

THEORETICAL ASPECTS OF THE INDUCED POLARIZATION METHOD

GY. DANKHÁZI

Introduction

The physical principles of the induced polarization method were known as early as in the 19th century but the idea that this phenomenon might be applied as a geophysical prospecting tool first occurred in the late forties. The method was soon found to be a powerful technique for ore and—to a certain extent—ground-water prospecting and it has been further developed since then in two main directions. Recently two different versions of the induced polarization (IP, for short) methods are common: the “time-domain” and the “frequency-domain” methods.

While IP measurements proved to be highly efficient in ore prospecting they are still inferior to other techniques in certain respects. Namely, a serious disadvantage of the method is that it does not yield exact rock-physical parameters only anomalies, i.e. relative values which do not express actual physical properties of the rocks but depend on the parameters of the instrument used. If we perform repeated measurements with different instruments at the same site then all of them would correctly localize the individual anomalies, but with quite different amplitudes, as a rule.

To make this clear let us recall the basic difference between the measurement of the specific resistivity (SR) and that of the IP. The apparent SR is computed from the formula

$$\rho_a = k \frac{\Phi}{I}$$

where we have adopted (throughout these pages) standard notations. Assuming ideal conditions the ρ_a value measured is independent of the parameters of the instrument. Inspecting, on the other hand, a similar formula of the time-domain method (NILSSON, 1971)

$$IP = \frac{1}{\Phi(t_0)} \int_{t_1}^{t_2} [\Phi(t_0) - \Phi(t)] dt$$

where t_1 , t_2 are limits of integration with respect to the decay curve, $\Phi(t)$ is the potential measured at time t , and t_0 is the moment when the current is interrupted, it is instantly clear that t_1 and t_2 are instrumental parameters and there is no obvious argument to think the IP values themselves as being independent of t_1 and t_2 . As a matter of fact we would have come to the same conclusion if we had analyzed any other formula of the IP method.

*ELGI, Budapest

All in all, it can be said that when measuring SR the value of ρ_a is uniquely given by a quantitative relation while in IP measurements we have to restrict ourselves to empirical formulae which inherently depend on the instrument used. The question naturally arises whether there could be found an instrument-independent value measurable by IP methods which would already directly characterize certain physical properties—e.g. ore-content—of the rocks.

In the present paper we shall address ourselves to the solution of this problem. To have a convenient footing to start with we shall first review Maxwell's equations.

Theoretical difficulties of the description of the IP effect

Let us assume that in the rock investigated the inducing current is DC or a low-frequency AC (of some tens cps) and that the rock is electrically conductive. Under these circumstances the resulting electromagnetic field is described by the quasi-stationary Maxwell equations which read, in the MKS system of units, as follows:

$$\begin{aligned} \text{curl } \vec{H} &= \sigma \vec{E} & \text{a),} & & \text{curl } \vec{E} &= -\mu \frac{\partial \vec{H}}{\partial t} & \text{b),} \\ \text{div } \vec{E} &= 0 & \text{c),} & & \text{div } \vec{H} &= 0 & \text{d),} \end{aligned} \quad (1)$$

where \vec{E} and \vec{H} are the usual field-strengths, σ is conductivity and μ the magnetic permeability. Obviously, it cannot be expected that Eqs. (1) should give any information as regards the induced polarization since these equations do not contain quantities corresponding to the polarization of the rock. But, as a matter of fact, the existence of IP has been repeatedly checked by experiments and also Maxwell's equations are of general validity, so there must exist a general enough setting of these equations which would describe the IP effect and yet it would include the system of equations (1) as a special case.

If we wish to preserve the general validity of Maxwell's equations care must be taken that after any modification or extension of their range of validity the correctness of the resulting equations be rigorously checked. It is, of course, a sound generalization if we substitute the resultant of different specific conductivities into Eqs. (1). An extension of this kind was advocated by WAIT (1959) in his theoretical work on the foundation of the frequency-domain measurements and this has been applied ever since by many authors.

Starting out from theoretical considerations WAIT (1959) introduced the concept of complex impedance defined by the formula

$$\bar{\sigma} = \sigma + i\omega\epsilon \quad (2)$$

where ω denotes circular frequency and ϵ is the dielectric constant of the rock. For alternating currents the magnetic field strength is

$$\begin{aligned} \vec{H} &= \vec{H}_0 e^{i\omega t} \\ \text{i.e.} & \\ \frac{\partial \vec{H}}{\partial t} &= i\omega \vec{H} \end{aligned} \quad (3)$$

Substituting Eqs. (2) and (3) into the system of equations (1) we obtain Maxwell's equations as given by Wait:

$$\begin{aligned} \text{curl } \vec{H} &= (\sigma + i\omega\epsilon)\vec{E} & \text{a),} & & \text{curl } \vec{E} &= -i\omega\vec{H} & \text{b),} \\ \text{div } \epsilon\vec{E} &= S & \text{c),} & & \text{div } \vec{H} &= 0 & \text{d),} \end{aligned} \quad (4)$$

where S denotes volumetric charge density. Eqs. (4b) and (4d) agree with the corresponding formulae of Eq. (1), Eq. (4a) needs no special explanation but several comments should be made about Eq. (4c). The volumetric charge density S figuring on the right-hand side of the above equation inevitably arises which means that a displacement current occurs in connection with the dielectric rock medium. So, in such a field, Ohm's law is violated since the conditions of a closed circuit are not met. This assumption, however, is open to serious objections especially if we perform, in thought, the following experiment.

Let us prepare a rock model by impregnating quartz sand with highly conductive electrolyte. It has been many times experienced with resistivity measurements that in such a field the current is divergence-free. If we mix metal filings into this model there immediately appears an induced potential which is probably due to the volumetric charge density S figuring in Eq. (4). This, however, would imply that the postulations assuring the validity of Ohm's law are not obeyed. So we are faced with a paradox situation. If the highly conductive rock-model and the metal filings both satisfy Ohm's law there seems to be no rational explanation why their mixture would fail in this respect.

We shall try to resolve this contradiction, in what follows.

An electro-dynamical rock-model for the IP effect

To begin with, we propose a simple physical pattern of the phenomenon without going too deep into the intricacies of a complete electro-chemical treatment. We shall use the rock-model described above.

Suppose that we measure, first of all, the specific resistivity of a pure quartz sand with some DC-method after a sufficiently long time from the initiation of the measuring current. We introduce then metal filings into the rock as to ensure a macroscopically homogeneous sample and repeat the previous measurement under identical conditions. Because of the introduction of the metal filings it would be natural to expect that the specific resistivity of the model should decrease, but we shall soon learn the very opposite if we perform the experiment: i.e. the specific resistivity will be found to increase and after interrupting the measuring current an induced potential will be detected. To explain these experimental findings we refer to Fig. 1.

The great circle in Fig. 1a. represents metal filings of molecular dimensions, say, positively charged, situated evenly in the above sample. The potential field of the positively charged metal filings causes the negative ions of the electrolyte to move towards the metal filings until they compensate for its potential field. The resulting



Fig. 1
1. ábra
Puc. 1

charge-configuration will be neutral, that is, it will not exert attraction on the remaining free ions of the electrolyte. If we deliver voltage to the sample then, besides the internal field, an additional external field acts on the configuration of Fig. 1a, and the ions will move further, as in Fig. 1b., until equilibrium is reached again. The charge-configuration rearranges itself to a dipole whose potential field keeps balance with the external field.

If the external field is switched off the dipoles gradually lose their charge and assume again the shape shown in Fig. 1a. So, we may propose the following mechanism for the increase of specific resistivity in electrically polarizable rocks resp. for the voltage-decay in them after the interruption of the inducing current: at the initiation of current the arising potential field acts on the totality of ions and the rock behaves as if it were non-polarizable. This is due to the fact that the external field which acts on those ions which are situated in charge-configuration is many times greater than the field pointing towards the centre of the configuration and so, not only free ions, but bound ones as well will participate in conduction. The bound ions move the slower the farther they get from the centre since, by their movement, they create a dipole whose field of force compensates for the external field. After some, definite, time bound ions cease to participate in electric conduction and only free ions will take part in it what explains why resistivity increases. The greater percentage of ions is bound in charge-configurations the greater the increase of resistivity will be.

At the interruption of the inducing current charges inside the dipoles strive to their original positions, a current begins to flow inside the dipoles and their potential field decreases. The direction of this current is opposed to the inducing current, the currents inside the dipoles close at the free ions which are adjacent to these dipoles as shown in Fig. 2. The direction of these latter currents is, however, identical with the inducing current with a magnitude equal to the dipole-currents.

Summarizing, the following conclusions can be drawn from what has been said above, for further use:

1. In electrically conductive and polarizable rocks only a definite part of the inducing current is converted to Joule heat due to ohmic conductivity.

2. The other part of the inducing current is stored, as in an accumulator, during the time of induction and after the interruption of the inducing current it is also converted to Joule heat through losses of resistivity.

3. The ratio of the two currents is determined by the number of points where dipoles are formed or, more exactly, by the ratio of the bound- and free ions.

These conclusions will be found as decisive in the subsequent theoretical considerations. But even now they suggest that there must be some kind of relationship between the polarization parameters and the ore-content of the rock. At this

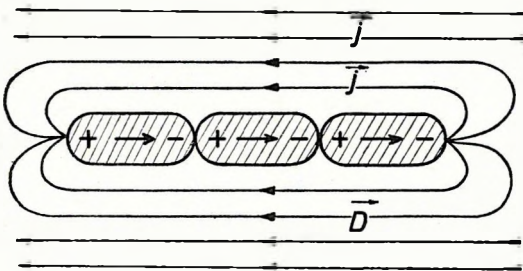


Fig. 2

2. ábra

Рис. 2

point of our exposition, however, it would be too early to make any definite statement without a thorough mathematical investigation of the above, physically suggestive, pattern.

Mathematical formulation of the theory of IP

To make mathematics as simple as possible, some idealizing assumptions will be made. It will be assumed that the investigated rock is macroscopically homogeneous, isotropic and of infinite extension and that the dipoles are brought about at evenly distributed points throughout the rock.

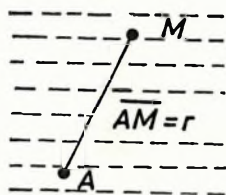


Fig. 3

3. ábra

Рис. 3

Suppose, that the inducing current is introduced at some point A into the rock and that the potential is measured at M (Fig. 3). Electrodes N and B are placed at infinity. The distance between electrodes A and M is r .

As it is well known, if the rock is not polarizable the current initiated at A is divergence-free at every other point. Denote the current density by \vec{j}_0 . Because of the dipoles formed also an additional current density, say \vec{j}_p , will occur and these current densities together account for the actual current in the rock. The polarization current density \vec{j}_p and the resulting polarization field-strength are both functions of the volumetric dipole density, denoted by S .

The polarization field strength \vec{P} and the dipole density S are in interrelation as shown by the function

$$\operatorname{div} \vec{P} = -S. \quad (5)$$

The current density \vec{j}_p can also be expressed by means of the field-strength vector \vec{P} . Indeed, the dipole momentum of a polarized volume element $d\tau$ is given by $\vec{P}d\tau$. If the average surface of a dipole is ds , its length dl , and the infinitesimal charge is e , then

$$\vec{P}ds dl = e dl.$$

Upon simplification, and differentiation with respect to time:

$$\frac{\partial \vec{P}}{\partial t} = \frac{1}{ds} \frac{\partial e}{\partial t} = \vec{j}_p. \quad (6)$$

A further relation, which will be used later, is the electro-dynamical equation of continuity:

$$\frac{\partial S}{\partial t} + \operatorname{div} \vec{j}_0 = 0. \quad (7)$$

After these preparations we proceed to write down the current which *really* flows in polarizable rocks, i.e. which already satisfies Ohm's law. On the strength

of our previous considerations it is plausible to expect that the density of this current, say \vec{j} , is of the form

$$\vec{j} = \vec{j}_0 - \vec{j}_p. \quad (8)$$

To prove this, we have to show that the divergence of j vanishes. Substituting Eq. (6) into Eq. (8):

$$\vec{j} = \vec{j}_0 - \frac{\partial P}{\partial t}$$

and taking the divergence of both sides

$$\operatorname{div} \vec{j} = \operatorname{div} \vec{j}_0 - \frac{\partial \operatorname{div} P}{\partial t}.$$

Further, making use of Eqs. (5) and (7), we have

$$\operatorname{div} \vec{j} = -\frac{\partial s}{\partial t} + \frac{\partial S}{\partial t} \equiv 0$$

i.e. the charge-density vector \vec{j} given by Eq. 8 is divergence-free and so the potential of point M is uniquely determined by this current. Equation (8) is already suitable to determine the direction of the current-density \vec{j}_p , namely that it acts against the inducing current, but it contains no information as to the magnitude of this density.

It has been recently found in a number of laboratory measurements that the density of the induced current is proportional to that of the inducing current, i.e.

$$|\vec{j}_p| = \alpha |\vec{j}_0|. \quad (9)$$

Some comments should be made about this relation. In the first place we note that Eq. (9) is an approximate relation, valid as long as the inducing current is small. For a sufficiently large inducing current the dipoles developed might disintegrate and Eq. (9) is no longer valid. As a matter of fact in actual field work we never encounter such extreme current densities (except in the immediate vicinity of electrodes) since the current-density decreases with the reciprocal of the second power of distance and we know from DAKHNOV (1959) that for current-densities of the order of 0.1 mA/cm² Eq. (9) already holds.

A second comment is that besides the disintegration of dipoles any deviation from idealized conditions might affect the validity of Eq. (9).

In field conditions the energy of the inducing current introduced into the soil will partly be consumed due to resistivity losses, another part will be spent for bringing dipoles about, but there always remains a piece of energy which may cause, through physico-chemical transformations, a permanent potential field. Within the scope of the present paper this phenomenon will be considered as a small perturbation which contributes to the noise-background of the measurement, and the validity of Eq. (9) will be taken for granted. It seems likely that a proper combination of time-domain and frequency-domain methods would be an efficient tool for suppressing this type of noise.

Returning to Eq. (8), we multiply both sides by the specific resistivity ϱ_0 of the rock:

$$\varrho_0 \vec{j} = \varrho_0 \vec{j}_0 - \varrho_0 \vec{j}_p. \quad (10)$$

The dimensions of both sides are that of field-strength, the terms on the right-hand side are the inducing-, resp. polarization field-strengths. Upon comparing this with the electrodynamic equation

$$\vec{D} = \vec{E} + \vec{P}$$

and taking into account Eq. (9) we get

$$\vec{D} = \varrho_0 \vec{j}, \quad \vec{E} = \varrho_0 \vec{j}_0 \quad \text{and} \quad \vec{P} = -\varrho_0 \vec{j}_p. \quad (11)$$

Since ϱ_0 , the specific resistivity of the rock, is constant, the divergence of vector \vec{D} is zero everywhere i.e. this field-strength vector obeys the postulate required by Ohm's law. So, to arrive at the desired form of Maxwell's equations which would already describe the polarization effect in conductive media we have to change \vec{E} into \vec{D} in Eq. (1). Before doing this, however, it will be profitable to write down a further relation, from Eqs. (10)–(11):

$$\vec{D} = \vec{E} - \kappa \vec{E} = \vec{E}(1 - \kappa). \quad (12)$$

According to our previous considerations κ may be called *polarization susceptibility*. Its numerical value expresses that from all ions participating in electrolytic conduction in a conductive rock medium, what amount (i.e. how many per cent) does contribute to dipole formation. κ , as a physical parameter is of paramount importance since it gives account of the density of those points (metal-filings in the above example) where dipoles arise.

If we divide Eq. (12) by ϱ_0 and introduce the notation

$$\varrho = \frac{\varrho_0}{1 - \kappa}$$

then, making use of the electrodynamical relation $D = \varepsilon E$ Eq. (12) can be rewritten as

$$\vec{D} = \frac{\varrho_0}{\varrho} \vec{E} \quad (13)$$

whence

$$\varepsilon = \frac{\varrho_0}{\varrho} \quad (14)$$

and, further, from Eq. (13)

$$\kappa = \frac{\varrho - \varrho_0}{\varrho}. \quad (15)$$

Here we must stop again to explain the physical meaning of ϱ and ε . As we have just seen, the polarization susceptibility κ is a measure of the volumetric density of those points where dipoles are brought about. If there are no such points, $\kappa = 0$, i.e. from Eq. (13) $\varrho = \varrho_0$. For κ greater than zero, $\varrho > \varrho_0$ which is plausible since certain ions do not take part in conduction. Here, ϱ is the specific resistivity given by DC methods after the formation of dipoles while ϱ_0 is the resistivity which would have been measured if all ions had participated in conduction. The case $\kappa = 1$ never occurs in practice since this would imply that all ions are situated in charge-

configurations, i.e. $\rho = \infty$. As for ε , we know that in conductive media $\varepsilon = 1$. From Eq. (14) we learn that this could only happen if there is no polarization i.e. if $\rho = \rho_0$. In case of polarization $\varepsilon < 1$: in a conductor just opposite to insulators, the dielectric constant may be less than unity. Conductors which have a disposition for polarization show a behaviour which is reminiscent of diamagnetic materials.

Let us suppose, in what follows, that we can somehow measure the values ρ_0 and ρ at point M of Fig. 3. Then we know that

$$\rho_0 = 4\pi r \frac{\Phi_0}{I} \quad \text{and} \quad \rho = 4\pi r \frac{\Phi}{I}$$

where I is the strength of the inducing current. Making use of Eq. (15)

$$\kappa = \frac{\Phi - \Phi_0}{\Phi} \quad (16)$$

already given by Seigel (*cf.* WAIT, 1959). He, as a matter of fact, denoted this value by m and termed it "chargeability". This terminology may be objected since κ is a well-known electrodynamical constant, identical with the polarization susceptibility, as it has been correctly termed by BLEIL (1953).

Because the polarization susceptibility, κ , is less than one, it would be more convenient for practical purposes to use its hundredth as a unit ($c\kappa$, i.e. centikappa) since this would give the percentage of all ions which are bound in dipoles.

Maxwell's equations in polarizable rocks

After these preparations we can write down Maxwell's equations for a polarizable conductor:

$$\begin{array}{ll} \text{curl } \vec{H} = \sigma_0 D & \text{a)} \\ \text{curl } \vec{D} = -\mu \frac{\partial \vec{H}}{\partial t} & \text{b)} \\ \text{div } \vec{D} = 0 & \text{c)} \\ \text{div } \vec{H} = 0 & \text{d)} \end{array} \quad (17)$$

where σ_0 is the specific conductivity of the rock (with respect to the totality of ions). The system of equations (17) is formally the same as Eq. (1) and in case of no polarization it evidently regresses to the latter. Eqs. (17c) and (17d) deserve no further comment, since (17c) has already been proved and (17d) has remained unchanged. The validity of (17a) and (17b) can also be simply checked.

Indeed, for a current of density \vec{j} flowing through a surface element ds we have

$$\int_s \vec{j} \, ds = \oint \vec{H} \, dl$$

i.e., by Stokes' theorem

$$\int_s \vec{j} \, ds = \int_s \text{curl } \vec{H} \, ds.$$

But, by Eq. (11)

$$j = \frac{1}{\rho_0} \vec{D} = \sigma_0 D$$

and the equality of the above integrals implies that

$$\text{curl } \vec{H} = \sigma_0 D.$$

To prove Eq. (17b) we follow a similar line of thought. We first express the potential by means of the magnetic flux Ψ , as

$$\Phi = - \frac{\partial \Psi}{\partial t}.$$

Since \vec{D} is divergence-free, we may write

$$\Phi = \oint_l \vec{D} \cdot d\vec{l}.$$

Recalling, that

$$\Psi = \mu \int_s \vec{H} \cdot d\vec{s}$$

we have

$$\oint_l \vec{D} \cdot d\vec{l} = -\mu \frac{\partial}{\partial t} \int_s \vec{H} \cdot d\vec{s}.$$

Applying again Stokes' theorem and equating the quantities behind the integral sign we have, indeed, that

$$\text{curl } \vec{D} = -\mu \frac{\partial \vec{H}}{\partial t}.$$

The system of equations (17) is already free of those contradictions which have been encountered in connection with Eqs. (4a) and (4c), so we are justified in expecting that their solution would yield *quantitative* relations between the induced polarization and other physical parameters of the rock.

Differential equation of a homogeneous polarizable conductive field

Since a general treatment of quasi-stationary fields would cause considerable mathematical difficulties we shall deal, in what follows, only with such cases where stationarity can be assumed as a reasonable approximation. So, we shall drop out of our calculations the equations describing the relation between \vec{D} and \vec{H} , the displacement current will be neglected either and it will be taken for granted that vector D can be expressed as gradient of the potential.

That is, we shall be concerned with a reduced set of equations

$$\frac{\partial S(r, t)}{\partial t} + \text{div } j_0 = 0, \quad (18)$$

$$\text{div } \vec{D} = 0. \quad (19)$$

We first inspect Eq. (18). According to Eqs. (5) and (10)

$$\operatorname{div} \vec{P} = -S(r, t) = \operatorname{div} (-\varkappa \varrho_0 \vec{j}_0) = -\varkappa \varrho_0 \operatorname{div} \vec{j}_0$$

since \varkappa and ϱ_0 are constants, i.e. Eq. (18) can be written as

$$\frac{\partial S(r, t)}{\partial t} + \frac{1}{\varrho_0 \varkappa} S(r, t) = 0. \quad (20)$$

We shall need a further relation between the dipoles' volumetric charge density $S(r, t)$, and the potential measured. We start out from Poisson's formula which asserts that

$$\Phi(r, t) = - \int \int \int \frac{\operatorname{div} \vec{P}}{R} d\tau$$

if there is no charge density on the surface, and R is the distance from the dipole to the point of observation. By Eq. (5)

$$\Phi(r, t) = \int \int \int \frac{S(r, t)}{R} d\tau. \quad (21)$$

Because of the homogeneity and isotropy of the rock, we can set

$$S(r, t) = S_A(t) \cdot W(r), \quad (22)$$

where $S_A(t)$ denotes the charge-density occurring at the origin of the system of coordinates and $W(r)$ is some factor of proportionality which depends on r . Inserting this into Eq. (21)

$$\Phi(r, t) = S_A(t) \int \int \int \frac{W(r)}{R} d\tau = S_A(t) Z(r) + \Phi_0(r) \quad (23)$$

where $Z(r)$ denotes the result of integration and $\Phi_0(r)$ is a constant which represents the constant field given by Eq. (19). Substituting Eq. (23) into Eq. (22):

$$\Phi(r, t) = \frac{S(r, t)}{W(r)} Z(r) + \Phi_0(r) \quad (24)$$

i.e.

$$S(r, t) = \frac{W(r)}{Z(r)} [\Phi(r, t) - \Phi_0(r)]. \quad (25)$$

Differentiating with respect to time

$$\frac{dS(r, t)}{dt} = \frac{W(r)}{Z(r)} \frac{d\Phi(r, t)}{dt}. \quad (26)$$

Substituting Eqs. (25) and (26) into Eq. (20) we obtain, upon simplifications, the differential equation

$$\varkappa \varrho_0 \frac{d\Phi(r, t)}{dt} + \Phi(r, t) = \Phi_0(r). \quad (27)$$

$\Phi_0(r)$ which figures on the right-hand side can be obtained from Eq. (19):

$$\operatorname{div} \vec{D} = -\operatorname{div} \operatorname{grad} \Phi = \Delta \Phi = 0.$$

The solution of the Laplace equation, $\Delta \Phi = 0$, is well known, it is given by

$$\Phi_0(r) = \frac{I \varrho}{4\pi r}. \quad (28)$$

The quantity ϱ which occurs here is naturally not identical with ϱ_0 . By Eq. (14)

$$\varrho = \frac{\varrho_0}{\varepsilon}$$

and Eq. (28) takes the form

$$\Phi_0(r) = \frac{I \varrho_0}{4\pi r \varepsilon} \quad (29)$$

where I is the current flowing in the rock. Putting this into Eq. (27), we get

$$\kappa \frac{d\Phi(r, t)}{dt} + \frac{1}{\varrho_0} \Phi(r, t) = \frac{I}{4\pi r \varepsilon} \quad (30)$$

which is a well-known differential equation whose solution is

$$\Phi(r, t) = \frac{I \varrho_0}{4\pi r \varepsilon} \left(1 - \kappa e^{-\frac{t}{\varepsilon \kappa}} \right). \quad (31)$$

Eq. (31) describes the behaviour of the potential in a rock of specific resistivity ϱ_0 and polarization susceptibility κ , if we introduce a constant current I into it. In non-polarizable rocks, $\varepsilon = 1$ and $\kappa = 0$, and the potential is, as it must be, simply

$$\Phi(r, t) = \frac{I \varrho_0}{4\pi r}.$$

The same formula applies at the moment of initiation of current, i.e. for $t = 0$, since $1 - \kappa = \varepsilon$. Consequently, at the moment of initiation the current also affects those ions which are built in the ion-configurations and all of them will participate in conduction; later—when the dipoles will have been brought about—they gradually lose their role in conduction and the potential exponentially tends to a limit value belonging to $t = \infty$, i.e. to

$$\Phi(r, t = \infty) = \frac{I \varrho_0}{4\pi r \varepsilon}$$

which corresponds to the case when only free ions transfer the current. The time-constant of the potential rise is given by the product $\varrho_0 \kappa$, i.e. the maximum value is attained (with a fair approximation) after a time $3\varrho_0 \kappa$. Assuming 10 ohm and 10 $c\kappa$ this time is 3 sec. This, of course, casts a new light on Eq. (16) in case if Φ_0 is measured at $t = 0$ and Φ somewhat later at $t > 3\varrho_0 \kappa$.

The differential equation (30) also describes the behaviour of the potential after the interruption of the current if we set $I = 0$. In this case the solution of (30) is

$$\Phi(r, t) = \frac{I_0 \kappa}{4\pi r \epsilon} e^{-\frac{t}{\epsilon_0 \kappa}} \quad (32)$$

i.e. at the moment of interruption ($t = 0$) the induced potential decreases to a fraction κ of its original value and keeps decreasing to zero, afterwards.

Obviously, if $\kappa = 0$ also $\Phi(r, t) = 0$, i.e. in non-polarizable rocks no decay of potential will be experienced. It is also evident from (32) that for a large specific resistivity and large polarization susceptibility the time-constant could be as large as 10 seconds or more. Figure 4 shows potential functions for some values of κ .

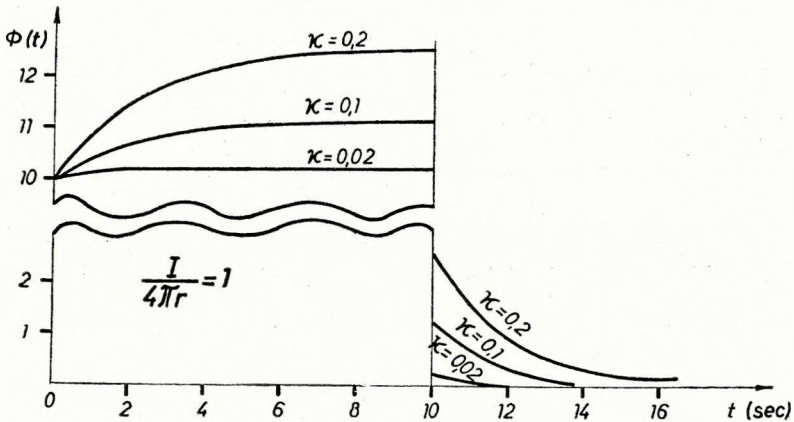


Fig. 4
4. ábra
Рис. 4

Measurement of the polarization susceptibility

Equations (31) and (32) suggest a quantitative method for the estimation of the physical parameter κ from the potential values measured at different time instants in a homogeneous isotropic medium. We shall use the following notations: after a time t_g from the initiation of the inducing current the potential measured at point M is Φ_g , while after a time t_1 from the interruption we denote it by Φ_1 . Inserting Φ_g and t_g into Eq. (31) we obtain, after rearrangements and multiplication by

$$-\frac{4\pi r \epsilon}{I}$$

that

$$\epsilon_0 - \frac{\Phi_g 4\pi r \epsilon}{I} = \epsilon_0 \kappa e^{-\frac{t_g}{\epsilon_0 \kappa}}$$

From Eq. (32)

$$\frac{4\pi r \varepsilon}{I} = \frac{\rho_0 \kappa}{\Phi_1} e^{-\frac{t_1}{\rho_0 \kappa}}$$

and, after some easy algebraic calculus

$$\kappa = \frac{\Phi_1}{\Phi_g e^{-t_1/\rho_0 \kappa} + \Phi_1 e^{-t_g/\rho_0 \kappa}} \quad (33)$$

In order to eliminate κ from the exponent we perform a further measurement at a time $t_2 = 2t_1$ after the interruption of the current. Writing down Eq. (32) for the new potential value, Φ_2 , as well, we have

$$\frac{\Phi_2}{\Phi_1} = \frac{e^{-2t_1/\rho_0 \kappa}}{e^{-t_1/\rho_0 \kappa}}$$

Taking the logarithm of both sides

$$\ln \frac{\Phi_2}{\Phi_1} = -\frac{t_1}{\rho_0 \kappa}$$

Substituting this into Eq. (33) we get an explicit term for κ

$$\kappa = \frac{\Phi^2}{\Phi_g \Phi_2 + \Phi_1^2 e^u} \quad (34)$$

where

$$u = \frac{t_2}{t_1} \ln \frac{\Phi_2}{\Phi_1}$$

If also $t_1 = t_g$ could be ensured κ would be given by the simple formula:

$$\kappa = \frac{\Phi_1^2}{\Phi_2(\Phi_g + \Phi_1)}$$

It must be noted that Eq. (34) has been derived for homogeneous media, that is, it is not necessarily valid under actual field conditions. There are, however, some significant conclusions which must be pointed out in connection with this formula. We note first that κ , as given by Eq. (34), is independent of the electrode configuration i.e. it can be determined under laboratory conditions on rock-samples or cores. If the investigated rock pattern is inhomogeneous (e.g. stratified) then the volumetric charge density S and the potential $\Phi_0(r)$ are complicated functions and the differential equation which would replace Eq. (30) could be only solved with sophisticated mathematical techniques. We shall not dwell upon this question any more in the present paper but we turn our attention to the frequency-domain method, in a homogeneous field.

Solution of the differential equation for AC

Let us consider an AC of circular frequency ω and amplitude I_v , i.e. let

$$I = I_v \sin \omega t.$$

Substituting this value into the differential equation (30), it becomes

$$\kappa \frac{d\Phi(r, t)}{dt} + \frac{1}{\varrho_0} \Phi(r, t) = \frac{I_v}{4\pi r \varepsilon} \sin \omega t. \quad (35)$$

The time-dependent term on the right-hand side of this equation suggests the use of Laplace transformation. Making use of the identities

$$\mathfrak{L} \left[\frac{d\Phi}{dt} \right] = p\Phi(p) - \Phi(0)$$

and

$$\mathfrak{L} [\sin \omega t] = \frac{\omega}{p^2 + \omega^2}$$

the Laplace transform of Eq. (35) will be

$$p\kappa \left[\Phi(p) - \Phi_0 \left(\frac{\omega}{p^2 + \omega^2} \right) \right] + \frac{1}{\varrho_0} \Phi(p) = \frac{I_v}{4\pi r \varepsilon} \cdot \frac{\omega}{p^2 + \omega^2}.$$

Multiplying both sides by ϱ_0 and substituting the value of Φ_0 —that is the voltage at the initiation of current—into this equation

$$p\kappa\varrho_0\Phi(p) - \frac{I_v p\kappa\varrho_0\omega}{4\pi r(p^2 + \omega^2)} + \Phi(p) = \frac{I_v\varrho_0\omega}{4\pi r(p^2 + \omega^2)}$$

which gives after some easy manipulations and making use of Eq. (14) that

$$\Phi(p) = \frac{I_v\omega}{4\pi r(p^2 + \omega^2)} \cdot \left(\frac{\varrho + \varrho\kappa\varrho_0^2}{p\kappa\varrho_0 + 1} \right) = \frac{I_v\omega}{4\pi r} \left[\frac{\varrho + p\kappa\varrho_0^2}{p^3\varepsilon(\varrho - \varrho_0) + p^2 + p\omega^2\varepsilon(\varrho - \varrho_0) + \omega^2} \right].$$

Finally, adding some further algebra and reference to Eqs. (15), Eq. (14) give

$$\Phi(p) = \frac{I_v\varrho_0}{4\pi r\varepsilon} \left[\frac{1 + p\kappa\varepsilon\varrho_0}{p^3\varrho_0\kappa + p^2 + p\omega\varrho_0\kappa + \omega^2} \right]$$

which may conveniently be written as

$$\Phi(p) = \frac{I_v\varrho_0}{4\pi r\varepsilon\omega} \left[\frac{1 + p\kappa\varepsilon\varrho_0}{\left(\frac{p^2}{\omega^2} + 1 \right) (\varrho_0\kappa p + 1)} \right].$$

Looking up any textbook which contains inversion formulae of the Laplace transform we find that

$$\mathfrak{L}^{-1} \left\{ \frac{1 + ap}{\left(\frac{p^2}{\omega^2} + 1 \right) (Tp + 1)} \right\} = \frac{\omega(T - a)e^{-\frac{t}{T}}}{1 + T^2\omega^2} + \omega \left(\frac{1 + a^2\omega^2}{1 + T^2\omega^2} \right)^{1/2} \sin(\omega t + \varphi)$$

with

$$\varphi = \text{arctg } a\omega - \text{arctg } T\omega.$$

Making the substitutions $a = \kappa \rho_0 \varepsilon$, $T = \kappa \rho_0$, dividing by ω and setting $1 - \varepsilon = \kappa$ we have

$$\Phi(r, t) = \frac{I_p \rho_0}{4\pi r \varepsilon} \left[\frac{\omega \rho_0 \kappa^2 e^{-t/\rho_0 \kappa}}{1 + \rho_0 \kappa^2 \omega^2} + \left(\frac{1 + \kappa^2 \varepsilon^2 \omega^2 \rho_0^2}{1 + \kappa^2 \omega^2 \rho_0^2} \right)^{1/2} \sin(\omega t + \varphi) \right]. \quad (36)$$

To determine the phase angle φ , we shall use the identity

$$\text{arctg } x - \text{arctg } y = \text{arctg } \frac{x - y}{1 + xy}$$

which gives, for $xy > -1$, that

$$\varphi = -\text{arctg } \frac{\rho_0 \omega \kappa^2}{1 + \varepsilon \kappa^2 \rho_0 \omega^2}. \quad (37)$$

Equations (36) and (37) are the basic formulae of the frequency domain method. Before a detailed study of these equations let us discuss the first term in Eq. 36. Since $\kappa < 1$, its second power, κ^2 , is small and it will be further diminished by the exponential factor $e^{-t/\rho_0 \kappa}$. So, we shall neglect this term in our subsequent considerations, i.e. we shall assume that the measurement is started after the lapse of a definite time (corresponding to two- or three times the time-constant $\rho_0 \kappa$) after the initiation of the inducing current. With this simplification Eq. (36) becomes

$$\Phi(r, t) = \frac{I_p \rho_0 (1 + \kappa^2 \omega^2 \varepsilon^2 \rho_0^2)^{1/2}}{4\pi r \varepsilon (1 + \kappa^2 \rho_0^2 \omega^2)^{1/2}} \sin(\omega t + \varphi). \quad (38)$$

Consider now the behaviour of $\Phi(r, t)$ for high frequencies. If $\rho_0 \omega \kappa \gg 1$ then the second terms dominate inside the brackets, in Eq. (38), and, approximately

$$\Phi(r, t) = \frac{I_p \rho_0}{4\pi r} \sin \omega t. \quad (39)$$

The phase angle, φ , is zero to a very good approximation. Thus, for very high frequencies the rock behaves as if it were non-polarizable, i.e. we simply measure the specific resistivity ρ_0 of the rock. This might be due to the fact that the ions taking part in ion-configurations will follow the changes of the inducing current and in the dipole the forces acting toward the centre are negligible as compared to the external field.

On the other extreme, if the measuring frequency is very low, i.e. $\rho_0 \omega \kappa \ll 1$, we have

$$\Phi(r, t) = \frac{I_p \rho_0}{4\pi r \varepsilon} \sin \omega t. \quad (40)$$

The phase shift φ can be neglected also in this case, since $\varrho_0\omega\kappa^2 = (\varrho_0\omega\kappa)\kappa$, i.e. $\varrho_0\omega\kappa^2 < \varrho_0\omega\kappa$ because of $\kappa < 1$.

From Eq. (40) we conclude that for very low frequencies the value of the potential depends, besides the specific resistivity ϱ_0 , on the dielectric constant ε as well, i.e. an apparent specific resistivity greater than ϱ_0 will be found. This can be explained by the fact that the ions bound in charge-configurations practically do not participate in conduction, they are in dipole state for the best part of the period.

The third case to be discussed is when

$$\varrho_0\omega\kappa = 1.$$

Then

$$\Phi(r, t) = \frac{I_v \varrho_0}{5.6 \pi r \varepsilon} (1 + \varepsilon)^{1/2} \sin(\omega t + \varphi)$$

and

$$\varphi = -\operatorname{arctg} \frac{1 - \varepsilon}{1 + \varepsilon}. \quad (41)$$

That is, for $\kappa\varrho_0\omega = 1$ both an increase of potential and a phase-shift will be experienced. It is worth noting that in this special case φ does not depend on specific resistivity.

We shall seek now the maximum of the phase-shift φ . We have already seen that for very low and very high frequencies there is practically no phase shift. Differentiating the quantity in the argument of the arctg function in Eq. (37) and making the derivative equal with zero:

$$\frac{\varrho_0\kappa^2}{1 + \varepsilon\kappa^2\varrho_0^2\omega^2} - \frac{2\varrho_0^3\kappa^4\varepsilon\omega^2}{(1 + \varepsilon\kappa^2\varrho_0^2\omega^2)^2} = 0.$$

The root of this equation is given by

$$\kappa\varrho_0\omega = \frac{1}{\sqrt{\varepsilon}}. \quad (42)$$

Since ε is near to one even for well-polarizable rocks, the phase shift φ attains its maximum for a value $\kappa\varrho_0\omega \approx 1$.

We proceed now to express κ in case of the frequency-domain method. Substituting the values of $\Phi(r, t)$ and I to the formula describing resistivity one gets, upon simplifications

$$\varrho(\omega) = \frac{\varrho_0}{\varepsilon} \mathfrak{G}^{1/2} \frac{\sin \omega t + \varphi}{\sin \omega t} \quad (43)$$

where

$$\mathfrak{G} = \frac{1 + \varepsilon^2\kappa^2\varrho_0^2\omega^2}{1 + \kappa^2\varrho_0\omega^2}.$$

This equation contains two unknown quantities, ϱ_0 and ε . For large enough measuring frequencies Eq. (43) can be simplified as

$$\mathfrak{S}_h = \varrho_0 \sin \omega t$$

while for very low frequencies we have

$$S_t = \frac{Q_0}{\epsilon} \sin \omega t.$$

From these formulae α is readily expressed:

$$\alpha = \frac{Q_t - Q_h}{Q_t}.$$

This formula works well under laboratory conditions and gives an expedient for the determination of α . In field work, however, we cannot neglect the interaction of \vec{D} and \vec{H} in Eq. (17) in the high-frequency case and the use of very low frequencies would be also problematic from a purely technical point of view. In routine mea-

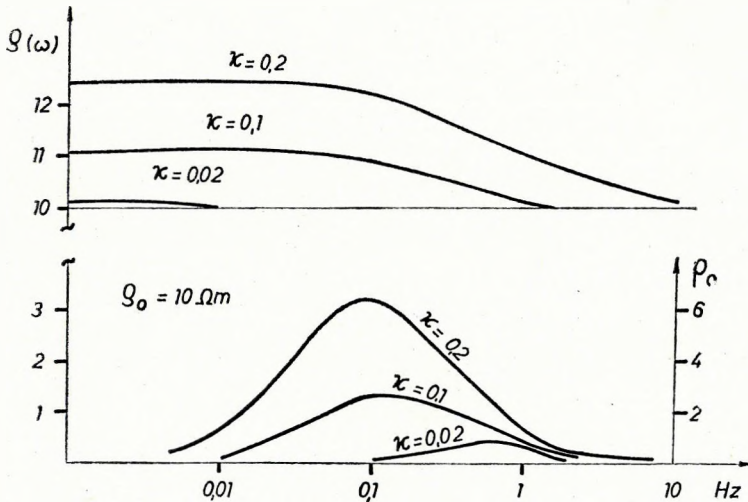


Fig. 5
5. ábra
Рис. 5

surements one has to use Eq. (43) and determine α from this formula by means of nomograms or other computational techniques.

Figure 5 gives a few illustrative examples of the change of specific resistivity and the phase angle in function of frequency.

Summarizing, it can be concluded that in homogeneous media the induced potential allows a quantitative description and the rock-physical parameter α can be determined. Equations (31), (32), (36) and (37) give a fair estimation for the order of magnitude of the induced potential and the shape of the curves shown in Figs. 4 and 5 are in accordance with experimental findings.

Experimental verification of our theoretical conclusions

In order to check numerically the formulae proposed we shall make use of the results of NILSSON'S (1971) laboratory measurements. Nilsson investigated the behaviour of the electrode-potential by a technique assuring nearly ideal conditions (see Fig. 6). The metals investigated were nickel and silver. The metal sheet gathers around itself the ions of opposite charge, these cluster in dipoles since the field is homogeneous—the only non-vanishing component of the current-density vector is the normal one—and the phenomenon of electric polarization can be quantitatively studied. Nilsson measured the real- and imaginary parts of the complex impedance

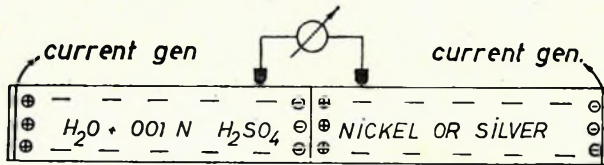


Fig. 6

6. ábra

Puc. 6

and presented their plots in his paper. We note that the graph of the real component resembles $\rho(\omega)$ and that of the imaginary component $\varphi(\omega)$ of our Fig. 5.

We have utilized Nilsson's measured values as follows. First, the values of ρ_0 and ρ could be determined from the real component since

$$\begin{aligned}\rho(\omega) &\rightarrow \rho \quad \text{if } \omega \rightarrow 0 \\ \rho(\omega) &\rightarrow \rho_0 \quad \text{if } \omega \rightarrow \infty\end{aligned}$$

and the phase-shift is zero. Making use of Eq. (15) we obtain κ from ρ and ρ_0

for nickel	for silver
$\rho \rightarrow R = 2.3 \text{ kohm}$	$R = 1.6 \text{ kohm}$
$\rho_0 \rightarrow R_0 = 0.9 \text{ kohm}$	$R_0 = 0.9 \text{ kohm}$
$\kappa = 0.61$	$\kappa = 0.44$

According to Eq. (42) the phase angle has its maximum at $\rho_0 \kappa \omega \approx 1$, further, by Eq. (41), this maximum value of φ solely depends on κ . But this phase-angle can be obtained from the measured values as well, since, *per definitionem*

$$\varphi = \arctg \frac{Im(z)}{Re(z)}$$

and its maximum occurs for the maximum of $Im(z)$. The phase angles, determined by the different methods, show a fair agreement:

for nickel	for silver
calculated 25°	16°
measured 23°	14°

This computation deserves a further comment. It has been indicated above that the value of α increases with the ore-content of the rock. In Nilsson's experiment, however, the measurements were performed under identical conditions and the computed α values still differed. This shows that not only the ore-content but its quality as well might influence the value of α , that is the equivalence principle also holds for the measurements of α .

An equivalent electric circuit for the IP effect

Figure 7 presents the equivalent electric circuit for IP. The equation governing the circuit agree with our previous equations if we set

$$c = 4\pi r \varepsilon,$$

$$R_1 = \frac{\rho_0}{4\pi r}, \quad R_2 = \frac{\rho - \rho_0}{4\pi r}.$$

This model corresponds to the electrode configuration shown in Fig. 3.

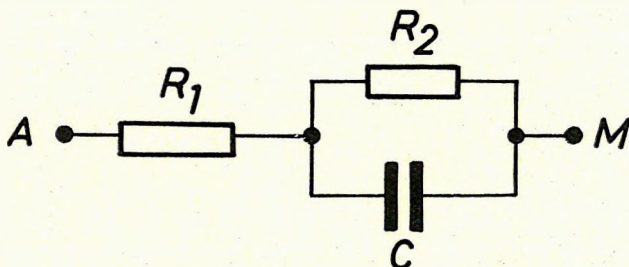


Fig. 7
7. ábra
Рис. 7

IP in a layered half-space

In this section we shall briefly sketch how our principal equations should be modified for the case of an inhomogeneous (layered) medium.

As a matter of fact, for the time-domain method even the two-layered case would raise considerable difficulties. For, in order to describe the potential functions we had to determine the functional form of the charge-density $S(x, y, z, t)$ and the value of the constant potential field, construct differential equations from these functions and, finally, extract the time constants from the solution of these equations. This program, however, remains a task of future research.

The frequency-domain method in itself allows a simple interpretation: let us consider two successive layers. If the layers are non-polarizable, the normal component of current density satisfies the equation

$$\vec{j}^{01n} - \vec{j}^{02n} = 0$$

where the indices refer to the respective layers. Letting ϱ_{01} and ϱ_{02} denote the specific resistivities, we have

$$\frac{1}{\varrho_{01}} \left(\frac{\partial \Phi}{\partial n} \right) - \frac{1}{\varrho_{02}} \left(\frac{\partial \Phi}{\partial n} \right) = 0.$$

In case of polarization the normal component of current density satisfies

$$\vec{j}_{1n} - \vec{j}_{2n} = 0$$

because of $\text{div} \vec{j} = 0$. Introducing specific resistivities ϱ_1 and ϱ_2 :

$$\frac{1}{\varrho_1} \left(\frac{\partial \Phi}{\partial n} \right) - \frac{1}{\varrho_2} \left(\frac{\partial \Phi}{\partial n} \right) = 0$$

that is, the normal component of the charge-density vector continuously changes at the boundary of specific resistivities. Consequently, for very low frequencies polarizable rocks show an increased resistivity while in the high frequency limit they reveal the real values of resistivities no matter whether the rocks are layered or not.

This conclusion might give a significant hint for future research since it seems to suggest that the familiar sets of curves used for SR interpretation could be applicable for the calculation of families of curves for the measurement of polarization susceptibility.

Conclusions

We have reviewed the basic physical ideas underlying the IP method and have shown that within the scope of a consistent electrodynamical treatment quantitative relations could be established between the polarization constants— α and ε —and the parameters of measurement (voltage, frequency, etc.). The orders of magnitude and the time- and amplitude-behaviour predicted by our theory show excellent agreement with recent experimental findings (ZONGE et al., 1970; NILSSON, 1971).

We consider Eq. (17) as the principal result of our paper since it makes possible a quantitative treatment of the induced potential and we can expect that a number of important problems which have arisen about the IP method in recent years will be tackled by means of these techniques.

REFERENCE LIST

- BLEIL, D. F., 1953: Induced Polarization: A Method of Geophysical Prospecting. *Geophysics*, XVIII, 3, 636–661.
- NILSSON, B., 1971: A new combined resistivity- and induced polarization-instrument and a new theory of the induced polarization-phenomenon. *Geoexploration*, 9, 55–56.
- WAIT, J. R., 1959: *Overvoltage Research and Geophysical Application*. Pergamon Press, London.
- ZONGE, K. L.—SAUCK, W. A.—SUMMER, I. S., 1970: Comparison of time, frequency and phase measurements in induced polarization. 40th Annual International SEG Meeting.

DANKHÁZI GYULA

A GERJESZTETT POTENCIÁL ELMÉLETI ALAPJAI

A dolgozat egy anyagi modellből kiindulva tárgyalja a gerjesztett potenciál jelenlétét. A modell szerint a pozitív vagy negatív töltéssel rendelkező anyagi részecskék maguk köré gyűjtik a környezetükben tartózkodó ellentétes töltésű szabad elektronokat vagy ionokat, így kifelé semleges viselkedést mutató töltéskonfigurációk jönnek létre. A gerjesztő áram hatására ezek a töltéskonfigurációk polarizálódnak, dipólusokká alakulnak. A gerjesztő áram bekapcsolása pillanatában az elektromosan vezető közegben minden ion — tehát a konfigurációkban levő is — részt vesz az elektromos vezetésben, a dipólusok kialakulása után azonban már csak a konfigurációkban le nem kötött elektronok vagy ionok közvetítik az áramot. A vezető közeg ellenállása tehát a dipólusok kialakulásával párhuzamosan növekszik, majd a dipólusok kialakulása után egy megnövekedett értéket vesz fel. A gerjesztő áram kikapcsolása után a dipólusok fokozatosan elvesztik töltésüket, a környezetükben elhelyezkedő elektronokon vagy ionokon kisülnek, így a gerjesztő áram irányával azonos irányú fokozatosan csökkenő áramot tartanak fenn.

A modell szerint a közegben ténylegesen folyó divergenciamentes áram a gerjesztő áram és a polarizációs áram különbsége. Ezt a megállapítást a dolgozat bizonyítja és ebből kiindulva megadja a Maxwell-egyenleteknek azokat az alakjait, amelyek a polarizációra hajlamos elektromosan vezető közegekben is helyesen írják le az elektromágneses tér viselkedését. Ilyen közegekben a dielektromos állandó értéke egynél kisebb.

A Maxwell-egyenletek birtokában, a kontinuitási egyenlet felhasználásával, a továbbiakban a dolgozat a homogén, végtelen kiterjedésű közeget (közeteket) vizsgálja, felírja a potenciálra vonatkozó differenciálegyenletet, majd ezt a gerjesztő áram bekapcsolásakor és kikapcsolásakor, továbbá szinuszos váltakozó áramú gerjesztésre is megoldja. A kapott eredmények a gyakorlatban szerzett tapasztalatokkal igen jó egyezést mutatnak, sőt Nilsson (1971) ideális feltételek között végzett méréseivel kielégítő számzerű egyezést is adnak.

A dolgozat befejezésül a *frequency-domain* mérések kiértékelésére vonatkozóan tesz néhány megjegyzést.

ДБ. ДАНКХАЗИ

ТЕОРЕТИЧЕСКИЕ ОСНОВЫ ВОЗБУЖДЕННОЙ ПОЛЯРИЗАЦИИ

В работе рассматривается явление возбужденной поляризации исходя из вещественной модели. По модели вещественные частицы, имеющие положительный или отрицательный заряд, собирают вокруг себя свободные электроны или ионы, имеющиеся в их окружности, в связи с чем создаются конфигурации заряда с нейтральным поведением смотря снаружи. На воздействие возбуждающего тока эти конфигурации зарядов поляризуются и превращаются в диполи. В момент включения возбуждающего тока в электрически проводящей среде все ионы — следовательно и те, которые находятся в конфигурациях — участвуют в электрическом введении, но после образования диполей ток передается только электронами или ионами, не связанными в конфигурациях. Следовательно, сопротивление проводящей среды увеличивается параллельно с образованием диполей, а затем оно приобретает определенное увеличенное значение. После выключения возбуждающего тока диполи постепенно теряют свой заряд, располагающиеся в окружности электроны или ионы разряжаются, в связи с чем они сохраняют ток, сила которого постепенно уменьшается и направление которого аналогично направлению возбуждающего тока.

Исследования на модели показывают, что бездивергентный ток, фактически протекающий в горных породах, представляется разностью между возбуждающим и поляризующим током. Это заключение подтверждается в настоящей работе. Исходя из этого получают формы уравнения Максвелла, правильно описывающие поведение электромагнитного поля даже в электрически проводящих средах, склонных к поляризации. В подобных средах величина диэлектрической постоянной меньше единицы.

Опираясь на уравнения Максвелла, с использованием уравнения непрерывности, в работе рассматриваются однородные среды бесконечного протяжения (горные породы), записывается дифференциальное уравнение, касающееся потенциала, а затем это уравнение решается для случая включения и выключения возбуждающего тока, а также для случая возбуждения синусоидального переменного тока. Полученные результаты хорошо согласуются с полученными на практике данными, и дают удовлетворительное численное совпадение с результатами измерений *Нильсона* (1971), проведенными при идеальных условиях.

В заключение в работе излагаются некоторые замечания относительно интерпретации измерения области частот.

