Magyar Állami Eötvös Loránd Geofizikai Intézet
GEOFIZIKAI KÖZLEMÉNYEK
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THEORETICAL LIMITATIONS OF THE HARMONIC ANALYSIS OF EARTH TIDES

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So much has recently been said about the increased accuracy of gravimeters and horizontal pendulums that we often forget that it is theory itself which limits the resolution power of most of our methods of harmonic analysis of tidal phenomena. True, it is generally understood when analyzing the lunisolar effect that due care has to be taken of phenomena modifying the resulting amplitude ratios (δ or γ) and phase shift ($\varkappa$). The first group of these phenomena consists of disturbances of exterior origin such as the effect of worldo ceans, temperature or atmospheric pressure. An other type of disturbances has to do with the instrument itself (non-linearity, instrumental delay and drift, etc.). A great number of papers have been devoted to these questions and it has been pointed out that a proper compensation-for these effects substantially improves the final values of δ, γ and $\varkappa$.

In the present paper we shall assume ideal conditions that is no exterior disturbances, and an ideal instrument, and we are going to show what degree of accuracy can be expected from the different methods of harmonic analysis and how this accuracy depends on the approximations and restrictions of the given methods.

As a matter of fact all of the methods in use date back to Doodson (1921) who first succeeded to present the lunisolar effect as a sum of 386 elementary waves. Recently Cartwright and Tayler (1971) extended this series development to 512 terms, making use of latest astronomical data. However, as shown by Иванова (1972) there is but a slight deviation between the lunisolar effect and its approximation by Doodson's series, the greatest error being less than 0,25 μgal for the vertical component, affecting the fourth decimal of amplitude ratio and the second of phase shift. So, even for the increased standards of measurements Doodson's series development proves accurate.

The first methods of harmonic analysis had been elaborated long before the computer era. Most of the restrictions served computational convenience as e.g. the required continuity of registrations or the preference of some given periods of registration time (of the order of 1 month). Some of these restrictions result in rather drastic effects: Перцев's 29-day method, e.g. was shown by Иванова *(op. cit)*. to distort the third decimal of apmlitude ratios, the first one of phase shift and to cause as much as 2–3% error in case of K_1 waves. Very likely the same goes for Lecolazet's method even though it takes into account a greater number of terms.

The current use of computers has rendered possible a simultaneous processing of time series, continuous or discontinuous ones as well. The majority of techniques

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Manuscript received: 1, 6,1973.

are based on the principle of least mean square errors. A particularly powerful method is that of Венедиков (1966) which is claimed by its author to possess definite advantages as compared to Перцев's or Lecolazet's methods.

Owing to the extremely small frequency differences between tidal waves it has been necessary, for purely technical reasons, to assume in all methods, no matter whether classic or recent, that in the neighbourhood of the investigated waves the other waves are characterized by the same δ (or γ) and $\varkappa$ values. Even though we realize the necessity of this condition we have to challenge its validity since it is well known that due to the effect of the liquid core of the Earth the δ (or γ) value cannot be constant in the vicinity of K_1 waves. In Table I we have compiled δ values for the vicinity of the K_1 wave according to Молоденский's second model and following the scheme of Венедиков *(op. cit.)*. For series shorter than 6 months we obtain $\delta = 1.1471$ and it is only for longer sequences that we get reasonable coincidence between reality and analytic results (for $\delta = 1.1429$ the deviation is 0.0001). Перцев's investigations have revealed that especially for semidiurnal waves the value of δ is strongly influenced by the effect of oceans even in the interior of continents. At some stations the correction of δ may rise to 0.030. Assuming the extreme case that the δ value of the wave of Doodson number 255545 lying near M_2 has not changed, the amplitude ratio for M_2 determined from the group will

I. táblázat
Table I
Таблица I

L (6 hó) L (6 months) Ц (6 месяцев)	L (6 hó) L (6 months) Ц (6 месяцев)	Darwin jelöléssel Darwin's notation Дарвиновское обозначение	Doodson-szám Doodson number Аргументное число	Amplitúdó Amplitude Амплитуда	δ
↑		π_1	162 556	1 029	1.164
			163 545	199	1.159
		P_1	555	17 584	1.158
	↑		164 554	147	1.155
			556	423	1.153
			165 545	1 050	1.145
		K_1	555	53 050	1.143
	↓		565	7 182	1.142
			575	154	1.140
		ψ_1	166 554	423	1.193
↓		φ_1	167 555	756	1.178

only change by 0.001. These examples are meant to show that while the above-mentioned restrictions certainly imply some limitations they can cause only minor errors, however.

The gravitational tide is described by the expression

$$Y^*(t) = \sum_{i=1}^{\alpha} \delta_i A_i \cos(\Phi_i + \omega_i t + \varkappa_i),$$

which is easily linearized to

$$Y^*(t) = \sum_{i=1}^{\alpha}[\delta_i(\cos \varkappa_i)A_i \cos(\Phi_i + \omega_i t) - \delta_i (\sin \varkappa_i)\, A_i \sin(\Phi_i + \omega_i t)]. \tag{1}$$

In Eq. (1) the observed value $Y(t)$ is approximated by $Y^*(t)$: A_i is the amplitude of the i-th elementary wave, Φ_i; its phase, ω_i its angular velocity, t time. Since A_i, Φ_i and ω_i can be theoretically determined our task is to find the values of $\delta_i \cos \varkappa_i$ and $\delta_i \sin \varkappa_i$. In order to reduce the number of unknowns VENEDIKOV introduced wave groups by

$$Y^*(t) = \sum_{i=1}^{m}\delta_i \cos \varkappa_i \sum_{j=m_i}^{n_i} A_j \cdot \cos(\Phi_j + \omega_j t) - \sum_{i=1}^{m}\delta_i \sin \varkappa_i \sum_{j=m_i}^{n_i} A_j \sin(\Phi_j + \omega_j t). \tag{2}$$

Here m is the number of groups, m_i being the serial number of the first, n_i that of the last wave belonging to the i-th group. As a matter of fact by solving Eq. (2) we determine the unknown parameters for wave groups instead of individual waves which means the introduction of a further assumption. Suppose we are given $Y(t)$ in the interval $[-L, L]$. To proceed further we have to minimize the error

$$\int_{-L}^{L}\{Y(t) - \sum_{i=1}^{\alpha}[\delta_i \cos(\varkappa_i + \Phi_i)A_i \cos \omega_i t - \delta_i \sin(\varkappa_i + \Phi_i)A_i \sin \omega_i t]\}^2\, dt =$$

$$= \int_{-L}^{L}[Y(t) - \sum_{i=1}^{\alpha}(A_i^* \cos \omega_i t + B_i^* \sin \omega_i t)]^2\, dt = \min.$$

Setting the partial derivatives $\dfrac{\partial V(t)}{\partial A_i^*}$, $\dfrac{\partial V(t)}{\partial B_i^*}$ equal to zero:

$$\sum_{j=1}^{\alpha} a_{ij}A_j^* + \sum_{j=1}^{\alpha} b_{ij}B_j^* = c_i, \tag{3}$$

where

$$\left.\begin{aligned} a_{ij} &= \frac{1}{L}\int_{-L}^{L}\cos \omega_j t \cdot \cos \omega_i t\, dt \\ b_{ij} &= \frac{1}{L}\int_{-L}^{L}\sin \omega_j t \cdot \cos \omega_i t\, dt \\ c_i = u_i &= \frac{1}{L}\int_{-L}^{L} Y(t) \cdot \cos \omega_i t\, dt \end{aligned}\right\} \quad i = 1, 2, \ldots, \alpha.$$

$$\left.\begin{aligned} a_{ij} &= \frac{1}{L}\int_{-L}^{L} \cos\omega_j t \cdot \sin\omega_i t\, dt = 0 \\ b_{ij} &= \frac{1}{L}\int_{-L}^{L} \sin\omega_j t \cdot \sin\omega_i t\, dt \\ c_i = V_i &= \frac{1}{L}\int_{-L}^{L} Y(t)\sin\omega_i t\, dt \end{aligned}\right\} \quad i = \alpha+1, \ldots, 2\alpha.$$

Thus Eq. (3) has decomposed into two systems of equations

$$\sum_{j=1}^{\alpha} a_{ij} A_j^* = u_i, \tag{4}$$

$$\sum_{j=1}^{\alpha} b_{ij} B_j^* = v_i.$$

Upon integration Eqs. (4) becomes

$$\sum_{j=1}^{\alpha} A_j^* \left\{ \frac{\sin[(\omega_j-\omega_i)\cdot L]}{(\omega_j-\omega_i)\cdot L} + \frac{\sin[(\omega_j+\omega_i)\cdot L]}{(\omega+\omega)\cdot L} \right\} = u_i,$$

$$\sum_{j=1}^{\alpha} B_j^* \left\{ \frac{\sin[(\omega_j-\omega_i)\cdot L]}{(\omega_j-\omega_i)\cdot L} - \frac{\sin[(\omega_j+\omega_i)\cdot L]}{(\omega_j+\omega_i)\cdot L} \right\} = v_i.$$

Since L is large we can put

$$\frac{\sin[(\omega_j+\omega_i)\cdot L]}{(\omega_j+\omega_i)\cdot L} \ll \frac{\sin[(\omega_j-\omega_i)\cdot L]}{(\omega_j-\omega_i)\cdot L} = a_{ij}.$$

$$\sum_{j=1}^{\alpha} A_j^* \cdot a_{ij} = u_i, \tag{5}$$

and

$$\sum_{j=1}^{\alpha} B_j^* a_{ij} = v_i.$$

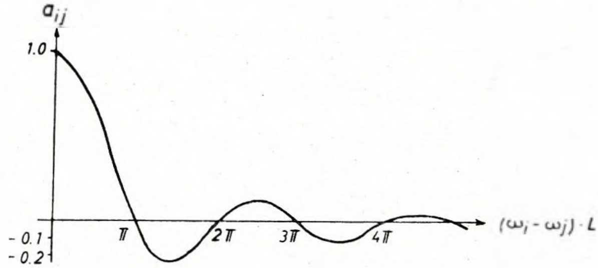
If $\omega_j = \omega_i$ then $a_{ij} = 1$. The plot of a_{ij} in function of $(\omega_j - \omega_i)L$ is shown in Figure 1.

As we have shown above Eq. (5) gives a means to determine the unknowns for individual waves. Consequently, Eq. (2) contains a further hypothesis: we assume that the value of the largest wave of the group coincides with that of the group, i.e.:

within any group $(m_i \leq j \leq n_i)$ $\quad a_{ij} = 1$;

outside the group $(j < m_i$ or $j > n_i)$ $\quad a_{ij} = 0$.

To have a feeling of the immediate consequences of these assumptions we compute the deviation of the δ value determined from the group from δ of a parti-



1. ábra
Figure 1
Рис. 1

cular wave belonging to the group. Assume that $\delta = 1.000$ and $\varkappa = 0{,}000°$. For an arbitrary time and latitude (say, T = January 1, 1900, 0^h U.T. and $\varphi = 45°$) the amplitude values are, taking into account the values of a_{ij}:

$$u_i = \sum_{j=r_i}^{p_i} A_j \cos \Phi_j \frac{\sin (\Delta\omega_j L)}{\Delta\omega_j L},$$

$$v_i = -\sum_{j=r_i}^{p_i} A_j \sin \Phi_j \frac{\sin (\Delta\omega_j L)}{\Delta\omega_j L},$$

where $p_i \geq n_i$ and $r_i \leq m_i$. (The introduction of this inequality is due to the fact that in some cases the waves considered by us are outside the generally used group limits.)

On the other hand the respective values as given by the usual approach are

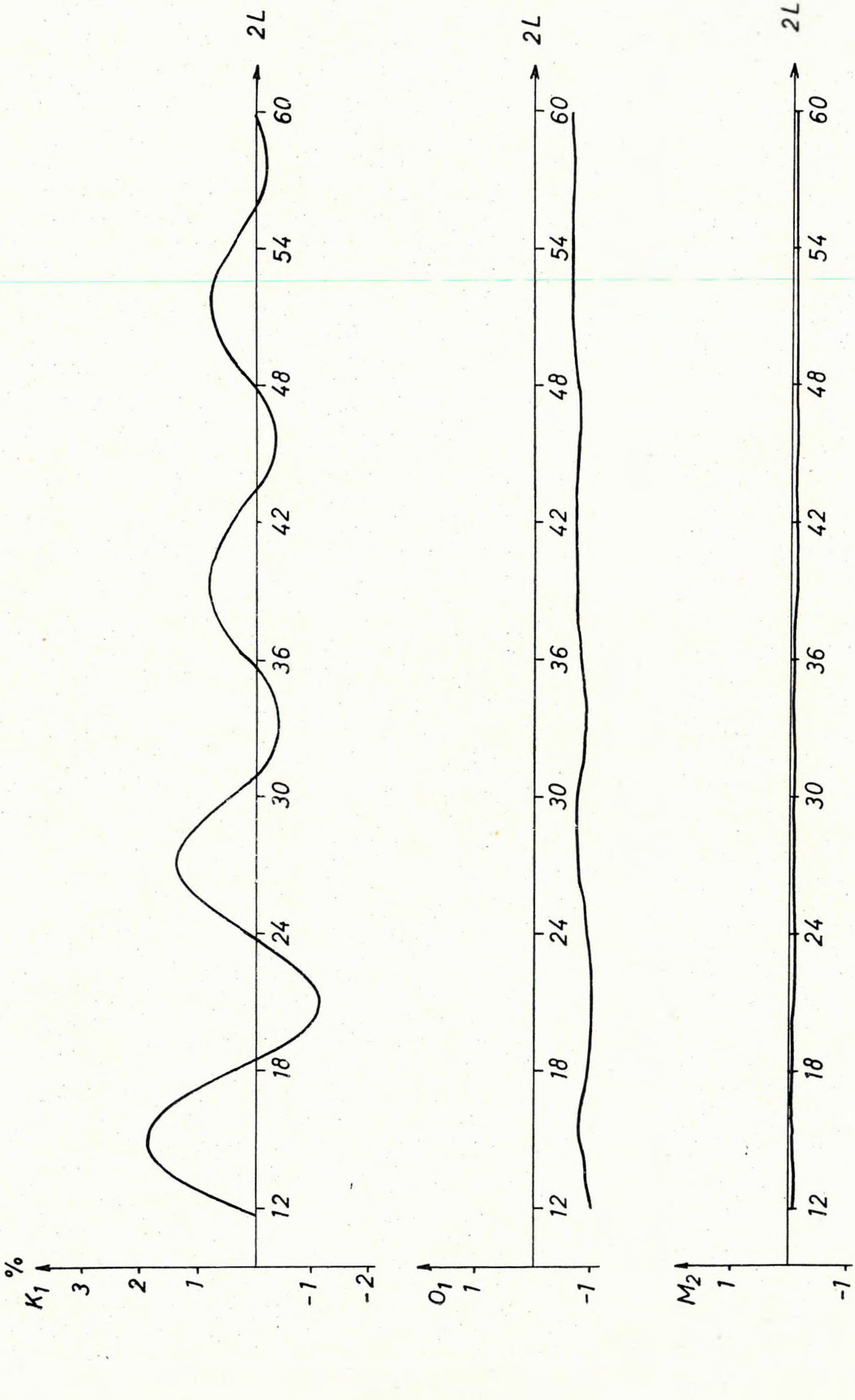
$$\bar{u}_i = \sum_{j=m_i}^{n_i} A_j \cos \Phi_j,$$

$$\bar{v}_i = -\sum_{j=m_i}^{n_i} A_j \sin \Phi_j.$$

The amplitude distortion is characterized by the expression

$$\sqrt{u_i^2 + v_i^2} / \sqrt{\bar{u}_i^2 + \bar{v}_i^2}$$

whose time-behaviour is shown by Figure 2 for K_1, O_1 and M_2 waves. (Time scale given in month.) The function has a rather intricated shape and for certain values of L the distortion reaches 1–2%. Since the computation of a_{ij} is simple, this side-effect can be eliminated by multiplying the unknowns $\delta_i \cos \varkappa_i$ and $\delta_i \sin \varkappa_i$ by a_{ij} in Eq. (2). An alternative approach could be to adjust the observed and theoretical curves independently of each other,



2. ábra
Figure 2
Рис. 2

IRODALOM

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VARGA PÉTER

A FÖLDI ÁRAPÁLY HARMONIKUS ANALÍZISÉNEK NÉHÁNY KORLÁTJA

A Föld árapályának analízisére kidolgozott módszerek, számítástechnikai okok miatt, bizonyos feltételezéseket tartalmaznak. A cikk néhány ilyen feltételezés hatását vizsgálja.

Az árapály sorfejtés hullámaihoz tartozó elméleti amplitúdó- (A) és fázis- (Φ) értékek pontosan írják le az abszolút merev Föld felszínén lejátszódó luniszoláris változásokat. Doodson, Cartwright és Tayler sorfejtése, a jelenlegi mérési pontosság mellett, megfelelőnek tekinthető.

Az árapályhullámok frekvenciái között levő igen kis különbségek miatt az összes analizáló eljárás kényszerűségből azt a feltételt tartalmazza, hogy a vizsgált árapályhullámok környezetében egyforma amplitúdóhányadosú (δ vagy γ) és fáziskülönbségű ($\varkappa$) hullámok vannak jelen.

Ez a kényszerfeltétel elvileg nem helyes, mert például a K_1 hullám környezetében az amplitúdó hányadosok a Föld cseppfolyós magjának hatása miatt — nem egyformák. 6 hónapnál rövidebb sorozatoknál δ_{K_1} értéke kb. 0,4%-kal tér el a valóságostól. Hosszabb sorozatoknál azonban az eltérés minimális.

Ha a legkisebb négyzetek elvének alkalmazásával egyenlítenek ki, a hullámokat csoportokra osztják és feltételezik, hogy az egyes csoportokra jellemző δ és γ értékek a csoportban található legnagyobb hulláméival egyeznek meg.

Ha az árapály poliharmonikus folyamatát nem csoportonként, hanem hullámonként egyenlítjük ki, a szomszédos hullámok hatása a vizsgálatra összetett, mert egyrészt a csoporton kívül levő hullámok is hatást gyakorolnak, másrészt pedig a csoporton belül levő hullámok torzító hatása is fellép. Ezért nem mondhatjuk egyszerűen, hogy a csoporthoz tartozó δ, γ és $\varkappa$ értékek az egyes hullámokhoz tartozókkal megegyeznek. A torzulás mértéke a körfrekvenciacsúcs-távolság ($\Delta\omega$) és a vizsgált sorozat hosszának ($2\,L$) függvénye.

Az elméleti árapályt i hullámonként kiegyenlítve és figyelembe véve e hullám m_i-től n_i-ig terjedő környezetét, a következő összefüggéseket kapjuk:

$$u_i = \sum_{j=m_i}^{n_i} A_j \cdot \cos \Phi_j \frac{\sin(\Delta\omega_j L)}{\Delta\omega_j \cdot L},$$

$$v_i = -\sum_{j=m_i}^{n_i} A_j \cdot \sin \Phi_j \frac{\sin(\Delta\omega_j L)}{\Delta\omega_j \cdot L},$$

ahol

$$\Delta\omega_j = 0, \quad \text{ha} \quad j = i.$$

A szokásos módon számított értékek pedig:

$$\overline{u}_i = \sum_{j=m_i}^{n_i} A_j \cos \Phi_j ,$$

és

$$\overline{v}_i = -\sum_{j=m_i}^{n_i} A_j \sin \Phi_j .$$

Innen a torzulás jellemzésére a

$$\frac{\sqrt{u_i^2 + v_i^2}}{\sqrt{\overline{u}_i^2 + \overline{v}_i^2}}$$

kifejezést kapjuk.

Ez utóbbi kifejezés értékének időbeli menete K_1, O_1 és M_2 hullámoknál a 2. ábrán látható. A függvényértékek bonyolultan változnak, és bizonyos L értékeknél a torzulás $1-3\%$ is lehet; ezért egy az adott hullámhoz tartozó δ, γ érték számításánál hatását nem szabad figyelmen kívül hagyni.

П. ВАРГА

О НЕКОТОРЫХ ОГРАНИЧЕНИЯХ ГАРМОНИЧЕСКОГО АНАЛИЗА ЗЕМНЫХ ПРИЛИВОВ

В методах, разработанных для анализа земных приливов, для облегчения вычислительных работ, предусмотрены некоторые допущения. В настоящей работе рассматриваются следствия таких допущений.

Теоретические величины амплитуд (A) и фаз (Φ) волн, связанных с разложением приливов, точно описывают лунно-солнечные вариации, происходящие на поверхности абсолютно жесткой Земли.

В связи с весьма незначительными разностями частот приливных волн во всех методах анализа предполагается, что в окрестности изучаемых приливных волн имеются волны с одинаковыми частными амплитуд (δ или γ) и одинаковой разностью фаз ($\varkappa$).

Данное принудительное условие теоретически не является правильным, так как напр. в окрестности волны K_1 частные амплитуд неодинаковы в связи с эффектом жидкого ядра Земли. В рядах, более коротких 6 месяцев величина δ_{K_1} отклоняется от фактической величины приблизительно на 0,4 %. Однако в более длинных рядах имеются лишь незначительные отклонения.

При применении для выравнивания, метода наименьших квадратов, волны разбиваются на группы причем, предполагается, что величины δ и γ, характерные для отдельных групп, совпадают с величинами, свойственными наибольшей волне в данной группе.

Если полигармонический процесс прилива выравнивается не по группам, а по волнам, влияние соседних волн на результаты анализа является сложным, поскольку как волны, находящиеся вне группы, так и волны в пределах группы оказывают искажающее влияние. Поэтому нельзя просто утверждать, что величины δ, γ и $\varkappa$, характерные для группы, совпадают с этими же величинами, характерными для отдельных волн. Степень искажения зависит от расстояния пика круговой частоты ($\Delta\omega$) и от длины изучаемой серии ($2L$).

Выравнивая теоретический прилив по волнам i и учитывая его окружность от m_i до n_i, получаем следующие соотношения:

$$u_i = \sum_{j=m_i}^{n_i} A_j \cos \Phi_j \frac{\sin(\Delta\omega_j L)}{\Delta\omega_j L}$$

$$v_i = -\sum_{j=m_i}^{n_i} A_j \cos \Phi_j \frac{\sin(\Delta\omega_j)L}{\Delta\omega_j L}$$

где

$$\Delta\omega_j = 0 \quad \text{если} \quad j = i.$$

Величины, подсчитанные обычным способом, имеют вид

$$\overline{u}_i = \sum_{j=m_i}^{n_i} A_j \cos \Phi_j$$

и

$$\overline{v}_i = - \sum_{j=m_i}^{n_i} A_j \sin \Phi_j .$$

По этим соотношениям для искажения получается выражение:

$$\frac{\sqrt{u_i^2 + v_i^2}}{\sqrt{\overline{u}_i^2 + \overline{v}_i^2}}$$

Временной ход этого выражения для волн K_1, O_1 и M_2 показан на рис. 2. Величины функций изменяются сложно и при определенных величинах L искажение может достигать 1—3 %. В связи с этим, при вычислении величин δ, γ данной волны необходимо учитывать искажение.