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CURVED RAY ALGEBRAIC RECONSTRUCTION TECHNIQUE APPLIED IN MINING GEOPHYSICS

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A proper understanding of the stress conditions in mines is of utmost importance for economy and for safety. Because the velocity of seismic waves in rocks is a function of pressure, the velocity distribution can be used to predict stress conditions.

A method is described that can be considered as a novel version of the Algebraic Reconstruction Technique. The procedure starts out from some initial velocity field and performs ray tracing based on the vectorial form of the Snellius–Descartes principle. By comparing the measured and computed travel times the velocity field is modified and new raypaths are computed until the deviations become less than some prescribed tolerance. The velocity field obtained represents the velocity distribution of the site studied.

Repeatedly performed transmission measurements yield information on the possible changes of the pressure conditions.

Introduction

Economic and safety considerations have made it an important task in several Hungarian coal mines to get detailed knowledge of the changes in the state of the surrounding rock formations. Because of mining activities the stability of these rock masses that have evolved through geological times becomes disturbed; these stress changes then cause different kinds of destructive phenomena. The proper tracing of these phenomena in space and time can be done only by a joint application of the different measuring methods. The Research Department of the Mecsek Coal Mines have elaborated a mining detection and control system [1] involving a method which contains—among other features—the repeated application of seismic transmission. This transmission technique is based on a measuring arrangement where the spatial domain to be studied lies within the sources and detectors; the acquisition of information is generally based on transmitted seismic waves [2], [3]. The processing and interpretation of the measurement results raise a number of mathematical and physical problems in view of which Mecsek Coal Mines and ELGI agreed to cooperate in an endeavour to solve these problems. The method elaborated and some preliminary results are presented here.

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1. Basic principles of the transmission ("transillumination") method

The propagation of elastic waves (direction, velocity, energy absorption and spectrum) is determined by the parameters of the medium; during propagation the waves accumulate the integral effects of all these parameters. So, at least in principle, there should be a way to determine the physical properties of a given rock formation from the observed parameters of the transmitted waves.

The transit time between a source-detector pair is given by

$$T_1 = \int_{R_l} \frac{ds}{V_l(\underline{r})}, \qquad T_2 = \int_{R_t} \frac{ds}{V_t(\underline{r})}. \tag{1a, 1b}$$

Observed amplitudes are described by

$$A_l = A_{ol}\Gamma_l \exp\left(-\int_{R_l} \gamma_l(\underline{r})\, ds\right), \tag{2a}$$

$$A_t = A_{ot}\Gamma_t \exp\left(-\int_{R_t} \gamma_t(\underline{r})\, ds\right), \tag{2b}$$

where

— R_l and R_t are the paths of propagation of the longitudinal and transverse waves, respectively;

— $V_l(\underline{r})$ and $V_t(\underline{r})$ the corresponding (scalar) velocities;

— A_l, A_t the observed amplitudes;

— A_{ol}, A_{ot} the generated amplitudes;

— Γ_l, Γ_t spherical divergence;

— $\gamma_l(\underline{r})$, $\gamma_t(\underline{r})$ are absorption coefficients.

In homogeneous media the determination of the constants V_l, V_t and γ is straightforward.

In inhomogeneous media the inversion of the above integrals raise two problems:

a) For given raypaths there exists an infinity of functions satisfying the integral expressions; this means that the distribution of the parameters cannot be determined,

b) In inhomogeneous media the raypaths R_l, R_t also depend on the velocity fields.

2. The reconstruction algorithm

2.1. General properties of the reconstruction technique

In recent years the problem has been approached by several variants of three basic methods (direct matrix method, analytical methods and the so-called Algebraic Reconstruction Technique (ART)) [4]. The common root of these methods goes back to the classical work of RADON [5] who justified that any two-dimensional domain can unambiguously be reconstructed by an infinite series of its one-dimensional projections. All known methods, however, assume straight-line integration paths, which means that they are not generally applicable for sufficiently complex geologies. While Bois [6], [7] did use curved raypaths, his iterative velocity determination is based on the rather inconvenient direct matrix method [8]. Our algorithm has been developed on the basis of the works of Bois and of Gordon and Herman [8], [9], see *Fig. 1*.

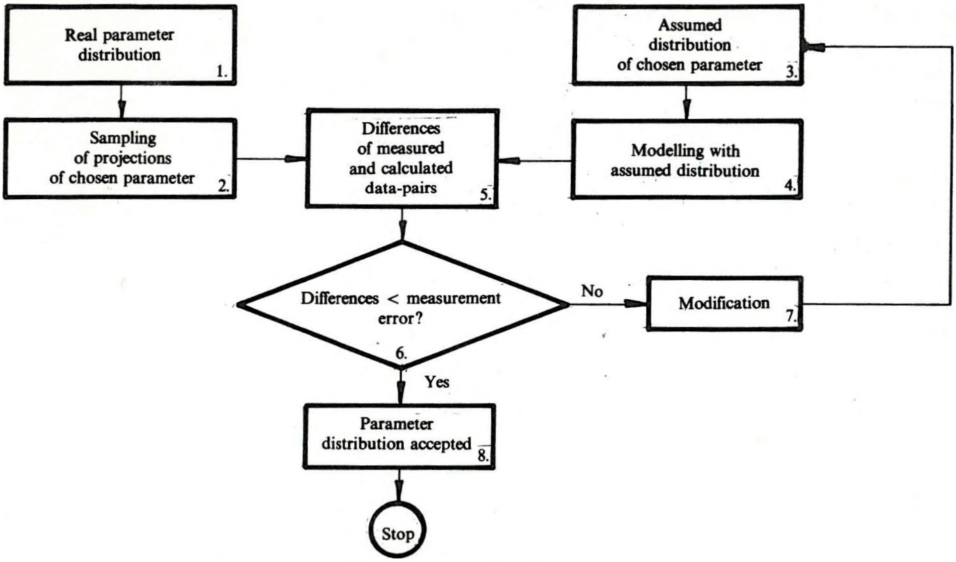


Fig. 1. Flow chart of the process

1. ábra. A kifejlesztett eljárás folyamatábrája
1 — a valóságban kialakult paramétereloszlás; 2 — egy választott paraméter vetületeinek mérése (mintavételezés); 3 — a választott paraméter feltételezett eloszlása; 4 — a mérés modellezése a feltételezett eloszlással; 5 — a mért és számított értékpárok különbsége; 6 — különbség < mérési hiba? 7 — módosítás; 8 — a paraméter eloszlása megfelel a valóságnak

Фиг. 1. Блок-схема разработанной процедуры
1 — Дествительное распределение параметров; 2 — Измерение проекции одного выбранного параметра (дискредитация); 3 — Предпологаемое распределение выбранного параметра; 4 — Моделирование измерения с помощью предпологаемого распределения; 5 — Разность вычисленных и измеренных параметров; 6 — Разность ошибка измерения? 7 — Исправление поля; 8 — Распределение соответствует истине

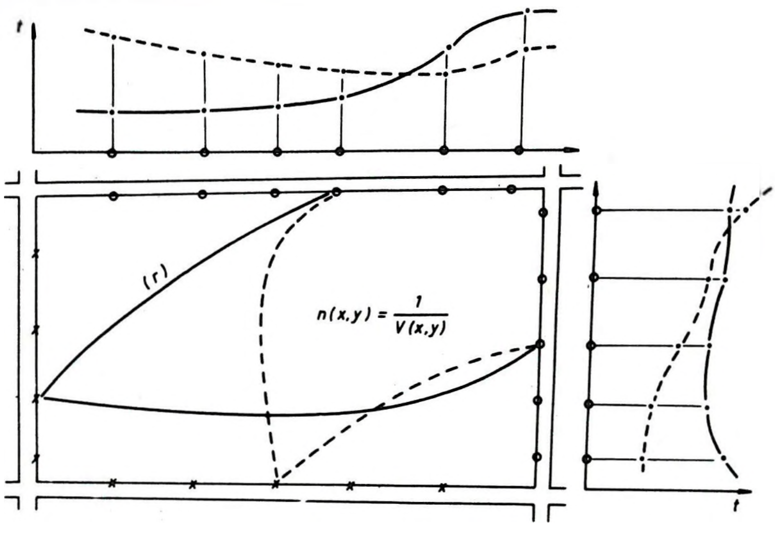
The spatial domain to be studied is "transilluminated" by transecting wave paths, sources and receivers being placed in the galleries.

The measurement results yield sampled values of the projections of distributions, burdened with measurement errors. From the finite number of values we can determine only a finite number of parameters of the velocity field—the number of determined parameters and the number of observed values not necessarily being equal (e.g. these parameters can be values of the velocity distribution at discrete grid points). The better the accuracy and the greater the quantity of the values along transecting raypaths the better will be the reconstruction.

According to the ART principle we compute by means of some supposed model the series of values corresponding to an assumed parameter distribution and to the measurement geometry. This data series is, of course, subject to modelling errors. Wave propagation is described by the laws of geometrical optics, absorption is computed by Eqs. 2a–2b.

The next step is to compare the measured and computed series of data. Our basic assumption—as in all iterative approaches of this kind—is that two distributions agree if and only if all of their projections agree.

If the deviations are greater than the measurement error the algorithm modifies the hypothetical parameter distribution toward the direction of a better agreement, then proceeds again with raypath modelling. So, from among the infinite number of possible fields it looks, by successive approximations, for that fitting best the measurements results.



$$t_r = \int_{(r)} n(x,y)\,ds \qquad r = 1,2 \ldots\ldots R(=\Sigma\cdot\Gamma)$$

Fig. 2. Characteristic scheme of a measuring pattern

2. ábra. Egy jellegzetes mérési elrendezés vázlata

Фиг. 2. Схема характеристической измерительной установки

The parameter field found will be considered as the best approximation of the real distribution, within the possibilities of the algorithm and the measurement errors.

Figure 2 presents a typical transmission measurement geometry used in mines. The projections to be used for the reconstruction are first-arrival times registered by the geophones, the parameter to be determined is the velocity distribution $V(x, y)$ inside the domain studied.

2.2. Computation of the transit times and amplitudes

By the basic equation of geometrical optics, as expressed by SOMMERFELD [13],

$$\operatorname{curl}(n\underline{s}) = 0, \tag{3}$$

where

$n(\underline{r}) = \dfrac{1}{V(\underline{r})}$ is the index of refraction,

$\underline{s}(\underline{r})$ is the normal to the wave-front.

Since, in the general case, this equation is not integratable, we have to compute separately all the wave paths corresponding to the different shot point-geophone pairs. Starting out from the shot points we launch a diversity of rays through the velocity field then select those that "hit" the individual geophones.

The rays can be computed, by Eq. 3, as

$$d\underline{s}(k) = \frac{1}{n(k)}\,[[\underline{s}(k) \mathrm{x} \operatorname{grad} n(k)] \times \underline{s}(k)], \tag{4}$$

$$\underline{s}(k+1) = \underline{s}(k) + d\underline{s}(k). \tag{5}$$

Here, k is the serial number of the steps of length Λ along the ray, the values $n(k)$ and grad $n(k)$ are computed by cubic interpolation from adjacent grid points;

$$n(k) = \sum_{hij} c_{hij}^{(k)} n_{hij}, \tag{6}$$

$$\operatorname{grad} n(k) = \sum_{hij} c_{hij}^{*(k)} \tag{7}$$

(the coefficients $c_{hij}^{(k)}$ and $c_{hij}^{*(k)}$ differ from zero in the neighbourhood of $4 \times 4 \times 4$ of point (k)). The vectorial form can easily be treated in 3 dimensions as well. The cubic interpolation fairly well describes the refraction, does not increase significantly the computation time and takes into account the fact that the wave only "feels" a neighbourhood of limited size of point (k).

For a more complex velocity field several rays could belong to the same shot point-geophone pair; from these that having the smallest transit time will be selected.

For a given ray (r) the transit time is given by

$$T_r^c = \Lambda \sum_{(r)} n(k) = \Lambda \sum_{k} \sum_{hij} c_{hij}^{(k)} n_{hij}. \tag{8}$$

On the basis of the raypaths the spherical divergence [Γ, (2a), (2b)] can also be determined; its value is proportional to the ray density in the immediate vicinity of the geophone. By these factors, and by the assumed distribution γ_{hij} the computed amplitude ratios for the raypaths of the first arrivals will be

$$F_r^c = \ln\left(\frac{A_r^c}{A_0}\right) = \ln \Gamma_r - \Lambda \sum_k \sum_{hij} c_{hij}^{(k)} \gamma_{hij}. \tag{9}$$

2.3. *Modification of the parameter distributions*

Modification of the hypothetical parameter distributions is carried out on the basis of the difference between the measured and computed results:

$$\Delta T_r = T_r^m - T_r^c; \qquad \Delta F_r = F_r^m - F_r^c. \tag{10}$$

From Eqs. (8), (9), by differentiation:

$$\delta T_r^c = \Lambda \sum_{hij} n_{hij} \left(\sum_k \delta c_{hij}^{(k)}\right) + \left(\sum_k c_{hij}^{(k)}\right) \delta n_{hij} \tag{11}$$

and

$$\delta F_r^c = \delta(\ln \Gamma_r) - \Lambda \sum_{hij} \left[\gamma_{hij} \left(\sum_k \delta c_{hij}^{(k)}\right) + \left(\sum_k c_{hij}^{(k)}\right) \delta \gamma_{hij}\right]. \tag{12}$$

The terms $\delta c_{hij}^{(k)}$ and $\delta(\ln \Gamma_r)$ describe the effect of the change in the raypath. Since these terms cannot be computed, they should be neglected. (This neglection is justifiable because the first arrival is the minimum time on all possible raypaths.)

Consequently

$$\delta T_r^c \cong \Lambda \sum_{hij} \left(\sum_k c_{hij}^{(k)}\right) \delta n_{hij} \tag{13}$$

and

$$\delta F_r^c \cong -\Lambda \sum_{hij} \left(\sum_k c_{hij}^{(k)}\right) \delta \gamma_{hij}. \tag{14}$$

The computed modification of the distributions at the grid point (hij) will be

$$\Delta n_{hij} = \sum_r^R d_{hij}^{(r)} \Delta n_{hij}^{(r)} = \frac{1}{\Lambda} \frac{\displaystyle\sum_r^R \sum_k \left(c_{hij}^{(k,r)}\right) \frac{\sum_k c_{hij}^{(k,r)}}{\sum_{hij}\left(\sum_k c_{hij}^{(k,r)}\right)^2} \Delta T_r}{\displaystyle\sum_r^R \sum_k \left(c_{hij}^{(k,r)}\right)} \tag{15}$$

and

$$\Delta\gamma_{hij} = \sum_{r}^{R} d_{hij}^{(r)} \Delta\gamma_{hij}^{(r)} = -\frac{1}{\Lambda} \frac{\sum_{r}^{R} \sum_{k} (c_{hij}^{(k,r)}) \frac{\sum_{k} c_{hij}^{(k,r)}}{\sum_{hij} \left(\sum_{k} c_{hij}^{(k,r)}\right)^2} \Delta F_r}{\sum_{r}^{R} \sum_{k} (c_{hij}^{(k,r)})}. \tag{16}$$

It can be seen that the modification in a given grid point is the weighted and normalized algebraic sum of the modifications computed along the individual ray-paths, on the basis of the difference between the theoretical and measured values. The weighting is made, basically, in inverse proportion to the "distance" of the given ray from the grid point.

The algorithm modifies the field values in the following way:

$$a) \quad \Delta n_{hij}^{w(r)} = 0 \quad \text{and} \quad \Delta\gamma_{hij}^{w(r)} = 0 \tag{17}$$

if for ray r

$$|\Delta T_r| < \mu(T) \quad \text{and} \quad |\Delta F_r| < \mu(F) \tag{18}$$

$\mu(T)$ and $\mu(F)$ are the errors of the time- and amplitude measurements, respectively.

$$b) \quad n_{hij}^{w+1} = \text{MIN}\,\{\text{MAX}\,[n_{\min};\, n_{hij}^{w} + \Delta n_{hij}^{w}];\, n_{\max}\} \tag{19}$$

and

$$\gamma_{hij}^{w+1} = \text{MIN}\,\{\text{MAX}\,[\gamma_{\min};\, \gamma_{hij}^{w} + \Delta\gamma_{hij}^{w}];\, \gamma_{\max}\}, \tag{20}$$

where

— w is the number of iterations;

— the indices "min" and "max" denote the plausible lower and upper boundaries of the given parameter;

— the operators MIN and MAX refer to the selection of the respective minimum and maximum values.

The above restrictions ensure, besides the physical reality of the computed fields (as, for example, $n > 0$) the rapid convergence of the algorithm [10]. In the course of the modifications it is obviously possible to take into account the boundary values of the distributions, known *a priori* from other measurements.

The change of the field due to a single iteration step is shown by *Fig. 3*. The reference times are given as if there were an infinitely large velocity jump concentrated to one grid square at the centre of a homogeneous velocity field (the unit of velocity is grid-size/time, the value of the homogeneous field is 3). The effect of this inhomogeneity is distributed in a star-like manner along the rays passing through the centre. The more distant rays, not affected by the velocity jump, try to counteract the spread of the modification, causing splits in the arms of the star.

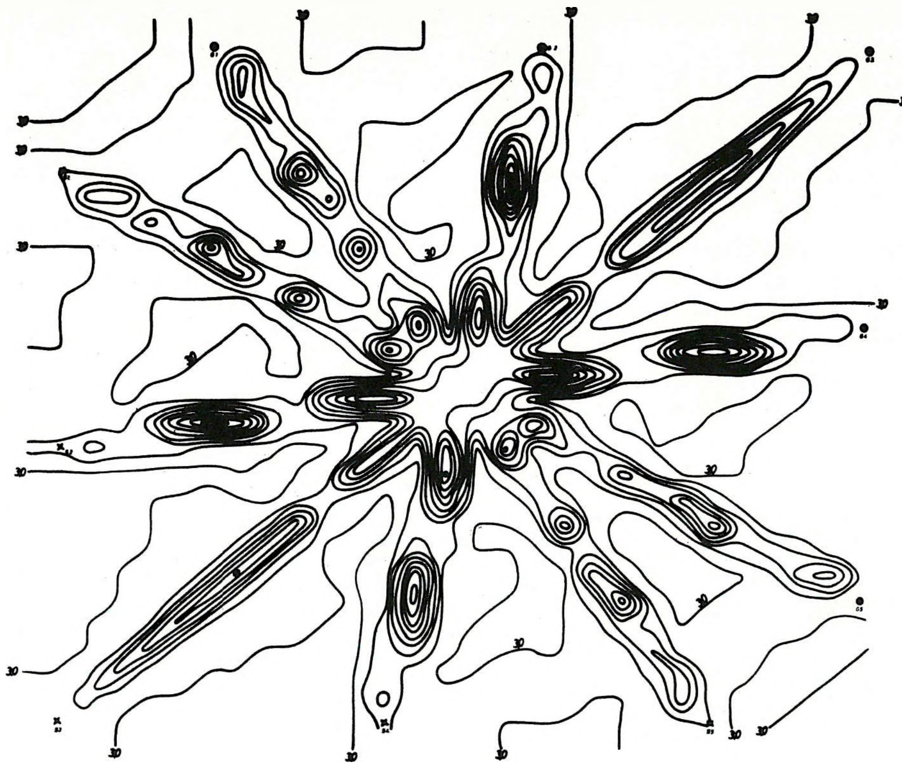


Fig. 3. Effect of one step in the iteration process *3. ábra.* Egy iterációs lépés hatása
Фиг. 3. Влияние шага итерации

3. Preliminary testing of the algorithm

The seismic waves giving the first arrivals propagate only in special cases in the plane of measurement (velocity fields perpendicularly homogeneous to the plane; measurements in waveguides, [11]). Even though our approach is applicable to 3-dimensional reconstructions on the strength of Eq. (4), this would necessitate spatial measurements. Since at present we have no reliably interpretable (i.e. sufficiently dense) spatial data and the algorithm works much more simply in the plane, we have restricted ourselves to reconstructing 2-dimensional distributions.

In order to judge the applicability of any iterative procedure, the following properties should be checked:

— convergence or divergence of the algorithm;
— rate of convergence;
— unambiguity, uniqueness of the limiting point.

In the literature the unambiguity of the seismic inverse problem has been proved rigourously only for some simple distributions [12]. In the case of iterative reconstruction techniques it is advisable to check the objectivity of the resulting distribution by a regeneration method [10]: the more surprising configurations of the result field should be deleted after which the iteration should be started again; if the deleted part reappears it is really due with a fair likelihood to the measured data.

A result from among our convergence studies is shown in Figs. 4, 5 and 6. The reference times refer to the velocity field of value 3, the initial distribution had the value of 4. The closeness of the approximation is measured by the quantities

$$(\overline{\delta t})^2 = \sqrt{\frac{1}{R}\sum_r^R \left(\frac{T_r^c - T_r^m}{T_r^m}\right)^2}, \tag{21}$$

$$(\overline{\delta t}) = \frac{1}{N}\sum_{ij} \left(\frac{V_{ij}^c - V_{ij}^m}{V_{ij}^m}\right), \tag{22}$$

$$(\overline{\delta V})^2 = \sqrt{\frac{1}{N}\sum_{ij} \left(\frac{V_{ij}^c - V_{ij}^m}{V_{ij}^m}\right)^2} \tag{23}$$

(cf. Fig. 4). It can be seen that the iteration gradually improves up to 4th–5th step. Similar results have been reported in [8], for straight raypaths.

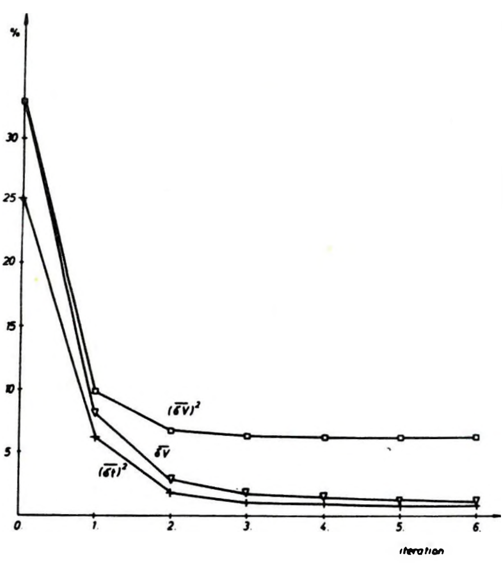


Fig. 4. Convergence curves of the process

4. ábra. Az eljárás konvergenciája

Фиг. 4. Сходимость процедуры

The histograms of velocity values *(Fig. 5)* show a similar convergence. The finite width of the histograms might be due to the following:

— the step size is of the order of 0.5 grid size; the computed times and the corresponding modifications have a statistical scatter;

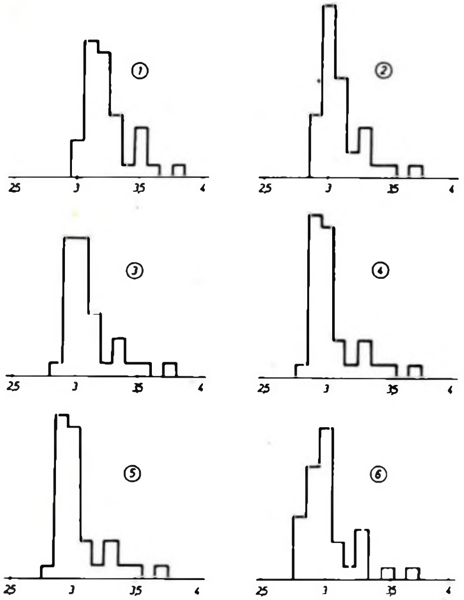


Fig. 5. Frequency histogram of velocity values

5. ábra. A sebességértékek gyakorisága (hisztogram)

Фиг. 5. Частота значений скоростей (гистограмма)

— there is a pronounced boundary effect due to the small size of the field.

The boundary effect is also striking in *Fig. 6a,* representing the velocity fields computed in iteration steps 1, 4 and 7: the algorithm cannot change properly the boundaries of the field and this causes an overshoot even at the very centre of the field.

To reduce these effects and to study the uniqueness of the solution we have also initiated another approximation of the reference field, starting out from the opposite side, with velocity 2. As seen, this series of iterations also tends to the reference field; the boundary effect and the overshoot are opposite in sign *(6b).* These results suggest that in actual cases it is worth while to compute 3–4 iteration steps starting out from initial distributions overestimated from below and from above, respectively, then carry out 1–2 more iterations with the average of the above results *(6c).*

Figure 7 illustrates a processing result. Reproduced by permission of the Research Department of the Mecsek Coal Mines. On the respective fields A, B and C we carried out, practically at the same time, separate measurements of the longitudinal first arrival times (all in all 26 shot points and 78 receivers). The boundary effects are due to the special measuring geometry; the main structure of the three fields, however, fits together fairly well.

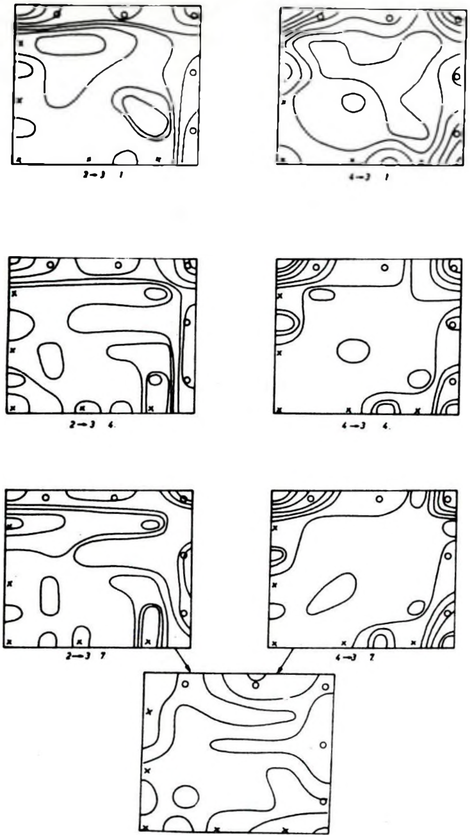


Fig. 6. Example of the reconstruction of a homogeneous velocity field

6. ábra. Kísérlet homogén sebességmező rekonstrukciójára

Фиг. 6. Проба восстановления поля однородных скоростей

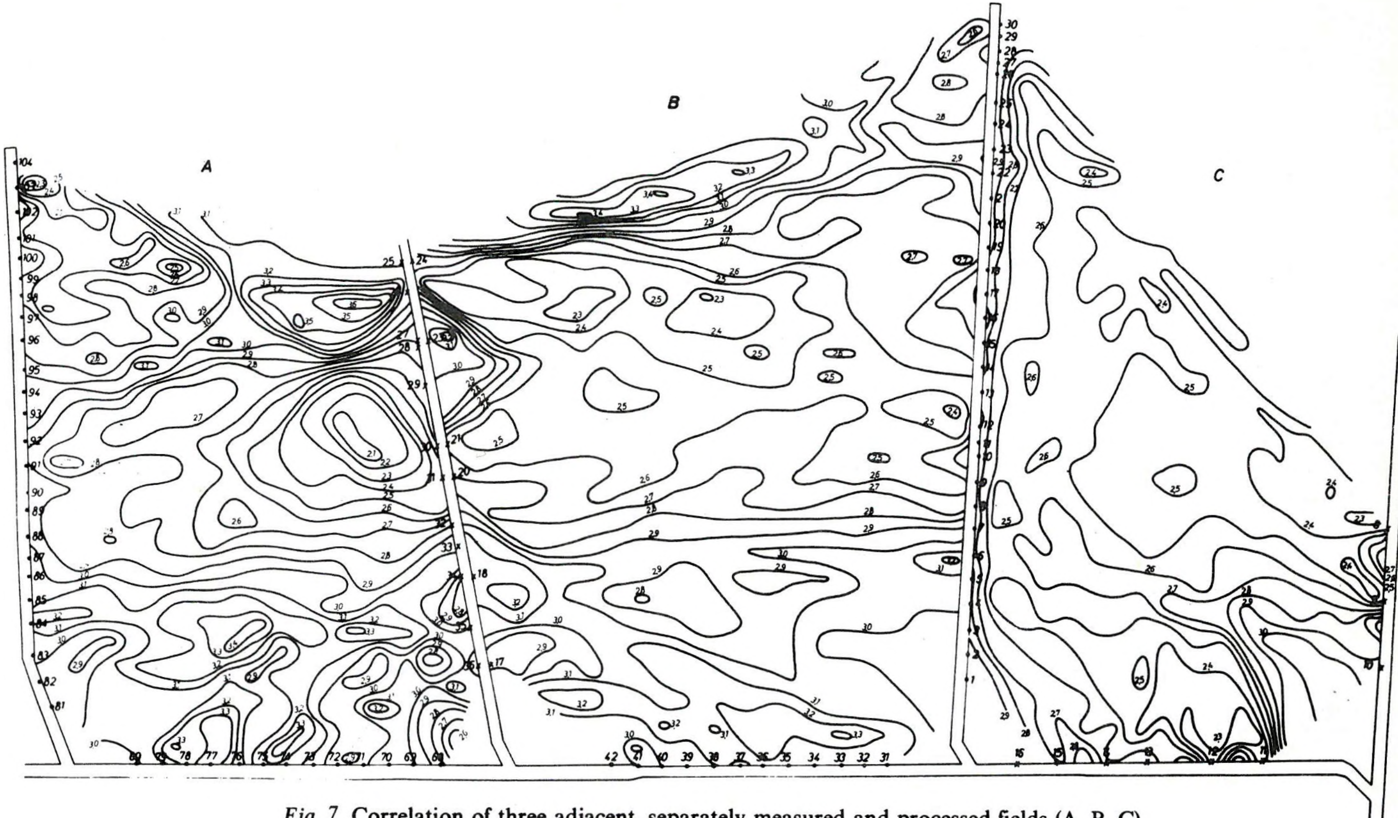


Fig. 7. Correlation of three adjacent, separately measured and processed fields (A, B, C).
(By permission of the Research Department of Mecsek Coal Mines)

7. *ábra*. Három, külön-külön végzett és feldolgozott mérés (A, B, C) korrelációja
(közölve a Mecseki Szénbányák Kutatási Osztályának engedélyével)

Фиг. 7. Корреляция трех, отдельно выполненных и обработанных измерений (A, B, C)
(С разрешением Отдела Исследований Мечекских Угольных Шахт)

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BÁNYABELI SZEIZMIKUS SEBESSÉGELOSZLÁS MEGHATÁROZÁSA A FESZÜLTSÉGELOSZLÁS MEGVÁLTOZÁSÁNAK KÖVETÉSÉHEZ

A bányabeli feszültségviszonyok ismerete biztonsági és gazdasági szempontból igen fontos. Mivel a szeizmikus hullámok sebességét a kőzetekben a nyomás befolyásolja, ezért a sebességeloszlás-adatokból következtethetünk a feszültségviszonyokra.

Az ismertetett módszer az ún. Algebraic Reconstruction Technique egyik változatának tekinthető. Egy kezdeti sebességmezőből kiindulva az eljárás sugárutakat követ a Snellius—Descartes-törvény vektorális formája alapján. A mért és számított futási idők összehasonlítása után — ha szükséges — módosítja a sebességmezőt, és újrakezdi a sugárút számítását. Az iterációs eljárás akkor fejeződik be, amikor a mért és számított időadatsor egy előre megadott értéknél kevésbé tér el egymástól. Az így kapott sebességmezőt fogadjuk el a vizsgált terület sebességeloszlásának. Rendszeresen végzett átvilágító mérésekkel és kiértékeléssel a vizsgált területen figyelemmel kísérhető a nyomásviszonyok alakulása.

Л. ХЕРМАН, Л. ДИАНИШКА, Й. ВЕРБЁЦИ

ОПРЕДЕЛЕНИЕ РАСПРЕДЕЛЕНИЯ СКОРОСТЕЙ СЕЙСМИЧЕСКИХ ВОЛН В ШАХТАХ ДЛЯ ПРОСЛЕЖИВАНИЯ ИЗМЕНЕНИЙ В РАСПРЕДЕЛЕНИИ НАПРЯЖЕНИЙ

Знание условий напряженности в шахтах представляет большой интерес с точки зрения безопасности и экономичности. Так как скорость сейсмических волн в горных породах подвергается влиянию давления, поэтому по данным распространения скоростей можно сделать вывод об условиях напряженности.

Излагаемый способ может рассматриваться как один из вариантов т. н. Algebraic Reconstruction Technique. Исходя из некоторого начального поля скоростей, процедура прослеживает траектории по векторальной форме закона Snellius-Descartes. После сопоставления измеренных и расчетных времен пробега — при необходимости — поле скоростей модифицируется и вычисление траектории начинается заново. Итерационная процедура заканчивается, когда расхождение между сериями измеренных и расчетных данных о временах пробега будет меньше заранее определенной величины. Регулярное выполнение работ по прослеживанию и интерпретации позволяет наблюдать изменения в условиях давления на изучаемом участке.