

## ESTIMATION ERROR OF INTERVAL VELOCITIES AND THE GEOLOGICAL MODEL

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The enormous volume of data obtained in course of geophysical measurements excludes the possibility of building up a geological model which was in complete conformity with all the measured data. Namely, this would imply that the number of parameters of the model ought to be equal to that of the measured data which would lead to an untractably large system of equations.

In seismic practice it has been known for a long time that instead of assuming an overcomplicated geological model structure in order to fit the measured data, it is always better to solve strongly overdetermined systems of equations. The discrepancies between the measured data and the theoretical results obtained from the simplified ideal model can be resolved by introducing the concept of noise. The less the known connection between the geological model and the measured data, the more the effect of the noise should be considered. This does not mean however that a proper understanding of the effect of the ever more intricated geological models would help us to reduce noise beyond limits. Since there is no cause-effect relation between the statistical error of measurement and the geological model, the concept of noise cannot be eliminated. On the other hand, there are no exact mathematical interpretation rules but for the most simple cases, i.e. there is an inverse correlation between the increasing number of parameters and the reliability of their estimation. Consequently, there are principal obstacles to the determination of very complex geological structures.

In this situation it seems to be a realistic compromise to adopt an interpretation strategy so that the deteriorating effect of noise on the evaluated parameters is minimal. More precisely, one has to find a geological model for which a maximum amount of information is contained in the measured data.

The information gained about the model parameters is defined as the change in their indeterminacy (entropy) as a consequence of their measurements. Beyond the solution of the given, undisputable equations, the above-said thoughts should serve as an objective mathematical model for the interpreter's professional conscientiousness. (Here we have in mind SLOTNICK's classical formulation: "The responsibility of the geophysicist rests in interpreting the data, making sections, drawing conclusions of a physical-geometric nature.... He should know, and be honest in transmitting, the value and limits to this conclusion. Then and only then does he do his full duty.")

The logically correct interpretation methods are based on three assumptions (for a more exact formulation see e.g. GOLTZMAN 1975, SALÁT and DRAHOS 1973, 1975):

1. Idealization of the measured data;
2. Idealization of the geological conditions, selection of an appropriate model;

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### 3. An appropriate transformation between the measured data and the geological model.

The aim of the present paper is to investigate the factors effecting the estimation of the interval velocities and to determine the dispersion of the values obtained as a function of the above-mentioned factors. The significance of the problem has already been emphasized by ANSTEY (1973). Model experiments will be presented, showing the connections between the statistical errors and the complexity of the geological model. The optimal number of parameters to be determined will be shown to depend on the quantity and quality of the measured data.

It will be assumed throughout the paper that the time-distance curves can be approximated by hyperboles. Some of the statistical characteristics of the parameters  $t_0$  and NMO and the variance of the stacking velocity  $v_s$  will be assumed to be known. The effect of noise will be considered through these factors. In the following examples seismic reflections will be characterized by the parameters of the hyperbole fitted to the local maxima of the traces. (Related investigations of the author were reported at the XX. Geophysical Symposium, in Szentendre, Hungary, 1975).

Next, the statistical properties of the errors of depth and interval velocity estimations will be investigated as a function of the noise. All computations will be performed for the fixed spread system E-60-2-12 (i.e. geophone spacing 60 m, offset = 2 times geophone spacing, fold-number = 12). The standard deviation of the arrival times (times of peaks) will be generally assumed as  $\sigma_t = 1 \text{ ms}$  unless otherwise mentioned. All results characterizing standard deviations of the different parameters will be proportional to  $\sigma_t$ .

According to the theory of stochastic processes  $\sigma_t$  can be expressed as

$$\sigma_t = \frac{f_n}{\sqrt{2n} (2\pi f_s)^2}$$

where  $f_n$  is the average noise frequency,  $n$  is the signal-to-noise power ratio,  $f_s$  is the average signal frequency. The above formula holds for large values of  $n$ . For low signal-to-noise ratios the formula for  $\sigma_t$  is more complicated.

### The Expected Value and the Variance of the Interval Velocity

The estimated value of the interval velocity depends on the chosen mathematical model. In the next paragraphs only a single model will be dealt with; it is expected however, that the fluctuations of the statistical parameters will be similar for the more general models as well.

To begin with let us recall some basic concepts about interval velocities. Suppose that the interval velocity between two reflectors is computed by the DIX formula (DIX, 1955):

$$v_{int}(i+1) = (v_{i+1}^2 t_{i+1} - v_i^2 t_i) / (t_{i+1} - t_i)$$

where  $v_{i+1}$  and  $v_i$  are the RMS velocities (i.e., practically, stacking velocities) and  $t_{i+1}$  and  $t_i$  are the zero-offset times of the respective horizons. It is well known (see AL-CHALABI, 1974) that for a horizontally layered medium the "interval velocity" computed between any two reflectors from the respective  $t_0$  and  $V_{RMS}$  parameters is the same as the RMS velocity between the two reflectors. The closer the computed

RMS velocity is to the effective average velocity of the set of layers in question, the more homogeneous this set is. Indeed, if this set consists of a series of equal velocity layers (i.e. if it is completely homogeneous), the RMS and average velocities will be equal. The ratio of these two velocities characterizes the homogeneity of the medium.

Because of the noise and the limited resolution power of the seismic method, the medium cannot be modelled by arbitrarily thin layers. In practice, we have to confine ourselves to the detection of the marked changes of the velocity functions, i.e. an optimal division of the depth section to a finite number of relatively thick layers should be sought for.

Let us consider the two-layered model shown in Fig. 1. Thicknesses and interval velocities of the two layers are  $h_1$ ,  $h_2$ ;  $v_1$  and  $v_2$ , resp. The theoretical derivation of the variance  $\sigma_{v_{int}}$  (2) of  $v_2$  is given in Appendix A. Since the variances are shown to depend very slightly on the actual values of  $v_1$ ,  $v_2$ , only the case  $v_1 = 2800$  m/s;  $v_2 = 3200$  m/s will be dealt with.

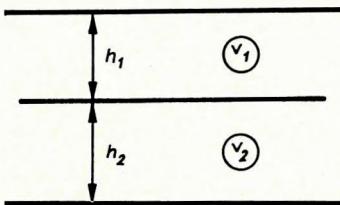


Fig. 1 Model applied for the study of the interval velocity  $v_2$   
1. ábra. Modell a  $v_2$  intervallumsebesség tanulmányozására  
Рис. 1. Модель для изучения интервальных скоростей  $V_2$

The dependence of  $\sigma_{v_{int}}$  (2) on  $h_1$ ,  $h_2$  will be investigated for the following two cases:  
A. The fluctuations  $\Delta t$  are completely non-correlated, i.e. the last four terms in Eq. A. 1. will vanish (Fig. 2);

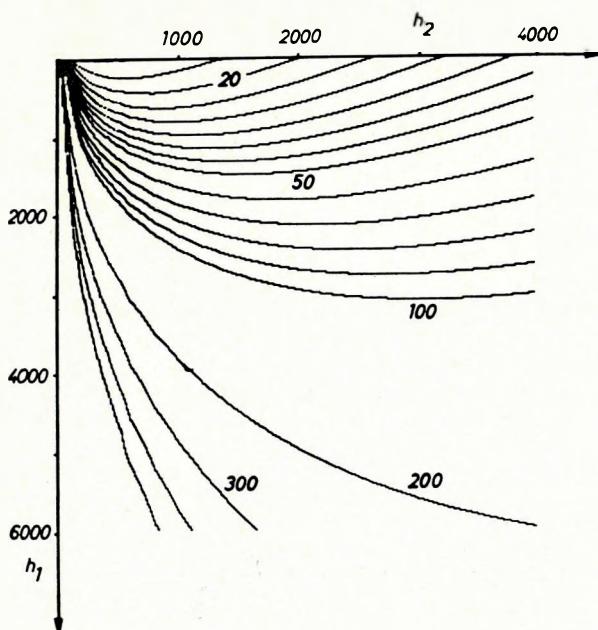


Fig. 2 Scatter of the interval velocity  $v_2$  in case of noncorrelated noises  
2. ábra. A  $v_2$  intervallumsebesség szórása korrelálatlan zajok esetén  
Рис. 2. Разброс интервальных скоростей  $V_2$  при наличии некоррелирующихся помех

B. The fluctuations are completely due to residual statics (Fig. 3).

In reality of course the fluctuating quantities contain static and non-correlated components as well. The variance of the interval velocity depends on the magnitudes and ratios of the two noise types (due to the independency of these different kinds of noise, their standard deviations simply add together). In case of  $h_1 = 0$  the two variances are almost equal.

The above results allow the following conclusions to be drawn:

a) The estimation of the interval velocity is less influenced by the errors of the static correction (case B) than by the inorganized noise (assuming an identical scatter  $\sigma_t$ ). The estimation of stacking velocity was deteriorated by both types of noise. By applying an automatic static correction program, the B type of noise can be eliminated and—in case of a good-quality material—we are usually left with a non-correlating noise of some  $\pm 1$  ms scatter.

b) In case of non-correlated noise the estimation of the velocity of thin layers becomes more and more illusory for increasing depths. For a more detailed analysis of interval velocities (e.g. to detect overpressured zones) an extreme smoothing

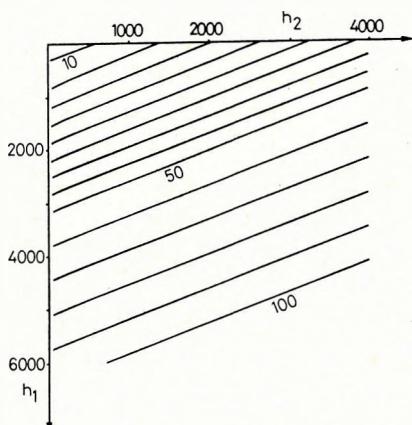


Fig. 3 Scatter of the interval velocity  $v_2$  in case of static noise

3. ábra. A  $v_2$  intervallumsebesség szórása statikus jellegű zajok esetén

Рис. 3. Разброс интервальных скоростей  $V_2$  при наличии статических помех

of these velocities should be carried out along the section. (This, of course, can be considered as a way to decrease  $\sigma_t$ ).

c) In case of non-correlating noise the optimal thickness  $h_2$  (for which the interval velocity can be estimated with a minimal error) increases proportionally with  $h_1$ .

#### Expected Value and Variance of the Estimated Depth Values

One of the most important transformed quantity is *depth*, which is the product of the average velocity and  $t_0$ . We shall confine ourselves to investigate the case of a horizontally layered medium in which case the stacking velocity  $v_s$ , derived from the travel-time curves, can be considered to be a good approximation of the RMS velocity. (For an exact definition of the different velocity concepts please refer to the works of TANER and KOEHLER 1969; SHAH 1973; AL-CHALABI 1974).

It is well-known (AL-CHALABI 1974) that the RMS ( $\approx$  stacking) and average velocities are connected by the formula

$$v_a = v_{RMS} / \sqrt{1+g},$$

where  $g$  is the *inhomogeneity factor*:

$$g = \frac{1}{D^2} \sum_{k=1}^{n-1} h_k \sum_{j=k+1}^n h_j \frac{(v_k - v_j)^2}{v_k v_j}; \quad g \geq 0$$

and  $D$  is the depth of the lower boundary of the  $n$ -th layer. (The above value of  $g$  can be easily derived from the identity:

$$v_{RMS}^2 - v_a^2 = \left[ \left( \sum_{k=1}^n t_k \right) \left( \sum_{k=1}^n v_k^2 t_k \right) - \left( \sum_{k=1}^n v_k^2 t_k \right)_i^2 \right] / \left( \sum_{k=1}^n t_k \right)^2$$

For a given value of  $g$ , the depth  $H$  can be expressed as:

$$H \approx \frac{v_a t_0}{\sqrt{1+g}}.$$

The computation of error is complicated by the fact that the three quantities shown in the above formula are not independent. The estimated value of  $g$  depends—apart from the expected errors of the estimated velocities and thicknesses of the individual layers—on the particular division of the measured data as well.

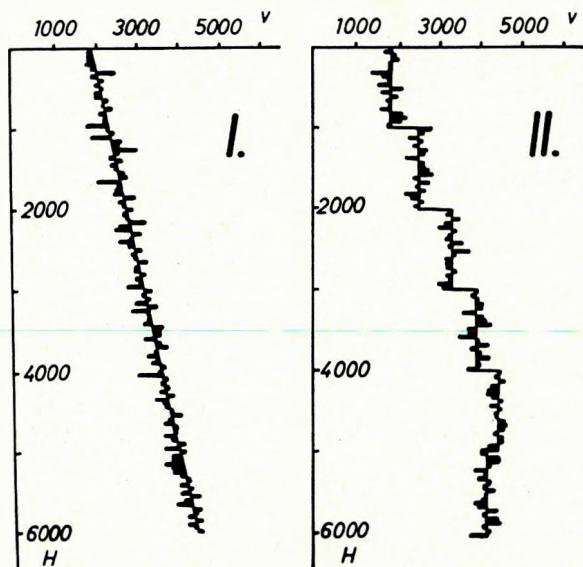
The problem will be studied on two statistical models. It will be aimed to get an at least qualitative estimation of the expected errors of  $g$  and  $H$  in some cases close to practical situations. At the same time the influence of the number  $N$  of the pair of parameters ( $H_i, v_i$ ) on the geological information gained will also be studied. *Geological information* will mean from here on the change of uncertainties (i.e. entropy) of the estimated parameters (e.g. layer thickness and interval velocity) during the process of measurements. To avoid the use of the complicated formulae describing the entropy in case of a large number of parameters, a more straightforward procedure will be applied. We shall study the RMS deviation  $\delta$  between the estimated parameters and the synthetic model parameters (interval velocities), as a function of the number of the parameters. This quantity, although less general than the entropy, adequately describes the quality of the estimated parameters. The solution to the inverse problem should be designed for models minimizing the error  $\delta$ .

The models are built up from 50 m thick, homogeneous layers. Interval velocities are normally distributed around their mean values, with standard deviation  $\sigma_v$ . Two (fixed) velocity models will be considered: a step-wisely changing model consisting of 1000 m thick constant sections, and a linear velocity-depth function (Fig. 4).

The arrival times of the reflections are computed from the model according to the formula:

$$t_i^2 = C_1 + C_2 x_i^2$$

(see TANER and KOEHLER 1969). To each  $t_i$  we add a non-correlated, Gaussian noise  $\Delta t_i$ ; of zero mean value and  $\sigma_t$  variance. Higher order terms, such as  $C_3$ , in the expression of  $t_i^2$  will be neglected.

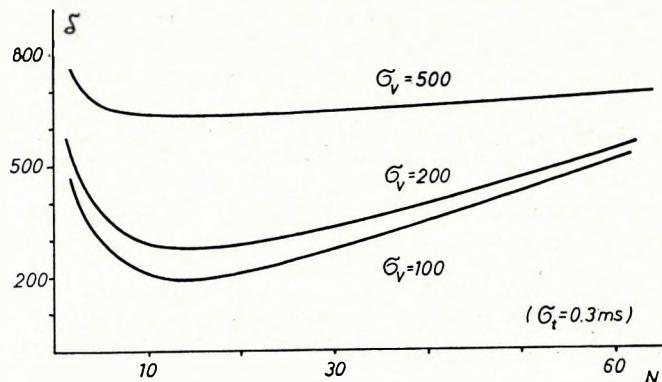


*Fig. 4* Two geological models for the study of the depth estimation  
*4. ábra.* Két geológiai modell a mélység becslésének tanulmányozásához  
*Рис. 4.* Две геологических модели для изучения оценки глубин

Next we take every  $m$ -th reflector ( $m = 1, 2, 3, \dots$ ) in turn and determine from numerous (30) realizations of the noise sequence  $\Delta t$  the expected values and variances of the parameters  $t_0, v_s, H, g$  and  $\delta$ , corresponding to the deepest reflector at 6000 m depth. The estimation of the parameters are performed by means of the Dix formula, using hyperboles fitted to the arrival times  $t_i + \Delta t_i$ . (In case of  $m = 20$ , the reflections taken into account correspond to the main velocity jumps in Model II).

From the results obtained, the following conclusions can be drawn:

a) In case of Model I  $\delta$  has a well-defined minimum with respect to the number of parameters  $N$ . The minimum occurs approximately at the same place for different values of  $\sigma_v$ . The existence of this minimum is due to two adverse effects: for large  $N$  the scatter of the interval velocities becomes too large, for small  $N$  the estimated values of velocities cannot describe sufficiently the main features of the model (Fig. 5).



*Fig. 5* Behaviour of  $\delta$  as a function of the number of parameter pairs ( $N$ ) for Model I, for fixed  $\sigma_v$   
*5. ábra.* A  $\delta$  mennyisége a paraméterpárok  $N$  számának függvényében az I. modell esetén, rögzített  $\sigma_v$  mellett

*Рис. 5.* Зависимость величины  $\delta$  от количества пар параметров ( $N$ ) для модели I при заданной величине  $\sigma_v$ .

*Рис. 5.* Зависимость величины  $\delta$  от количества пар параметров ( $N$ ) для модели I при заданной величине  $\sigma_v$ .

b) The optimal number of parameters depends on  $\sigma_v$ , and on the degree of smoothing along the section. In case if a more accurate measured data is available a coarser division of the model should be chosen (Fig. 6).

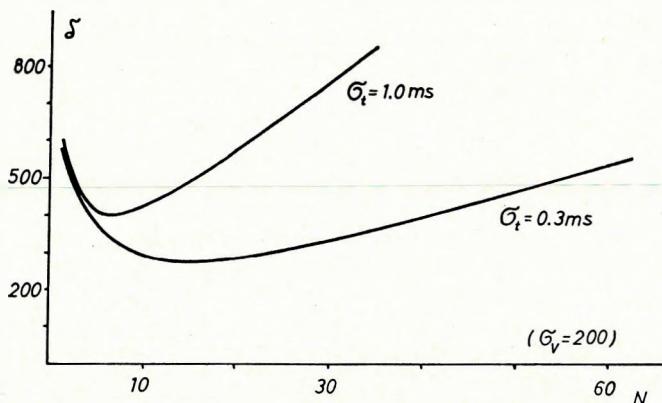


Fig. 6 Behaviour of  $\delta$  as function of the number of parameter pairs ( $N$ ) for Model I, for fixed  $\sigma_v$   
6. ábra. A  $\delta$  mennyisége a paraméterpárok  $N$  számának függvényében az I. modell esetén,  
rögzített  $\sigma_v$  mellett

Рис. 6. Зависимость величины  $\delta$  от количества пар параметров ( $N$ ) для модели I при  
заданной величине  $\sigma_v$ .

c) In case of Model II the quantity  $\delta$  reveals a clear-cut absolute minimum corresponding to the actual number of velocity jumps. For practical purposes it is advisable to use the flatter, a slightly disadvantageous, local minimum for model-building (Fig. 7).

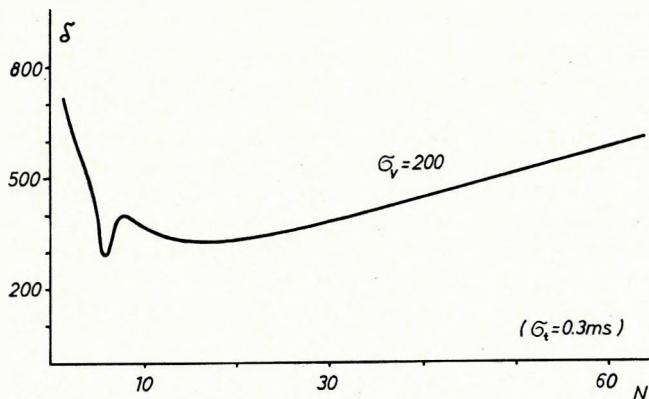


Fig. 7 Behaviour of  $\delta$  as a function of the number of parameter pairs ( $N$ ) for Model II  
7. ábra. A  $\delta$  mennyisége a paraméterpárok  $N$  számának függvényében a II. modell esetén  
Рис. 7. Зависимость величины  $\delta$  от количества пар параметров ( $N$ ) для модели II.

d) There is a close connection between the estimated depth  $H$  and the inhomogeneity factor  $g$ : the expected value of  $H$  is distorted by the error of factor  $g$  (Fig. 8).

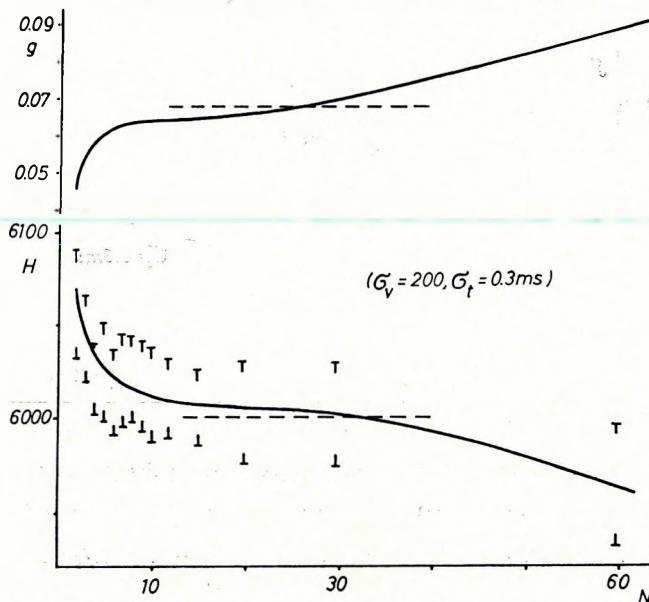


Fig. 8 The estimated inhomogeneity factor ( $g$ ) and depth ( $H$ ) as functions of the number of parameter pairs ( $N$ ), for Model I; — — — exact value

8. ábra. A becsült  $g$  inhomogenitás faktor és  $H$  mélység alakulása a paraméterpárok  $N$  számának függvényében (I. modell); — — — pontos érték

Рис. 8. Зависимость фактора оцененной неоднородности  $g$  и глубины  $H$  от количества пар параметров ( $N$ ) для модели I. — — — точные величины

Around the optimal number of parameters (see Fig. 6) the estimated  $g$  and  $H$  values are almost independent of  $N$ ; however  $g$  is less than its theoretical value. The decreased value of  $g$  is due to the fact that a correct interpretation necessarily results in a much more homogeneous, simpler model than the reality. If we do not want to fool ourselves with incorrect velocity estimations, it is not worthwhile to strive at a more accurate estimation of the inhomogeneity factor.

Summarizing the above results it turns out that the factor  $g$  is only of a secondary importance. If the number of parameters is nearly optimal, the only effect of  $g$  will be felt in a (slightly) distorted expected value of the estimated depth. As a first approximation the influence of the errors of  $g$  on the scatter  $\sigma_H$  of the estimated depths can be neglected:

$$E(\Delta H^2) \equiv \sigma_H^2 \approx \frac{1}{1+g} E\{(t_0 + \Delta t_0)(v_s + \Delta v_s) - t_0 v_s\}^2,$$

$$\sigma_H^2 \approx \frac{1}{1+g} [t_0^2 \sigma^2 + v_s^2 \sigma_{t_0}^2 + 2t_0 v_s E(\Delta t_0 \Delta v_s)].$$

The scatter of the depth values can be approximated by the above formula, as it is shown in Fig. 8.

On the basis of the investigated models ( $\sigma_t = 0.3$  ms) it seems that the velocities should be determined at every depth of 250–500 m, the samples to be taken should not necessarily be equally spaced in time. The value of the inhomogeneity factor can also be found from sonic logs. The importance of the optimal number of parameters, tailored to the accuracy of the measurement (i.e. to  $\sigma_t$ ), should especially be emphasized.

It is needless to say things are much more complicated on real seismic materials. The primary reflections should be picked from different types of organized noise and multiples. They are not uniformly distributed and by no means of equal energy along the sections.

## Appendix A

### Expected Value and Variance of the Estimated Interval Velocity

Let us consider a series of horizontal, homogeneous layers and adopt the usual assumption that stacking velocity  $v_s$  and RMS velocity are equal.

From the definition of the RMS velocity, the interval velocity of the layer situated between the  $i$ -th and  $(i+1)$ -st reflector can be expressed by the Dix formula as:

$$v_{int}(i+1) = \sqrt{\frac{v_{RMS}^2(i+1)t_0(i+1) - v_{RMS}^2(i)t_0(i)}{t_0(i+1) - t_0(i)}}$$

where the meaning of  $t_0(i+1)$ ,  $t_0(i)$ ,  $v_{RMS}(i+1)$  and  $v_{RMS}(i)$  should be clear.

Given that the above quantities are burdened with errors:

$$\begin{aligned} &[v_{RMS}(i+1) + \Delta v_{RMS}(i+1)], \quad [v_{RMS}(i) + \Delta v_{RMS}(i)], \\ &[t_0(i+1) + \Delta t_0(i+1)] \text{ and } [t_0(i) + \Delta t_0(i)], \end{aligned}$$

let us consider the derived value of

$$[v_{int}(i+1) + \Delta v_{int}(i+1)]$$

[For sake of simplicity  $v_i$  and  $t_i$  will be used instead of  $v_{RMS}(i)$  and  $t_0(i)$ ]

$$\begin{aligned} \Delta v_{int}(i+1) = & \sqrt{\frac{(v_{i+1} + \Delta v_{i+1})^2(t_{i+1} + \Delta t_{i+1}) - (v_i + \Delta v_i)^2(t_i + \Delta t_i)}{(t_{i+1} + \Delta t_{i+1}) - (t_i + \Delta t_i)}} - \\ & - \sqrt{\frac{v_{i+1}^2 t_{i+1} - v_i^2 t_i}{t_{i+1} - t_i}}. \end{aligned}$$

Expanding the first term into Taylor series, neglecting higher than first-order terms and substituting back the original expression for  $v_{int}(i+1)$  we obtain:

$$\Delta v_{int}(i+1) = \frac{1}{T v_{int}(i+1)} \left\{ v_{i+1} t_{i+1} \Delta v_{i+1} - v_i t_i \Delta v_i + \right.$$

$$+ \frac{\mathbf{v}_{i+1}^2}{2} \left( 1 - \frac{\mathbf{t}_{i+1}}{T} + \frac{\mathbf{v}_i^2 \mathbf{t}_i}{T \mathbf{v}_{i+1}^2} \right) \Delta \mathbf{t}_{i+1} - \frac{\mathbf{v}_i^2}{2} \left( 1 + \frac{\mathbf{t}_i}{T} - \frac{\mathbf{v}_{i+1}^2 \mathbf{t}_{i+1}}{T \mathbf{v}_i^2} \right) \Delta \mathbf{t}_i \Big\},$$

where  $T = t_{i+1} - t_i$ .

Let us compute now the expected value of  $\Delta v_{int}(i+1)$ :

$$\begin{aligned} E \Delta v_{int}(i+1) &= \frac{1}{T v_{int}(i+1)} \left\{ \mathbf{v}_{i+1} \mathbf{t}_{i+1} E(\Delta \mathbf{v}_{i+1}) - \mathbf{v}_i \mathbf{t}_i E(\Delta \mathbf{v}_i) + \right. \\ &\quad + \frac{\mathbf{v}_{i+1}^2}{2} \left( 1 - \frac{\mathbf{t}_{i+1}}{T} + \frac{\mathbf{v}_i^2 \mathbf{t}_i}{T \mathbf{v}_{i+1}^2} \right) E(\Delta \mathbf{t}_{i+1}) - \\ &\quad \left. - \frac{\mathbf{v}_i^2}{2} \left( 1 + \frac{\mathbf{t}_i}{T} - \frac{\mathbf{v}_{i+1}^2 \mathbf{t}_{i+1}}{T \mathbf{v}_i^2} \right) E(\Delta \mathbf{t}_i) \right\} = 0 \end{aligned}$$

since all terms on the right-hand-side have zero expected values. It should be mentioned that in spite of the above result the estimation of  $v_{int}(i+1)$  is generally not unbiased because of the hyperbolic approximation of the travel-time curves.

The variance of the estimation error  $\Delta v_{int}^2(i+1)$  is:

$$\begin{aligned} E(\Delta v_{int}^2(i+1)) &\equiv \sigma_{v_{int}}^2(i+1) = \frac{1}{T_{int}^2(i+1)} \left\{ \mathbf{v}_{i+1}^2 \mathbf{t}_{i+1}^2 \sigma_{v_{i+1}}^2 + \right. \\ &\quad + \mathbf{v}_i^2 \mathbf{t}_i^2 \sigma_{v_i}^2 + \frac{\mathbf{v}_{i+1}^4}{4} \left( 1 - \frac{\mathbf{t}_{i+1}}{T} + \frac{\mathbf{v}_i^2 \mathbf{t}_i}{T \mathbf{v}_{i+1}^2} \right)^2 \sigma_{t_{i+1}}^2 + \\ &\quad + \frac{\mathbf{v}_i^4}{4} \left( 1 + \frac{\mathbf{t}_i}{T} - \frac{\mathbf{v}_{i+1}^2 \mathbf{t}_{i+1}}{T \mathbf{v}_i^2} \right)^2 \sigma_{t_i}^2 + \\ &\quad + \mathbf{v}_{i+1}^3 \mathbf{t}_{i+1} \left( 1 - \frac{\mathbf{t}_{i+1}}{T} + \frac{\mathbf{v}_i^2 \mathbf{t}_i}{T \mathbf{v}_{i+1}^2} \right) E(\Delta \mathbf{t}_{i+1} \Delta \mathbf{v}_{i+1}) + \\ &\quad + \mathbf{v}_i^3 \mathbf{t}_i \left( 1 + \frac{\mathbf{t}_i}{T} - \frac{\mathbf{v}_{i+1}^2 \mathbf{t}_{i+1}}{T \mathbf{v}_i^2} \right) E(\Delta \mathbf{t}_i \Delta \mathbf{v}_i) - \\ &\quad - 2 \mathbf{v}_{i+1} \mathbf{v}_i \mathbf{t}_{i+1} \mathbf{t}_i E(\Delta \mathbf{v}_{i+1} \Delta \mathbf{v}_i) - \\ &\quad - \mathbf{v}_{i+1} \mathbf{v}_i^2 \mathbf{t}_{i+1} \left( 1 + \frac{\mathbf{t}_i}{T} - \frac{\mathbf{v}_{i+1}^2 \mathbf{t}_{i+1}}{T \mathbf{v}_i^2} \right) E(\Delta \mathbf{t}_i \Delta \mathbf{v}_{i+1}) - \\ &\quad - \mathbf{v}_{i+1}^2 \mathbf{v}_i \mathbf{t}_i \left( 1 - \frac{\mathbf{t}_{i+1}}{T} + \frac{\mathbf{v}_i^2 \mathbf{t}_i}{T \mathbf{v}_{i+1}^2} \right) E(\Delta \mathbf{t}_{i+1} \Delta \mathbf{v}_i) - \\ &\quad - \frac{\mathbf{v}_{i+1}^2 \mathbf{v}_i^2}{2} \left( 1 - \frac{\mathbf{t}_{i+1}}{T} + \frac{\mathbf{v}_i^2 \mathbf{t}_i}{T \mathbf{v}_{i+1}^2} \right) \left( 1 + \frac{\mathbf{t}_i}{T} - \right. \\ &\quad \left. - \frac{\mathbf{v}_{i+1}^2 \mathbf{t}_{i+1}}{T \mathbf{v}_i^2} \right) E(\Delta \mathbf{t}_{i+1} \Delta \mathbf{t}_i) \Big\} \end{aligned} \tag{A.1}$$

where the covariance-term  $E(\Delta \mathbf{t}_k \Delta \mathbf{v}_k)$  will be derived in Appendix B.

If the events corresponding to the  $i$ -th and  $(i+1)$ -st horizons are not correlated, the last four terms vanish from the expression of  $\sigma_{v_{int}}^2(i+1)$ . If, however, static correction errors also contribute to the terms  $\Delta t_{i+1}$  and  $\Delta t_i$ , the latter four terms cannot be neglected.

In this case the following inequalities hold:

$$0 \leq E(\Delta v_{i+1} \Delta v_i) \leq \rho \quad (\text{A.2})$$

$$0 \leq |E(\Delta t_i \Delta v_{i+1})| \leq |E(\Delta t_{i+1} \Delta v_{i+1})| \quad (\text{A.3})$$

$$0 \leq |E(\Delta t_{i+1} \Delta v_i)| \leq |E(\Delta t_i \Delta v_i)| \quad (\text{A.4})$$

$$0 \leq E(\Delta t_{i+1} \Delta t_i) \leq \sigma_{t_0}^2 \quad (\text{A.5})$$

where

$$\begin{aligned} \rho = & \frac{v_{i+1}^3 v_i^3}{x^4} \{ (t_i + NMO_i)(t_{i+1} + NMO_{i+1}) \sigma_{NMO}^2 + \\ & + [(t_i + NMO_i)NMO_{i+1} + (t_{i+1} + NMO_{i+1})NMO_i] E(\Delta t_0 \Delta NMO) + \\ & + NMO_{i+1} NMO_i \sigma_{t_0}^2 \} \end{aligned}$$

The inequalities (A.2)–(A.5) express the fact that the covariance terms are restricted between two kinds of degenerated limiting values.

On the left-hand-side of the above inequalities the equality sign holds for the case when the noise are completely non-correlated between successive horizons. The right-hand-side limit corresponds to the case of the identical noise for all horizons, i.e. static correction noise alone.

It should be noted that the above derivation is independent of the particular method of determination of the parameters of the hyperbole; any method of velocity determination can be described by means of  $\sigma_{t_0}$ ,  $\sigma_{v_s}$  and  $E(\Delta t_0 \Delta v_s)$ .

## Appendix B

### Covariance of the Estimation Errors of $t_0$ and $v_s$

Using the series development of  $v_s$  up to the first order terms:

$$E(\Delta t_0 \Delta v_s) = E \left\{ -\frac{v_s^3}{x^2} [(t_0 + NMO) \Delta NMO + NMO \Delta t_0] \Delta t_0 \right\},$$

$$E(\Delta t_0 \Delta v_s) = -\frac{v_s^3}{x^2} [(t_0 + NMO) E(\Delta t_0 \Delta NMO) + NMO E(\Delta t_0^2)]$$

or, in another form

$$E(\Delta t_0 \Delta v_s) = -\frac{v_s^3}{x^2} [(t_0 + NMO) E(\Delta NMO) + NMO \sigma_{t_0}^2]$$

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KÉSMÁRKY ISTVÁN

AZ INTERVALLUMSEBESSÉG BECSLÉSI HIBÁI  
ÉS A GEOLÓGIAI MODELL

Figyelembevéve a geofizikai mérési adatok nagy tömegét, belátható, hogy a kiértékelés célja nem lehet olyan geológiai modell meghatározása, mely minden mérési adattal teljes összhangban van. Ekkor ugyanis a modell a mérési adatokkal azonos számú paramétert tartalmazna. A szeizmikában ilyen nagyméretű egyenletrendszer megoldása számítástechnikai és egyéb okok miatt abszurdum, de más módszereknél is komoly buktatókkal rendelkezik.

Kiértékelési tapasztalatok, számos szerző munkája (és e dolgozat is) azt bizonyítják, hogy helytelen olyan bonyolult geológiai hatókat meghatározni, melyek hatása azonos a mért adatokkal. A gyakorlatban erősen túlhatározott egyenletrendszer megoldása a célszerű.

A mérési adatok és a feltételezett ideális geológiai alakzatok hatása közti ellentmondást a zajok fogalmának bevezetésével oldjuk fől. A zajokat statisztikus módszerekkel kezeljük. Annál többet vagyunk kénytelenek a zajok terhére írni, minél primitívebb esetekre vannak csak feltárva a különféle hatások és hatók között kapcsolatok. Az egyre bonyolultabb szerkezetek hatásának feltárássával azonban nem csökkenthető minden határon túl a „zajok” nagysága, több okból sem. Egyrészt a statisztikus mérési hibák már nincsenek kapcsolatban a hatókkal, ezek nem tüntethetők el. Másrészt matematikai összefüggések csak a valóságosnál lényegesen egyszerűbb esetekre ismeretesek csupán. A paraméterek növekvő száma és a becslések megbízhatósága között fordított arányosság szerű összefüggés van. Látható, hogy az egyre bonyolultabb geológiai alakzatok megismerése elvi korlátokba ütközik.

Ilyen helyzetben a kiértékelés legelfogadhatóbbnak tűnő célkitűzése az, hogy az így definiált zajok zavaró hatása minimális legyen a meghatározott geológiai paramétereire. (Két véletlen köztő optimum megkereséséről van tehát szó, a modellparaméterek száma szerint.)

Precízen megfogalmazva ez azt jelenti, hogy olyan ható paramétereit kell meghatározni, melyre a mérési adatok maximális információt szolgáltatnak. Információ alatt értjük a hatóparaméterek bizonytalanságának (entrópiájának) megváltozását a mérések következtében.

Az adott, vitán felüllálló egyenletek megoldásán túl a vázolt szemléletmódot lehetőséget nyújt a kiértékelő szakmai lelkismeretességének objektív matematikai modellezésére. A geofizikai mérések hasznosításának éppen az a leglényegesebb kérdése, hogy a fizikai módszerek mely határig segíthetik a geológiai kiértékelő munkát. Ezen túl már csak földtaní módszerekkel lehet következtetéseket levonni.

A logikailag helyes kiértékelési eljárások három alappillérrre építhetők. (Más, szabatosabb megfogalmazásai is léteznek, pl. F. M. Golcman 1975; Salát, Drahos 1973, 1975.)

1. A mért adatok idealizálása.
2. A geológiai viszonyok idealizálása, alkalmas modell megválasztása.
3. A két alapvető tartomány közti transzformáció megválasztása.

A dolgozat ismerteti a becsült intervallumsebességeket befolyásoló tényezőket és a meghatározott értékek szórásának alakulását ezek függvényében. A téma fontosságára több szerző is felhívta a figyelmet (Anstey 1973). Az elvégzett modellvizsgálatok alapján következtetések vonhatók le a statisztikus hibák hatásának és a modell minőségének kapcsolatáról. A meghatározandó geológiai paraméterek optimális száma érzékenyen függ a mérési adatok minőségétől, mennyiségtől.

A továbbiakban a menetidőgörbéről félteszük, hogy hiperbolával jól közelíthetők,  $t_0$  és  $NMO$  paramétereinek egyes statisztikai jellemzőit ismertnek vesszük, a  $v_s$  stacking sebesség szórását úgyszintén. A zajok hatásait e tényezőkön keresztül vesszük figyelembe. A bemutatandó példák esetében a csatornák lokális maximumaira illesztett hiperbola paramétereivel jellemezzük a reflexiókat.

A következőkben egyes geológiai paramétereire — az intervallumsebességekre és mélységekre — kapható becslések statisztikai tulajdonságait vizsgáljuk, a zajok függvényében. A terítési rendszert rögzítjük, számításainkat az E—60—2—12 rendszerre végezzük el. Az időmérési adatokat (a lokális maximumhelyeket) terhelő hiba  $\sigma_t$  szórását 1 ms-nak választjuk, hacsak külön nem jelezzük a kivételt ez alól. (A szórás jellegű eredmények  $\sigma_t$ -vel arányosak.)

$\sigma_t$  értéke a stochasztikus folyamatok elmélete alapján levezethető:

$$\sigma_t = \frac{f_z}{\sqrt{2n(2\pi f_j)^2}},$$

ahol  $f_z$  a zajok közepes frekvenciája,  $n$  a jel/zaj energia aránya és  $f_j$  a jel közepes frekvenciája\*. A fenti kifejezés  $n$  nagyobb értékei mellett érvényes, kis jel/zaj arány esetén az összefüggés bonyolultabb.

## И. КЕШМАРКИ

### ПОГРЕШНОСТИ ОЦЕНКИ ИНТЕРВАЛЬНЫХ СКОРОСТЕЙ И ГЕОЛИГИЧЕСКАЯ МОДЕЛЬ

Все более важным средством и целью интерпретации сейсморазведочных данных является определение интервальных скоростей.

В работе рассматриваются факторы, влияющие на точность оценки интервальных скоростей, а также зависимость разброса определенных величин от этих факторов.

Проведенные модельные исследования позволяют делать интересные выводы о связи между влиянием статистических погрешностей и качеством, детальностью модели.

Оптимальное число определяемых геологических параметров в значительной мере зависит от качества и количества данных наблюдений.

