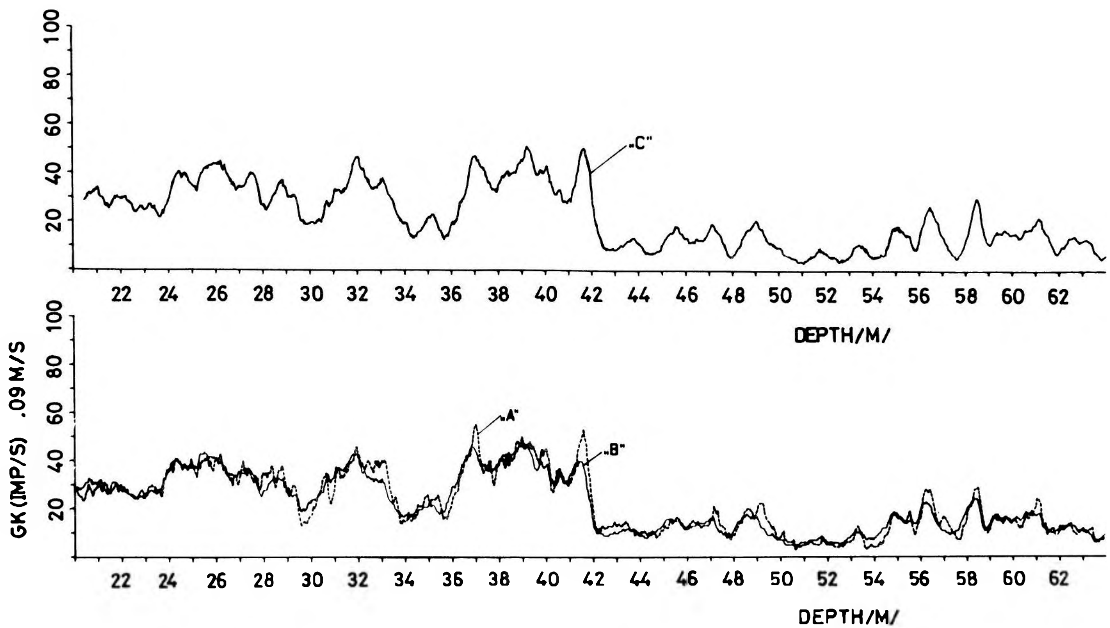


Fig. 2. Gamma-ray log. A — quasistatic measurement; B — measurement with probe velocity 0.09 m/s; C — after filter application

2. ábra. Term. gamma szelvény. A — kvázistatikus mérés. B — 0,09 m/s szondasebességgel való mérés; C — szűrő alkalmazása után

Рис. 2. Диаграмма ГК. А — квазистатическое измерение; В — измерение при скорости подъема 0.09 м/с С — После применения фильтра

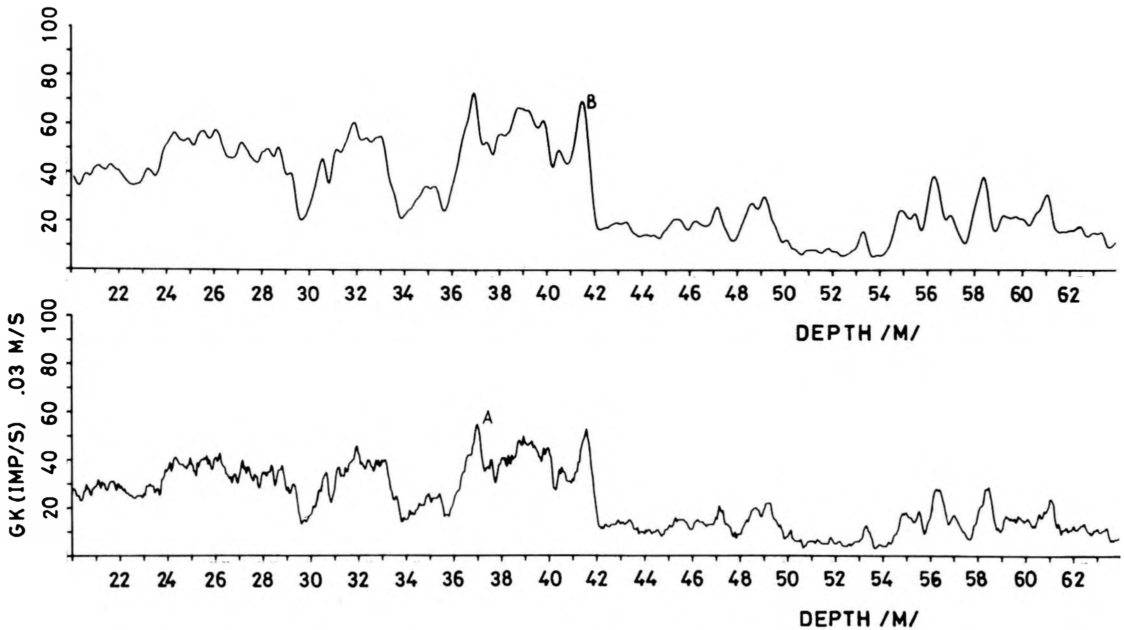


Fig. 3. Experimental application of the second deconvolution. A — primary gamma-ray log; B — GIR inversion

3. ábra. A második konvolúció kísérleti alkalmazása. A — elsődleges term. gamma szelvény B — GIR inverzió

Рис. 3. Экспериментальное применение второй деконволюции. A — дервичная диаграмма ГК; B — инверсия GIR of the calculated operator with the given function. It is evident that we have achieved the value 1 with the error of 0.013, which is certainly sufficient, no matter that some mild oscillations may occur.

Thereby, we have derived the operator for the second deconvolution step and we can proceed to the practical calculation.

Time	Given wavelet	Desired wavelet	Correlation functions		Symmetric filter	Shaping filter	Actual output	Shaping errors
	W	D	R	S	G	F	H	E
−5			.013		−.046	−.008	.000	3.E-04
−4			.053		.104	.030	−.001	−7.E-04
−3	.050		.194	.050	−.085	−.021	.001	6.E-04
−2	.130		.426	.130	.193	.001	−.001	−1.E-03
−1	.360		.827	.360	−1.054	−.400	.007	7.E-03
0	1.000	1.00	1.304	1.000	1.994	1.277	.987	−1.E-02
1	.360		.827	.360	−1.054	−.400	.007	7.E-03
2	.130		.426	.130	.193	.001	−.001	−1.E-03
3	.050		.194	.050	−.085	−.021	.001	6.E-04
4			.053		.104	.030	−.001	−7.E-04
5			.013		−.046	−.008	.000	3.E-04

4. Example of application

Let us adduce an example of the natural gamma activity measurement in *Fig. 2*. Curve *A* implies the measurement carried out so very slowly that it can be regarded as a static one. The used time constant is 5 s. Curve *B* represents the measurement with the velocity of 0.09 m/s. The decrease of the maxima can distinctly be observed as well as their shifting along the depth axis. Curve *C* represents the result of the operator application, as outlined earlier. It is apparent that the operator restored the values of the maxima and their positions along the depth axis very well.

An experimental application of the second deconvolution step is attempted in *Fig. 3*. The deconvolution operator demonstrated in the table in the preceding section has been applied to curve *A* in *Fig. 2*. For the α constant determination a section of the curve has been utilized at a depth of 41–42 m, where an intense step of the registered gamma-ray intensity occurs. We have estimated the distance of 0.3 m in which the intensity decreases to 0.36%. Then, the α constant is 3.33, which is the input value for the calculation of the double GIR exponential model. This again serves as the input for the MEREU program (see the table in the preceding section). It is clearly seen in *Fig. 3* that after the transformation the resulting curve exhibits an increase in the maxima of approx. 30%, as compared with the original transformed curve. Hence, the credibility of the transformation depends on our ability to estimate correctly the α constant and to accomplish the demand that the α constant be invariable along the borehole profile.

5. Conclusion

In conclusion we may say that the first transformation and its results are directly applicable in practice. The anomaly amplitude is restored and the anomalies are positioned at the proper depth. The second transformation, as stated above, is dependent on the credibility of the α constant determination.

REFERENCES

Conaway J. C., 1980: Exact inverse filters for the deconvolution of gamma-ray longs. Geoexploration, **18**, pp. 1–14.

George C. F., Jr., Smith H. W., Bostick F. X., Jr., 1962: The Application of inverse convolution techniques to improve signal response of recorded geophysical data. Proceedings of the I.R.E., November.

George C. F., Jr., Smith H. W., Bostick F. X., Jr., 1964: Application of inverse filters to induction log analysis. Geophysics, 29., 1, February, pp. 93–104.

Kanasewich E. R., 1973: Time sequence analysis in geophysics. The University of Alberta Press.

Kulhánek O., 1976: Introduction to digital filtering in Geophysics. Elsevier Scientific Publishing Company, Amsterdam–Oxford–New York.

Mereu R. F., 1978: A computer program to obtain the weights of a time—domain wave—shaping filter which is optimum. Geophysics, **43,** 1, February, pp. 197–215.

Rice R. B., 1967: Inverse convolution filters. Geophysics, **27,** 1, February, pp. 4–18.

Robinson E. A., 1967: Multichannel time series analysis with digital computer programs. Holden–Day, San Francisco, Cambridge, London, Amsterdam.

A GAMMA-SUGÁR SZELVÉNYEK KÉT LÉPÉSES DEKONVOLUCIÓJA

J. ŠAMŠULA, J. ZEMČÍKOVÁ

A szerzők természetes radioaktivitás meghatározást végeztek egy fúrásban gamma-szelvényezés útján. A kapott görbét befolyásolják a mérési folyamat és a műszer jellemzői, valamint a rétegek kölcsönhatásai, diszperziós és árnyékolási folyamatai. A cikk bemutat egy dekonvoluciós eljárást, amely lehetővé teszi a zavaró hatások csökkentését, feltéve, hogy bizonyos mérési körülmények ismeretesek. A korrigált értékek jó egyezést mutatnak a földtani viszonyokkal.

ДВУХСТЕПЕННАЯ ДЕКОНВОЛЮЦИЯ ГАММА-КАРОТАЖНЫХ ИЗМЕРЕНИЙ

Й. ШАМШУЛА, Я. ЗЕМЧИКОВА

Естественная радиоактивность горных пород в разрезе скважины была измерена при использовании ГК. На измеренную кривую ГК существенное влияние оказывают собственный измерительный процесс и аппаратура, с одной стороны, и взаимное рассеяние и экранирование в пластах с другой. В работе показаны возможности уменьшения этих влияний при помощи операторов деконволюции, допуская, что известны определённые условия измерения. Исправленные величины таким образом лучше соответствуют действительной геологической обстановке.