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GEOFIZIKAI KÖZLEMÉNYEK
25

METHODOLOGICAL BASIS OF A ϱ PROCESSOR FOR THE DIRECT DETERMINATION OF DENSITIES IN BORE-HOLES

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Introduction

In well-logging practice gamma-gamma logs are generally interpreted by using charts, graphically constructed from model measurements. To replace the rather time-consuming, tiresome manual interpretation procedure, there are two obvious possibilities:

1. Digitalisation of the logs, computer processing and automatic plotting of the resulting ϱ values,
2. Application of an analogue ϱ processor ensuring the direct registration of the density log.

The present paper is devoted to the basic theory underlying the construction of ϱ processors, including a mathematical approximation of the interpretation charts. Mud cake correction will also be approximated mathematically.

1. Basic Equation of the Two Detector Gamma-Gamma Method

The theoretical basis of the method is expressed by the equation describing the primary Compton scattering of gamma photons. We shall need two separate equations, for short and long probes:

$$N_s = K_s \varrho e^{-Q_s} \tag{1.1}$$

$$N_l = K_l \varrho e^{-Q_l}. \tag{1.2}$$

Taking the ratio of these equations we get

$$\frac{N_s}{N_l} = \frac{K_s}{K_l} e^{-(Q_s - Q_l)}, \tag{1.3}$$

which is the basic equation of the gamma-gamma method. In the above formulae:

N_s and N_l — counts of short and long probes, respectively;
K_s and K_l — constans depending on source strength N_0, solid angle $d\Omega$ and on the KLEIN–NISHINA–TAM differential scattering cross section $\sigma(\vartheta, \Phi)$;

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Q_s and Q_l depend:
- on the path of the primary and scattered rays in the rock (x_{s_1}, x_{l_1} and x_{s_2}, x_{l_2}) (see Fig. 1);
- on the density (ϱ) of the rock;
- on the average mass absorption coefficient of the rock (μ_{m_1}; μ_{m_2});
- on the thickness and density of the mud cake (t_{mc}, ϱ_{mc});
- on the average mass absorption coefficient of the mud cake (μ_{mc_1}; μ_{mc_2}).

(Index *1* refers to the primary, index *2* to the scattered ray).

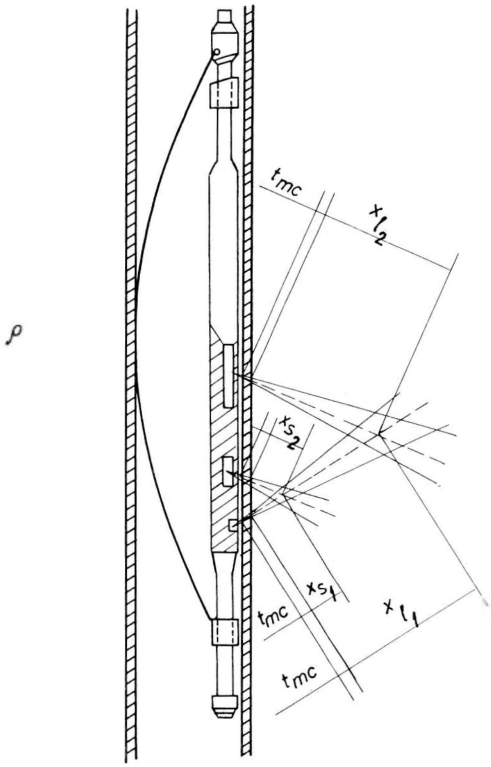

Fig. 1. Sketch of primary gamma scattering processes in bore-holes

1. ábra. A fúrólyukakban lejátszódó egyszeres gamma szórási folyamatok sematikus rajza

Рис. 1. Схематическое представление процессов однократного гамма-рассеяния, происходящих в скважинах

Let us write, in a general form,

$$Q = [(x_1 - t_{mc})\mu_{m_1} + (x_2 - t_{mc})\mu_{m_2}]\varrho + \\ + (\mu_{mc_1} + \mu_{mc_2})t_{mc}\varrho_{mc}. \tag{1.4}$$

In the absence of the mud cake Eq. (1.4) reduces to

$$Q = (x_1 \cdot \mu_{m_1} + x_2 \mu_{m_2})\varrho. \tag{1.5}$$

Introducing

$$C = x_1 \mu_{m_1} + x_2 \mu_{m_2} \tag{1.6}$$

and using indices s and l to short and long probes respectively, we get:

$$-(Q_s - Q_l) = -(C_s - C_l)\varrho \tag{1.7}$$

i.e.

$$\frac{N_s}{N_l} = \frac{K_s}{K_l} e^{-(C_s - C_l)\varrho} \tag{1.8}$$

2. One-Variable Linear Regression for the Approximation of the Base Line of the Interpretation Chart

Figure 2 shows the interpretation chart for the KRGG–2–120–60sY type radioactive probe. The chart consists of two parts:

a) The "base line" and the branching lines for mud cake correction (central part); *b)* a nomogram for direct read-out of the ϱ values (on the right).

Individual points of the base line are determined by the values of counts (N_s and N_l) measured by the short and long probes, respectively normalized to water, and by the density values (ϱ).

Let us try to fit an empirical formula to the base line. We start out from the general equation

$$N = K\varrho e^{-C\varrho}, \tag{2.1}$$

where: N — registered counts;
ϱ — density of the rock;
K, C — appropriate constants.

Writing Eq. (2.1) for an arbitrary ϱ and for $\varrho = 1$ (water), and taking the logarithm of their quotient we obtain the equation of the base line:

$$Y = \ln x - c(x-1) \tag{2.2}$$

with

$Y = \ln \dfrac{N}{N_w}$ — logarithm of the counts normalized to water.

Eq. 2.2 of course, cannot be used for direct regression because of the non-linearity of the function $\ln x$.

If, in a given interval (a, b) the function $\ln x$ is approximated by a straight line $\Phi(x) = Dx + \zeta$, D and ζ parameters can be determined by the least mean squares principles minimizing the integral

$$S = \int_a^b \{\ln x - (Dx + \zeta)\}^2 \, dx. \tag{2.3}$$

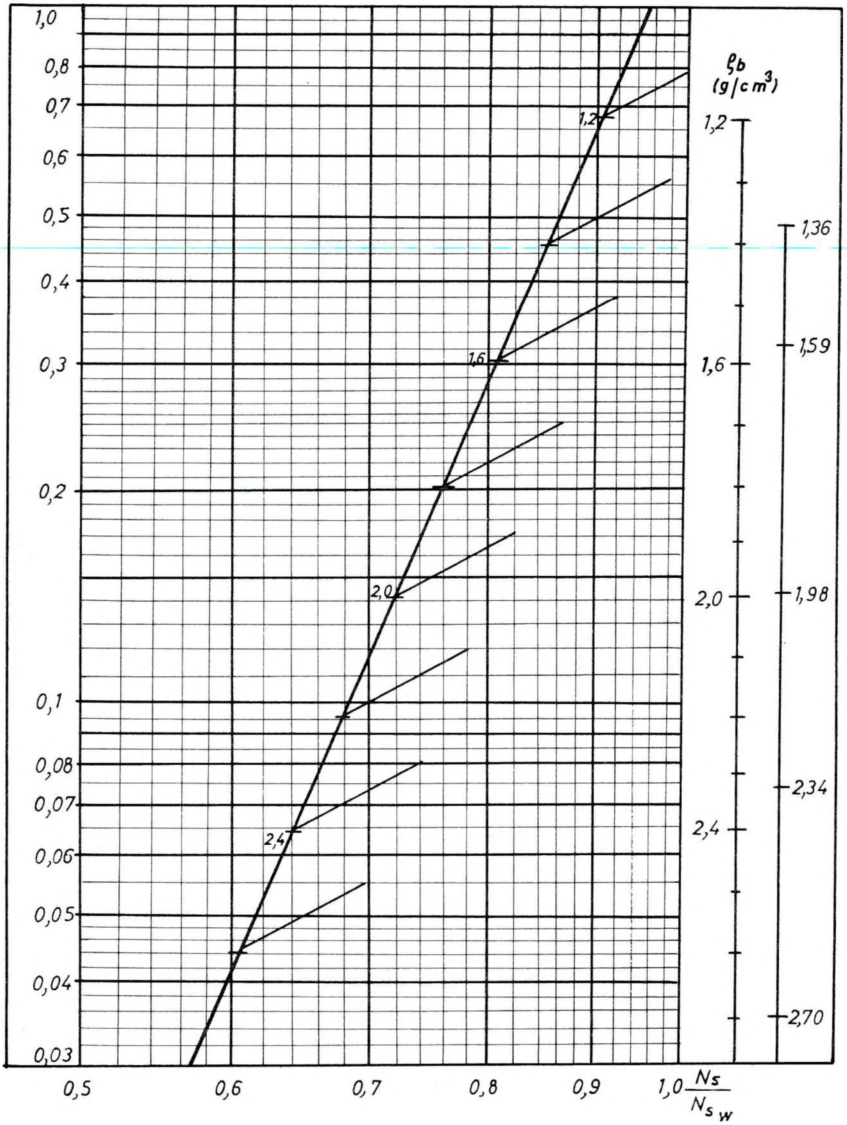

Fig. 2. Interpretation chart for the KRGG–2–120–60sY type radioactive logging equipment

2. ábra. KRGG–2–120–60sY típusú rádióaktív szonda kiértékelő diagramja

Рис. 2. Палетка для интерпретации данных, получаемых зондом РК типа KRGG–2–120–60sY

Substituting back the approximate relationship $\ln x \approx Dx + \zeta$ to Eq. (2.2) and regrouping the terms:

$$Y = \underbrace{(D-C)}_{m}x + \underbrace{(\zeta + C)}_{b} \tag{2.4}$$

we find that m and b can be obtained directly by linear regression. Indeed,

$$Y = \underbrace{\left(r \cdot \frac{S_y}{S_x}\right)}_{m}x + \underbrace{\left[\bar{Y} + \bar{x}r\frac{S_y}{S_x}\right]}_{b} \tag{2.5}$$

where:

$\bar{Y}$ and $\bar{x}$ are mean values of the dependent and independent variables;
S_y and S_x are the corresponding standard deviations;
r is the correlation coefficient.

Table I. contains the results of the linear regression for the KRGG–2–120–60sY probe. Calculations have been performed for two different diameters ($d_1 = 86$ mm and $d_2 = 214$ mm). The correlation coefficients obtained for the short and long probes show that there is a very strong connection between the values of counts normalized to water and the densities (ϱ). This, at the same time, proves the accuracy of the approximation of the base line, and the validity of the basic equation.

Table I

No.	$\varrho_i = x_i$	$d_1 = 86$ mm, $a_s = 13$ cm: $\frac{N_s}{N_{s_w}}$	Y_{s_i}	$a_l = 38$ cm: $\frac{N_l}{N_{l_w}}$	Y_{l_i}	$\varrho_i = x_i$	$d_2 = 214$ mm, $a_s = 13$ cm: $\frac{N_s}{N_{s_w}}$	Y_{s_i}	$a_l = 38$ cm: $\frac{N_l}{N_{l_w}}$	Y_{l_i}
1	1.00	1.0000	0.0000	1.0000	0.0000	1.0000	1.0000	0.0000	1.0000	0.0000
2	1.36	0.8485	−0.1642	0.4849	−0.7238	1.46	0.8989	−0.1086	0.4188	−0.8704
3	1.59	0.7839	−0.2435	0.3249	−1.1242	2.15	0.7633	−0.2701	0.1322	−2.0234
4	1.98	0.7558	−0.2799	0.1432	−1.9435	2.34	0.7576	−0.2776	0.1071	−2.2340
5	2.34	0.6580	−0.4186	0.0736	−2.6091	2.50	0.7010	−0.3552	0.0750	−2.5903
6	2.70	0.5896	−0.5283	0.0353	−3.3439	2.70	0.6253	−0.4695	0.0545	−2.9095
Mean value	1.8283		−0.2724		−1.6241	2.0233		−0.2465		−1.7713
S.D.	0.6336		0.1864		1.2432	0.6568		0.1693		1.1129
r		−0.9747		−0.9998			−0.9807		−0.9991	
m		−0.2872		−1.9627			−0.2524		−1.6894	
b		0.2538		1.9632			0.2645		1.6498	

Finally, writing Eq. (2.4) obtained by linear regression for both short and long probes

$$Y_s = m_s x + b_s, \tag{2.6}$$

$$Y_l = m_l x + b_l \tag{2.7}$$

the equation of the base line will be:

$$Y_s - Y_l = (m_s - m_l)x + (b_s - b_l) \tag{2.8}$$

or with the original notations:

$$\ln \frac{N_s}{N_{s_w}} - \ln \frac{N_l}{N_{l_w}} = (m_s - m_l)\varrho + (b_s - b_l). \tag{2.9}$$

3. An Approximate Mathematical Solution for Mud-Cake Correction

One among the factors influencing gamma-gamma measurements in bore-holes is the presence of mud-cake. As an effect of mud-cake, points to be interpreted do not lie exactly on the base line but somewhat displaced, upwards to the right, or downwards to the left. (This latter case occurs for mud-cakes of high density).

In what follows we derive an approximate mathematical solution for the continuous determination of the mud-cake correction ($\Delta\varrho$). The basic ideas of the method can be understood from the sketch of Fig. 3.

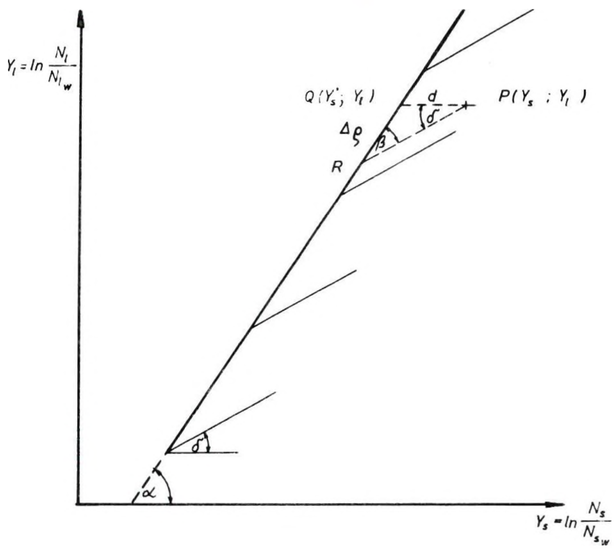

Fig. 3. Principle of the mathematical approximation of continuous mud-cake correction

3. ábra. Szemléltető rajz a folyamatos iszaplepény-korrekció közelítő matematikai módszerének megértéséhez

Рис. 3. Схема для иллюстрации приближенного математического метода непрерывного ввода поправок за глинистую корку

Consider the triangle PQR. By the sine theorem the mud-cake correction ($\Delta\varrho$) is given by

$$\Delta\varrho = d\frac{\sin\delta}{\sin\beta}, \tag{3.1}$$

where

$$d = Y_s - Y_s',$$
$$\beta = \alpha - \delta. \tag{3.2}$$

The sign of the correction $\Delta\varrho$ is either positive or negative, depending on whether the measured point ϱ lies on the right-hand side ($Y_s > Y_s'$) or left-hand side ($Y_s - Y_s'$) of the base line.

Of course, this approximation has definite limitations. If the measured point happens to be too far off the base line ($t_{mc} > 1 - 1{,}5$ cm) we are not justified by projecting it back along a straight line. In such cases approximations based on higher-order polynomials should be certainly better. This, however, is outside the scope of an analogue ϱ processor, because of the complicated circuitry required. Problems of this kind should be dealt with digital processing.

4. Methodological principles for constructing a ϱ Processor

The mathematical formulae derived in previous sections are simple enough to be realized by an analogue circuitry. The main task of this instrument is to register a continuous apparent density (ϱ_{app}) curve and mud-cake correction ($\Delta\varrho$) curve. Real density values are the sum of these curves:

$$\varrho = \varrho_{app} \pm \Delta\varrho. \tag{4.1}$$

Equation (4.1) is, methodologically, the basic equation of the ϱ processor.

By substituting $x = \varrho_{app}$ into Eq. (2.7), and making some rearrangements:

$$\varrho_{app} = m_l^* Y_l + b_l^*, \tag{4.2}$$

where

$$m_l^* = \frac{1}{m_l}; \quad b_l^* = -\frac{b_l}{m_l}.$$

The parameters m_l and b_l have to be determined by linear regression.

Mud-cake correction is computed by Eq. (3.1). The angles δ and β can be determined from the interpretation charts. The value of d comes from Eq. (3.2), Y_s' term can be determined by putting Y_l = const. into Eq. (2.8), i.e.:

$$Y_s' = M\varrho + B + Y_{l\text{const}}, \tag{4.3}$$

where

$$M = m_s - m_l,$$
$$B = b_s - b_l.$$

From Eq. (4.3):

$$d = Y_s - (M\varrho + B + Y_{l\text{const}}). \tag{4.4}$$

Introducing the notation

$$K = \frac{\sin \delta}{\sin \beta}.$$

Eqs. (4.2)–(4.3) together yield the basic equation of the ϱ processor:

$$\varrho_{\text{real}} = (m_l^* Y_l + b_l^*) + K\{Y_s - [(M\varrho_{\text{app}} + B) + Y_l]\}. \tag{4.5}$$

In the absence of mud-cake Y_s', $\Delta g = 0$, i.e. the counts measured by the long probe yield real density values.

5. Field Experiments with the KRGG–2–120–60sY type Probe and ACD–75 Type ϱ Processor

The aim of these experiments was to record continous density logs in situ, to check the reliability and limitations of the ϱ processor. Two wells: Cs–220 (Csordakút) and Na–221 (Nagyegyháza), were used for the experiments.

In well Cs–220 continuous density logging was performed from 120 to 172 m, logging repeated between 120–150 m for statistical reliability checks.

For statistical analysis 24 corresponding ϱ samples were used from each log and their deviations were classified into four intervals.

The percentage distribution of deviations is given in Table II.

Table II

Deviation intervals (g/cm^3)	No. of samples	%
0.00–0.05	18	75
0.05–0.07	5	21
0.07–0.1	0	0
greater than 0.1	1	4

As Table II shows, 75% of the deviations are within the accuracy limit of density determination.

In well Na–221 gamma-gamma and continous density logs were taken between 24–100 m, with repeated measurement from 25 to 75 m. Logs are presented in Fig. 4. The density values obtained by the ϱ processor were compared to those calculated manually using the chart of Fig. 2, for altogether 18 layers (Table III). It can be stated that—except for a few samples—there is a fair agreement between the two sets of ϱ values.

As already mentioned, the distance (d) of the measured point off the base line sets a natural limit to the accuracy of the ϱ values determined by the ϱ processor. In case of a high d value the projection along a straight line back to the base yields unrealistically high $\Delta\varrho$ correction. We realize that in such cases a projection along second—or higher—order curves would yield better results.

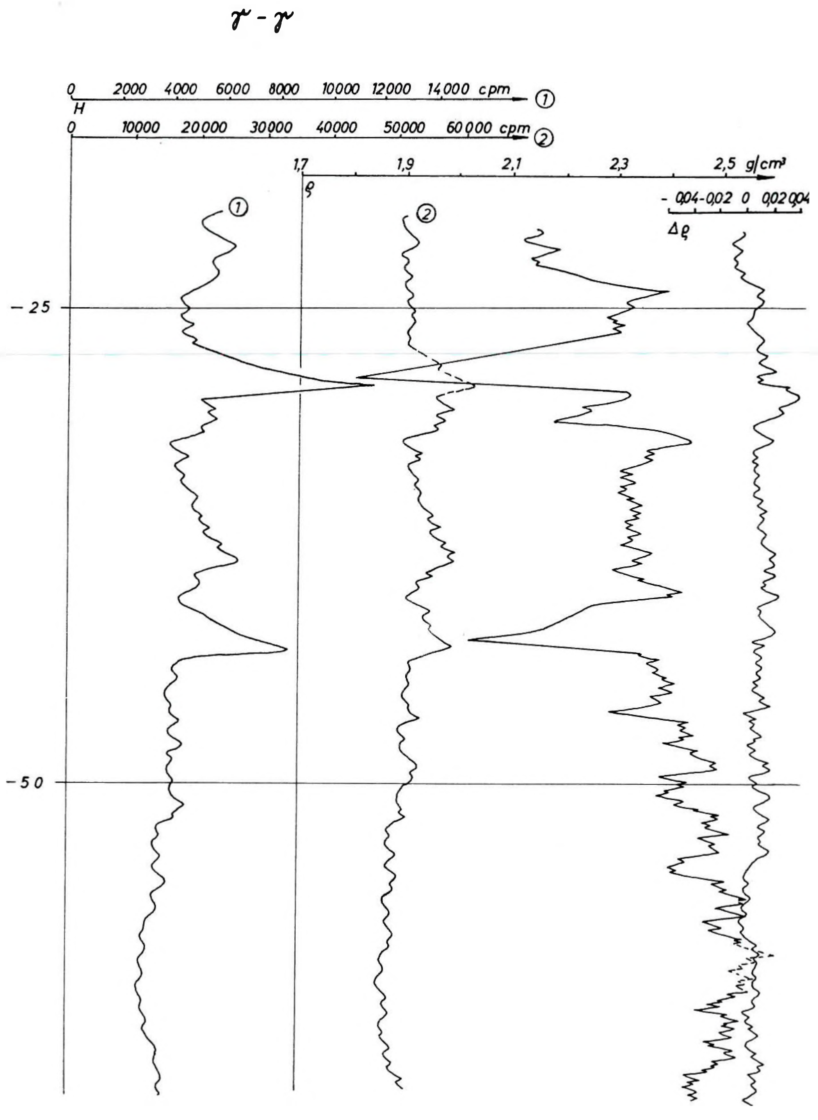

Fig. 4. Gamma-gamma logs, continuous density log (ϱ) and $\Delta\varrho$ log for the Na–221 bore-hole
1) long probe 2) short probe

4. ábra. Nagyegyháza–221 számú fúrásban felvett gamma-gamma, folyamatos sűrűség (ϱ) és $\Delta\varrho$ szelvények
1) hosszú csatorna; 2) rövid csatorna

Рис. 4. Кривые непрерывной плотности ГГК ϱ и кривые $\Delta\varrho$, полученные в скважине Надьэдьхаза–221
1 – длинный канал; 2 – короткий канал

In the next years, we should further refine the theoretical principles of the ϱ processor by developing the basic equation of primary scattering and introducing a correction term for secondary scattering effects. Theoretical calculations and model experiments will be performed to incorporate bore-hole diameter corrections into the basic equation of the ϱ processor. All these research works will be reported in forthcoming papers.

Table III

Probe type: KRGG–2–120–60 sY
Probe No: 7636
Isotope: Cs^{137} 14.9 mCi

Bore-hole Na–221.

		$a_s = 13$ cm		$a_1 = 38$ cm			ACD–75	manual
N°	Depth [m]	N_s [cpm]	$\frac{N_s}{N_{s_w}}$	N_l [cpm]	$\pm\sigma_l$ [cpm]	$\frac{N_l}{N_{l_w}}$	ϱ_b [g/cm³]	ϱ_b [g/cm³]
1	2	3	4	5	6	7	8	9
1	25.0	53 142	0.646	4 286	104	0.0701	2.34	2.33
2	26.0	51 429	0.625	4 429	105	0.0725	2.32	2.31
3	28.7	62 857	0.764	11 429	169	0.1871	1.86	1.80
4	29.5	57 143	0.695	5 000	112	0.0819	2.31	2.33
5	31.0	58 286	0.708	5 571	118	0.0912	2.23	2.19
6	32.0	53 142	0.646	3 857	98	0.0631	2.43	2.45
7	33.0	53 142	0.646	4 000	100	0.0655	2.37	2.37
8	35.0	54 286	0.660	4 857	110	0.0795	2.28	2.32
9	38.0	59 429	0.723	6 429	127	0.1053	2.20	2.37
10	40.0	52 571	0.639	4 286	104	0.0702	2.34	2.47
11	42.5	58 857	0.715	8 143	143	0.1330	2.03	2.04
12	45.0	52 000	0.632	3 857	98	0.0631	2.39	2.39
13	46.2	54 286	0.660	4 143	102	0.0678	2.39	2.28
14	48.0	51 429	0.625	4 286	104	0.0702	2.30	2.40
15	49.0	52 286	0.635	4 000	100	0.0655	2.38	2.50
16	54.5	48 571	0.591	3 429	93	0.0561	2.40	2.42
17	59.0	48 000	0.583	2 857	85	0.0468	2.53	2.60
18	62.0	48 000	0.589	2 786	83	0.0456	2.54	2.46

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ANDRÁSSY LÁSZLÓ

"ϱ" PROCESSZOR MÓDSZERTANI ALAPJAI A TÉRFOGATSÚLYÉRTÉKEK FÚRÓLYUKBAN TÖRTÉNŐ KÖZVETLEN MEGHATÁROZÁSÁRA

A kétdetektoros gamma-gamma mérőrendszerek és a „ϱ" processzor együttes alkalmazása fúrólyukakban közvetlenül iszaplepényre korrigált valódi térfogatsúlygörbe mérését és analóg formában történő regisztrálását teszi lehetővé.

A gamma-fotonok Compton szórását leíró egyszeres szórás alapegyenletéből kiindulva meghatároztuk a kétdetektoros gamma-gamma eljárás alapegyenletét. Következésképpen az említett egyenletek ismeretében — iszaplepénymentes feltételekre — logaritmizálás és egyszerű matematikai műveletek elvégzése után lineáris regresszió segítségével meghatároztuk a bázis egyenes egyenletét. Bemutatjuk a KRGG–2–120–60 sY típusú kétdetektoros gamma-gamma mérőrendszer bázis egyenesének egyenletét.

A méréseket befolyásoló iszaplepénykorrekció kiszámítására egy közelítő matematikai megoldást mutatunk, amely segítségével iszaplepényre korrigált valódi térfogatsúlyértékek határozhatók meg. Az eljárás t_{mc} = 1 – 1,5 cm-nél kisebb iszaplepényvastagságokra alkalmazható. Cs–220 fúrás szelvényanyagából elemző feldolgozást végeztünk a térfogatsúlygörbe és ismétlésének összehasonlítására. A Na–221 fúrás szelvényanyagából elvégeztük a kiértékelő diagram segítségével kézi úton és ACD–75 processzorral kapott térfogatsúlyadatok összehasonlítását.

Л. АНДРАШИ

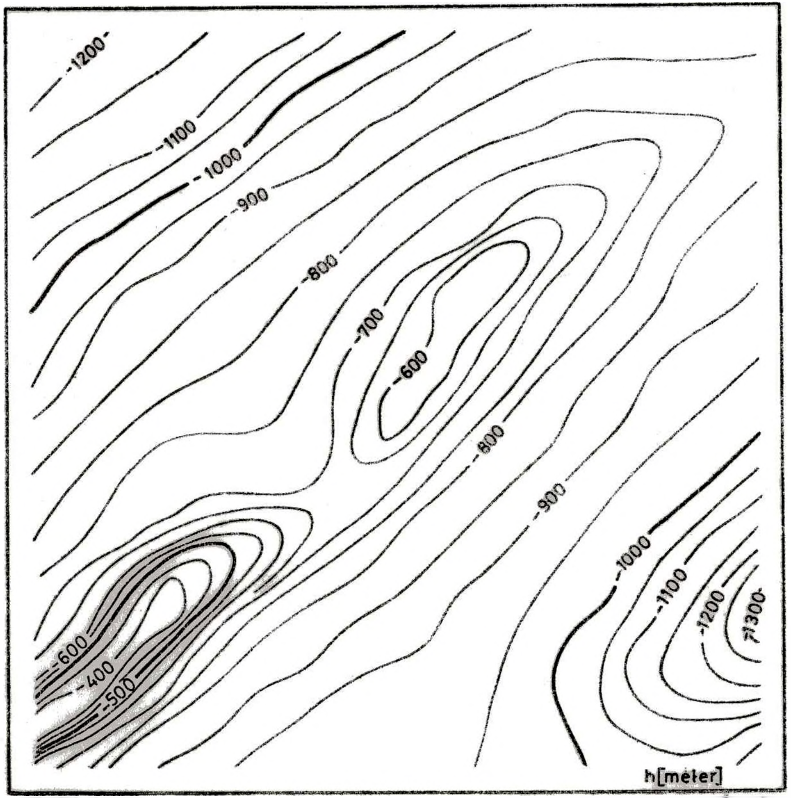
МЕТОДИЧЕСКИЕ ОСНОВЫ ПРОЦЕССОРА «ϱ» ДЛЯ ПРЯМОГО ОПРЕДЕЛЕНИЯ ВЕЛИЧИН ОБЪЕМНОГО ВЕСА В СКВАЖИНАХ

Совместное использование двухдетекторного зонда ГГК и процессора «ϱ» дает возможность получать и записывать в аналоговой форме кривые эффективного объемного веса, исправленные за глинистую корку непосредственно в скважинах.

Исходя из основного уравнения однократного рассеяния, описывающего комптоновское рассеяния гамма-фотонов, можно определить основное уравнение для метода двухдетекторного ГГК. Зная указанные уравнения, после логаритмирования и проведения простых математических процедур, для условий без глинистой корки – было определено уравнение основной прямой. В работе приводится уравнение основной прямой двухдетекторного зонда ГГК типа KRGG–2–120–60 sY.

Для вычисления поправок за глинистую корку, влияющую на результаты наблюдений, приводится приближенное математическое решение, позволяющее определить эффективные величины объемного веса, исправленные за глинистую корку. Данный метод может применяться при толщинах глинистой корки меньших t_{mc} = 1 – 1,5 cm.

Каротажные данные скважины Cs–220 были подвергнуты обработке и анализу для сопоставления кривой объемного веса с кривой повторных измерений. По каротажным данным скважины Na–221, с использованием палетки были сопоставлены данные об объемном весе, полученные в результате ручной интерпретации и обработки процессором типа ACD–75.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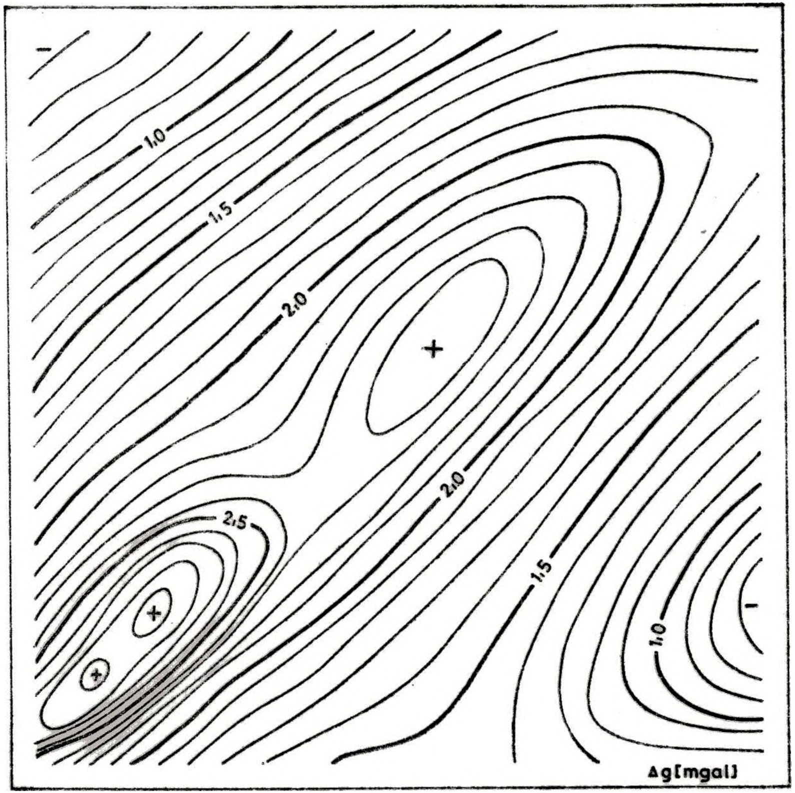
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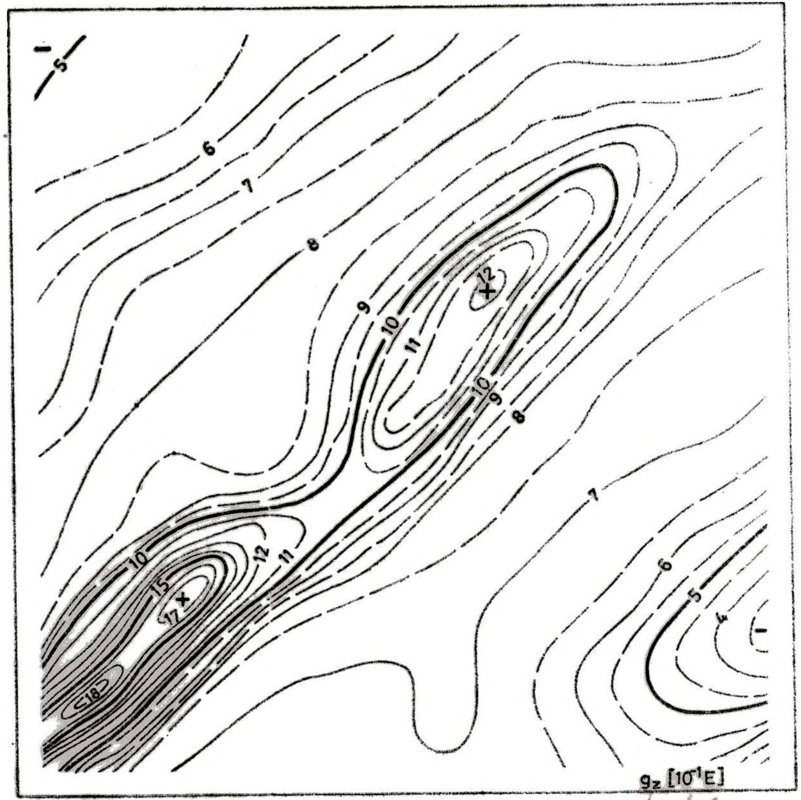
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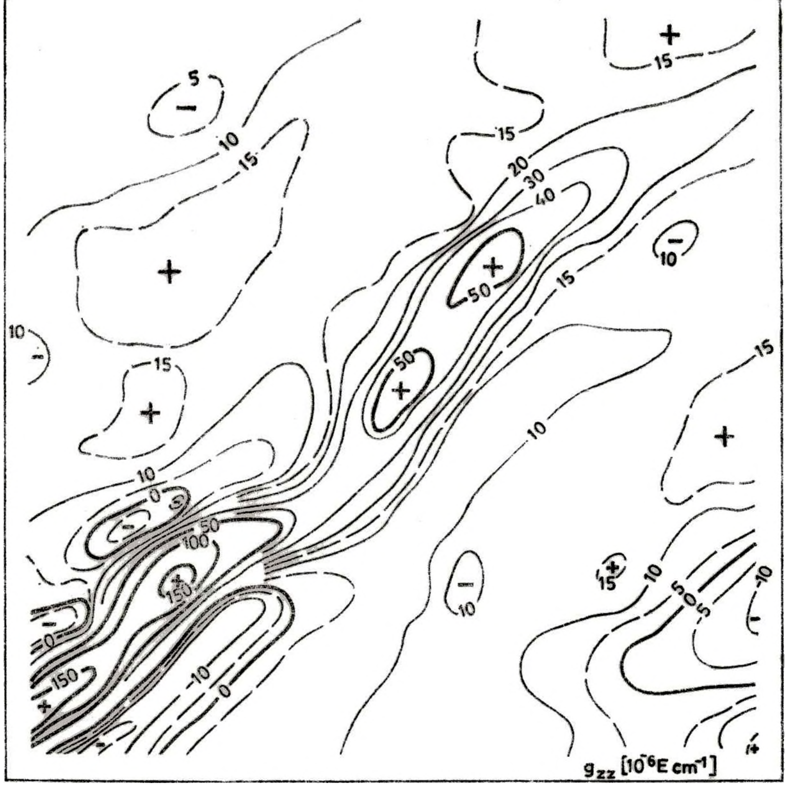
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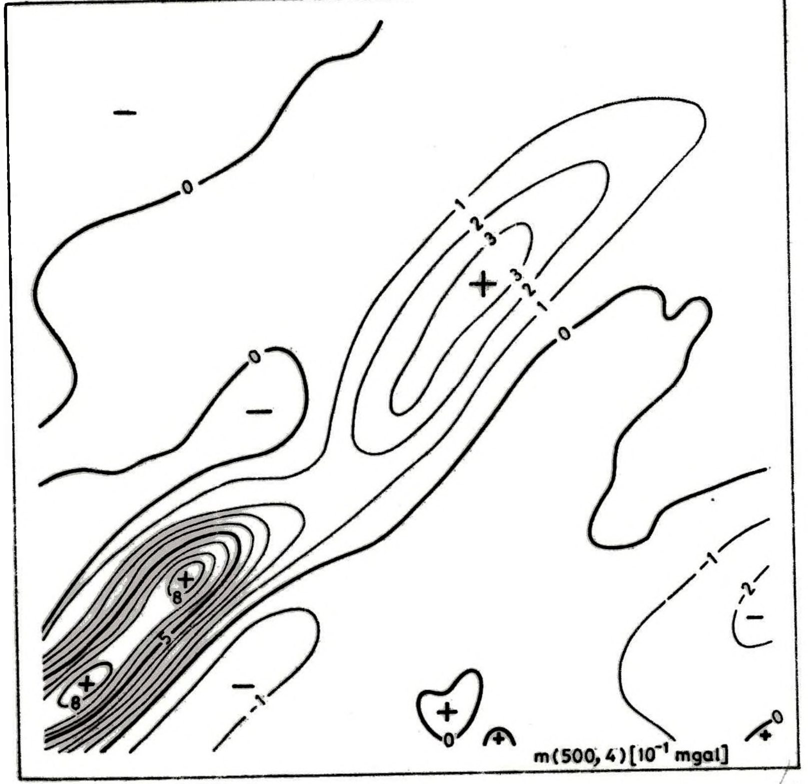

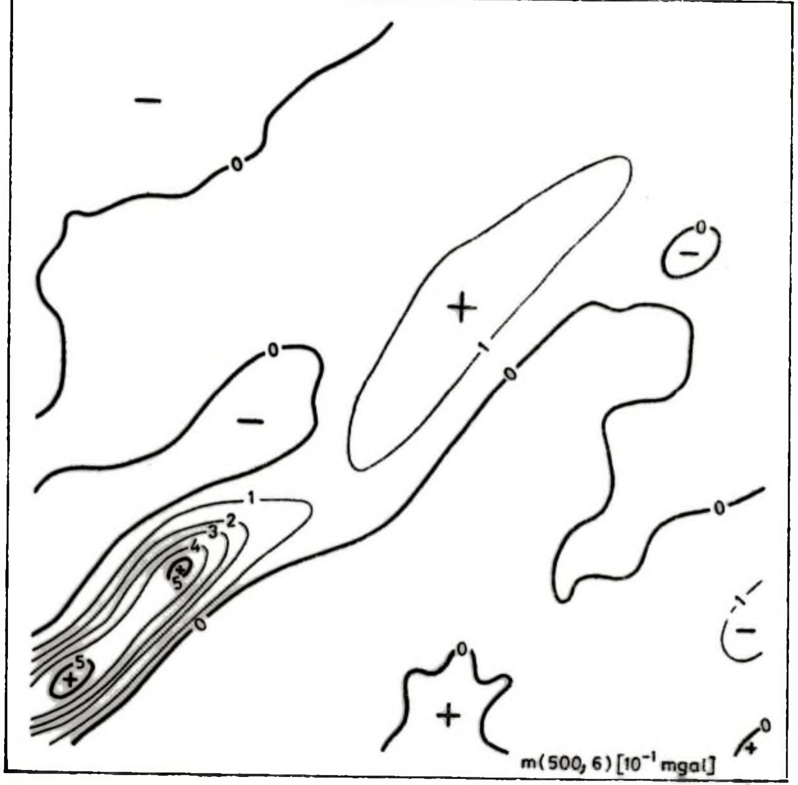

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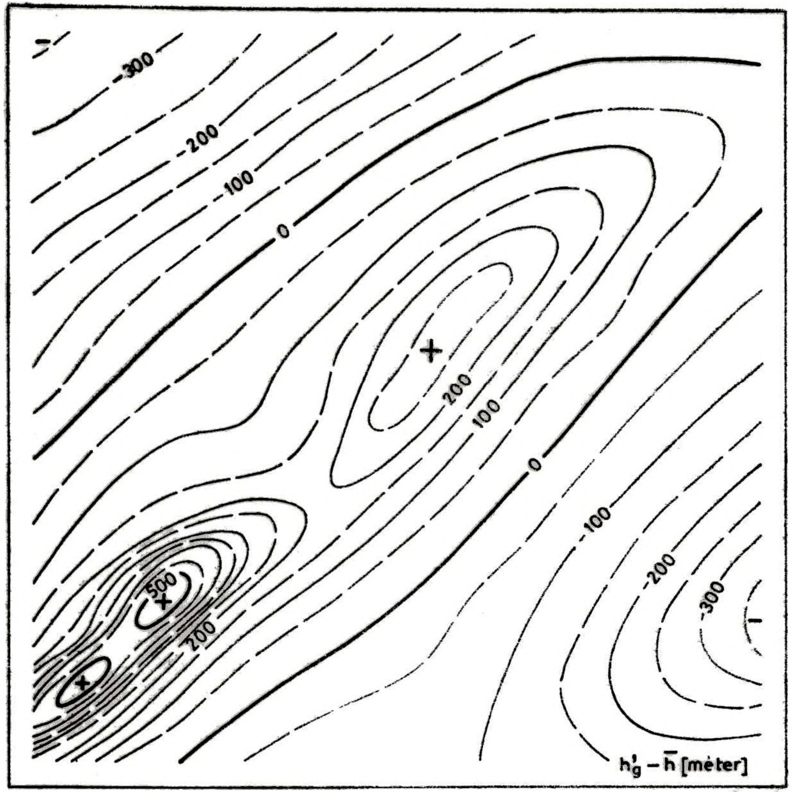

$h'_g = 2.5\Delta g + 1.7 g_z + 0.019 g_{zz}$.

$\bar{h} = -860$

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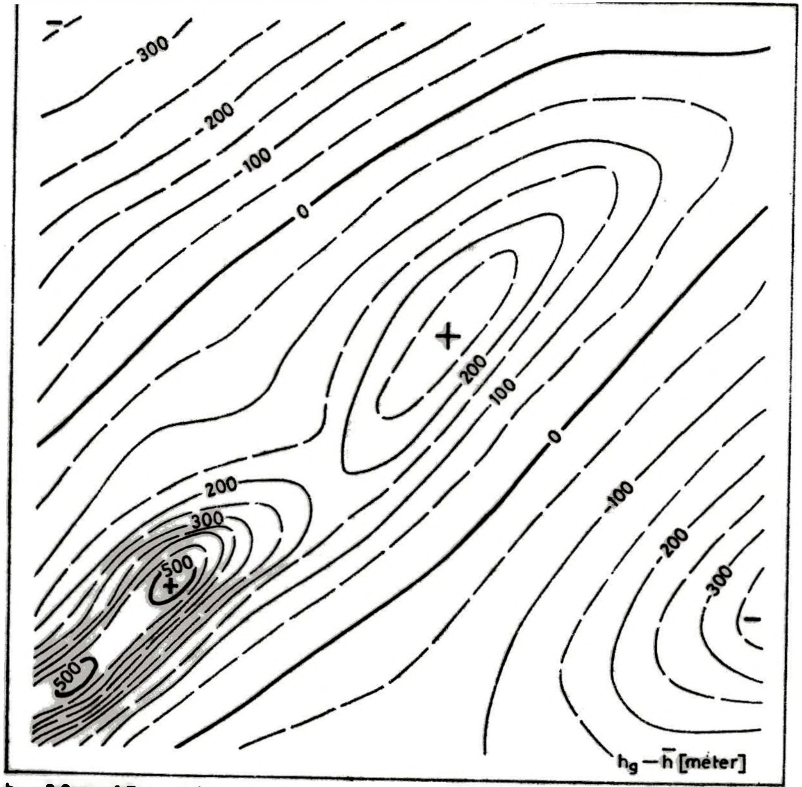

$h_g = 2.9\Delta g + 1.7m_3 - 4.5m_4 + 12.5m_6$

$\bar{h} = -860$

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$\Delta h' = h'_g - h$

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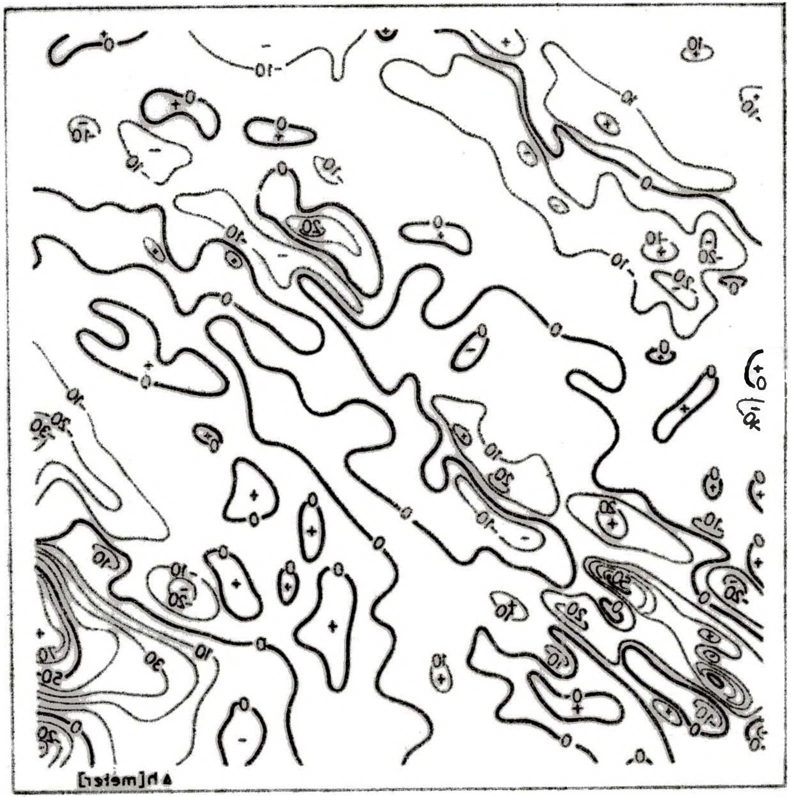

$\Delta h = h_g - h$