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THE EFFECT OF CHANGES IN WAVEFORM UPON CDP SUMMATION

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Essentially, stacking is a kind of filtering. It filters our random noises and, from among regular noises, multiple reflexions.

When using stacking for filtering out multiples, it is generally assumed that the waveform of multiples is entirely identical on the different channels. This, however, not always holds, not even as a rough approximation.

In the present paper, all the computations and notations are based on the velocity function and system of notations developed in the paper "Filtering effect of common-depth-point systems and their transfer functions" (Bodoky, 1970).

Computations were carried out for two types of stacking channels: a split-spread system (1,5, 2,5, 5,5, 6,5, 9,5, 10,5) and an offset-shotpoint system (12, 16, 20, 24, 28, 32). This special choice of stacking channels permits to obtain data, apart from the parameters investigated, about the role of the offset too.

In order to characterize the multiple transfer of various types of channels to be stacked, the function Φ is introduced. This is the ratio of transmitted, resp. total multiple energy:

$$\Phi(t_0, d) = \frac{\int_0^\infty [A(\omega) S(\omega, t_0, d)]^2 \, d\omega}{\int_0^\infty [f A(\omega)]^2 \, d\omega} \tag{1}$$

where t_0 = time of recording,
d = seismometer spacing,
ω = circular frequency,
$A(\omega)$ = spectrum of the arrivals,
$S(\omega, t_0, d)$ = the transfer function of stacking,
f = number of coverages.

The detailed form of the transfer function is given by

$$S(\omega, t_0, d) = \sum_{i=1}^{f} c_i e^{j\omega\tau_i(t_0, d)} \tag{2}$$

where c_i = weight of the i-th channel,
τ_i = residual moveout of the multiple reflexion arriving on the i-th channel.

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In the computation of the Φ function the assumption on the identical form of the arrivals appears as their expression with identical weights and identical spectra. Thus, if we are investigating what kind of transfer variations could arise from the different forms of arrivals, the variations of the Φ functions due to deviations in spectra and in weights must be investigated.

Since the $\Phi(t_0, d)$ functions, in the paper mentioned, are approximately independent of t_0, i.e. the multiple-transfer of the stacked channel types discussed there is approximately time-independent it was sufficient to carry out the computations for a single t_0 value, say $t_0 = 2$ sec.

The first problem is that, if the weights of the channels are not assumed to be identical, the number of possible cases is infinite; namely an infinite number of weight series can be ordered to the channels to be stacked. The probability of occurrence of a certain weight series is so small that the knowledge of its transfer properties gives no practically usable information. Therefore a statistic approach was chosen and instead of dealing with individual weight series, statistical properties (expected value and standard deviation) of the Φ functions were investigated as function of the scattering of the weight series.

The expected value of channel weights is invariably assumed to be unity. This ensures, without restricting generality that the denominator on the right-hand side of Formula (1) remains unchanged. (Namely, the constant f in the denominator is the value of the transfer function $S(\omega)$ for $\tau_1 = \tau_2 = \ldots = \tau_f = 0$).

If the weights c_i are treated as variables, Φ will be a function of $f+1$ variables of the type $\Phi(d, c_1, \ldots, c_f)$. However, in order to get rid of the great number of independent variables, instead of variables $c_1, \ldots, c_f$, their scattering $\sigma(c)$ will be treated as a single independent variable,

$$\sigma^2(c) = \frac{1}{f} \sum_{i=1}^{f} (c_i - 1)^2.$$

Denoting the weight series by C_k (where index k refers to some given series), the scattering of the series will be denoted by $\sigma(C_k)$. The same $\sigma(C_k)$ scattering may correspond to many weight series C_k. Consequently, the expected value and standard deviation of the $\Phi[\sigma(C_k)]$ function of a given stacking channel type are for a fixed $\sigma(C_k)$ value and given values (t_0, d), as follows:

$$E\{\Phi(C_k)_{\sigma=\text{const}}\} = \frac{1}{n} \sum_{k=1}^{n} \Phi(C_k)_{\sigma=\text{const}} \tag{3}$$

and

$$\sigma^2\{\Phi(C_k)_{\sigma=\text{const}}\} = \frac{1}{n} \sum_{k=1}^{n} [\Phi(C_k)_{\sigma=\text{const}} - E\{\Phi(C_k)_{\sigma=\text{const}}\}]^2 \tag{4}$$

where n is the number of all such possible weight series, the scattering of which is exactly the constant chosen.

The course of the calculations was the following: first the value of Φ was calculated, according to relations (1) and (2), for six weight series of identical scattering for a given place (t_0, d), then the values of $E(\Phi)$ and $\sigma(\Phi)$ were determined from the

Φ values obtained, according to Formulae (3) and (4). The six weight series of identical scattering were selected from the permutations of an arbitrarily chosen series.

In Formulae (3) and (4), n represents the number of all appropriate weight series. In practical calculations, this theoretical requirement is not met, due to the number and way of generating of the weigth series used; therefore our results are only approximate.

The calculations were carried out with seven different values of weight scattering, in function of seismometer spacing ($d = 10$ m sampling interval, from 0 m to 150 m).

The $E\{\Phi[\sigma(c), d]\}$ function of the split-spread stacking channel type is shown in Fig. 1, its $\sigma\{\Phi[\sigma(c), d]\}$ function in Fig. 2. The same functions for the offset-shotpoint stacking channel type are shown in Figs. 3 and 4.

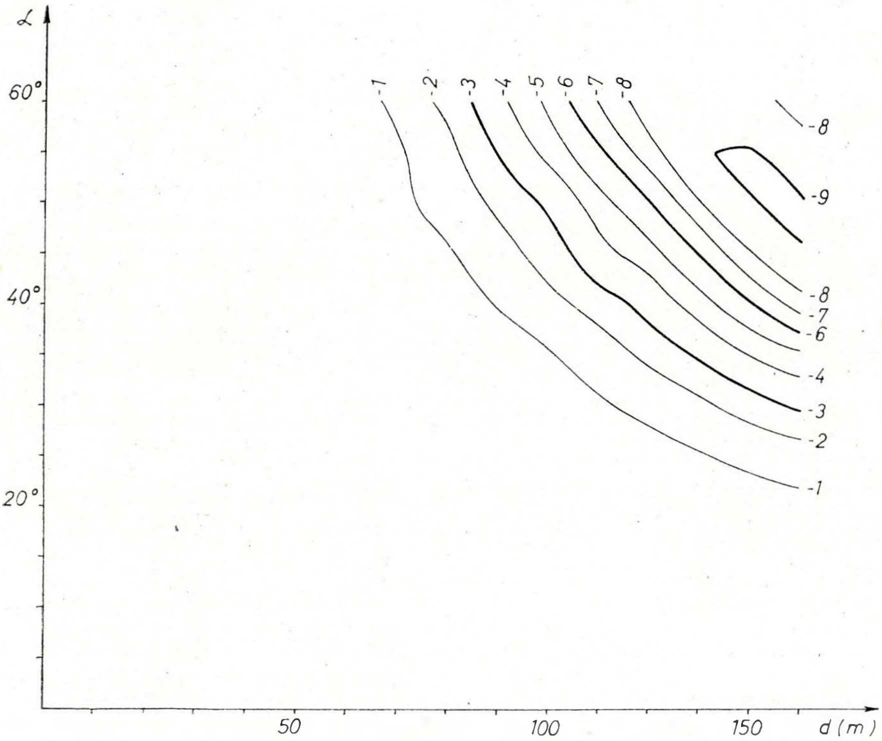

Fig. 1 The expected value (in dB) of the multiple-attenuation of the stacking channel type (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) as a function of the scattering of channel weights [$\sigma(c)$] and of seismometer spacing (d)

1.ábra. Az (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) összegcsatorna típus többszörös reflexió csillapításának várható értéke (dB-ben ábrázolva) a csatorna súlyok szórásának [$\sigma(c)$] és a geofontávolságnak (d) a függvényében

Рис. 1. Зависимость ожидаемой степени подавления кратных отражений (в дб) для суммотрассы типа (1,5; 2,5; 3,5; 6,5; 9,5; 10,5) от разброса весов каналов [$\sigma(c)$] и от шага сейсмоприемников (d)

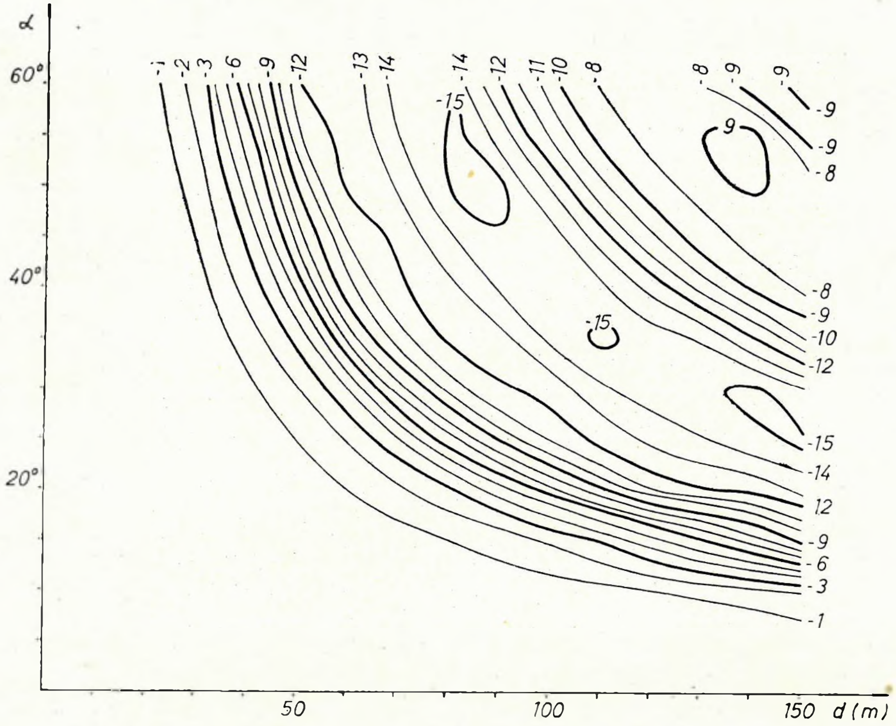

Fig. 2 The scattering of the multiple-attenuation of the stacking channel type (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) (in dB) as a function of the scattering of channel weights [$\sigma(c)$] and of the seismometer spacing (d)

2. ábra. Az (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) összegcsatorna típus többszörös reflexió csillapításának szórása (dB-ben ábrázolva) a csatorna súlyok szórásának [$\sigma(c)$] és a geofontávolságnak (d) a függvényében

Рис. 2. Зависимость разброса степени подавления кратных отражений (в дб) для суммотрассы типа (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) от разброса весов каналов [$\sigma(c)$] и от шага сейсмоприемников (d)

The investigations concerning weight scattering were carried out also for the scattering of the spectra of multiples to be stacked.

Under expected value and scattering of spectra, the expected value and scattering of the peak-frequency $a_{\max}$ of the spectra will be understood. For simplicity's and unambiguity's sake it was agreed upon that the spectra can be of a Ricker-wavelet spectrum form only. This type of spectrum is described by

$$A(\omega) = \frac{\omega^p}{p} e^{\frac{1}{2}\left(\frac{\omega}{p}\right)^2} \tag{5}$$

where $p = \pi a_{\max}$.

In order to make a comparison with the results of the paper used as initial material possible, the value of $E(a_{\max})$ was chosen for 30 cps.

Fig. 3 The expected value of the multiple-attenuation of the stacking channel type (12, 16, 20, 24, 28, 32) (in dB) as a function of the scattering of channel weights [$\sigma(c)$] and of the seismometer spacing (d)

3. ábra. A (12; 16; 20; 24; 28; 32) összegcsatorna típus többszörös reflexió csillapításának várható értéke (dB-ben ábrázolva) a csatorna súlyok szórásának [$\sigma(c)$] és a geofontávolságnak (d) a függvényében

Рис. 3. Зависимость ожидаемой степени подавления кратных отражений (в дб) для суммотрассы типа (12, 16, 20, 24, 28, 32) от разброса весов каналов [$\sigma(c)$] и от шага сейсмоприемников (d)

Starting out again from Formula (1), $A(\omega)$ occurs both in the numerator and denominator of the right-hand side. As previously, instead of $A(\omega)$, $E\{A(\omega)\}$ will be written in the denominator. In the numerator, the product $[A(\omega)S(\omega)]$ can be written, after some rearrangements:

$$[S(\omega)A(\omega)] = \left(\sum_{i=1}^{I} e^{j\omega\tau_i}\right) A(\omega)\, e^{j\omega\tau_i}.$$

(The weights c_i are taken as of unit value).

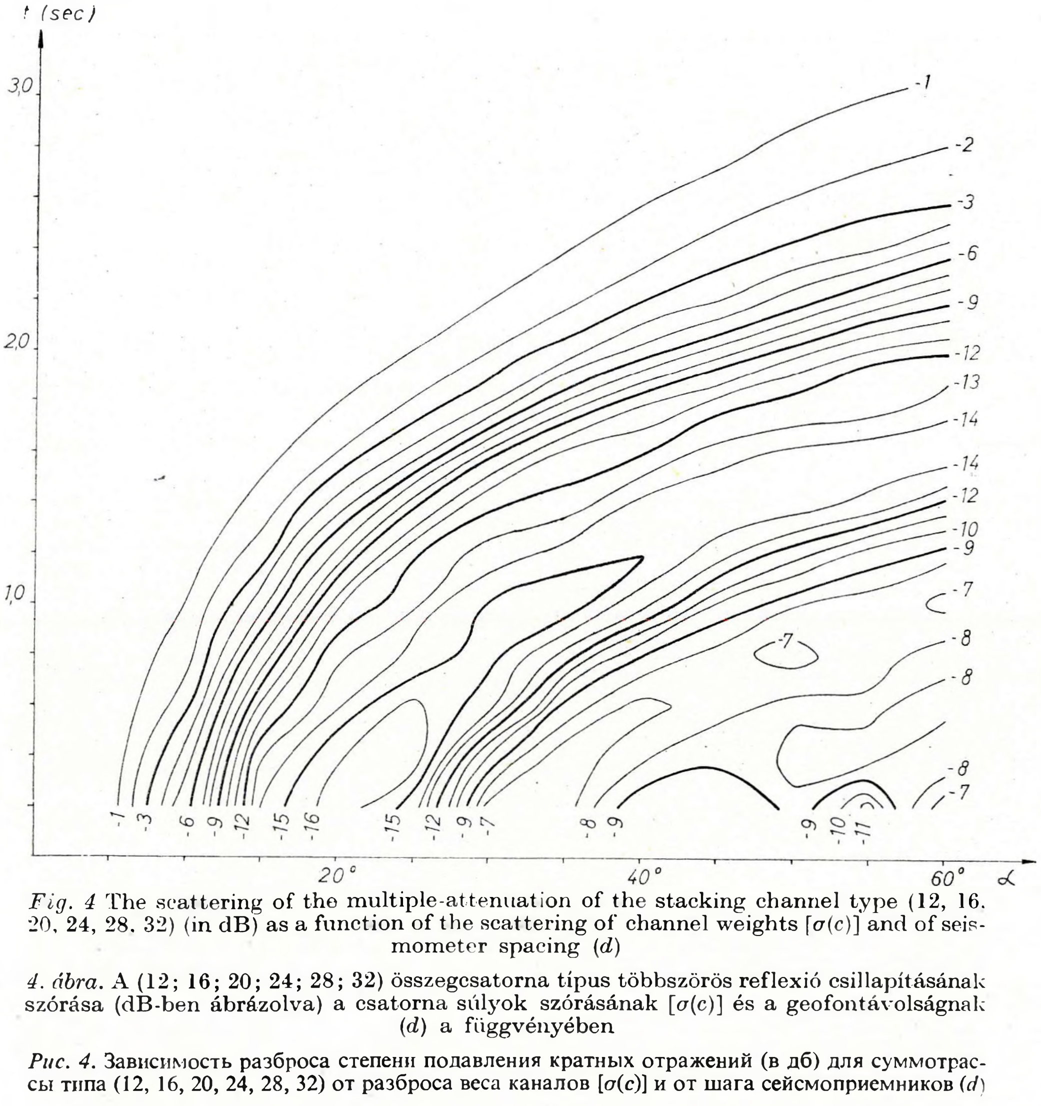

Fig. 4 The scattering of the multiple-attenuation of the stacking channel type (12, 16, 20, 24, 28, 32) (in dB) as a function of the scattering of channel weights [$\sigma(c)$] and of seismometer spacing (d)

4. ábra. A (12; 16; 20; 24; 28; 32) összegcsatorna típus többszörös reflexió csillapításának szórása (dB-ben ábrázolva) a csatorna súlyok szórásának [$\sigma(c)$] és a geofontávolságnak (d) a függvényében

Рис. 4. Зависимость разброса степени подавления кратных отражений (в дб) для суммотрассы типа (12, 16, 20, 24, 28, 32) от разброса веса каналов [$\sigma(c)$] и от шага сейсмоприемников (d)

Since it is assumed now that the arrivals have different spectra, also these spectra must be indexed. Thus, the right-hand side of (1) will be

$$\frac{\int\limits_0^\infty \left[\sum_{i=1}^{f} A_i(\omega)\, e^{j\omega\tau_i}\right]^2 d\omega}{\int\limits_0^\infty [fE\{A_i(\omega)\}]^2\, d\omega}.$$

Accordingly, Φ will be, on a given place (t_0, d) a function of the form $\Phi[A_1(\omega), A_2(\omega), \ldots, A_f(\omega)]$. Let also here possible spectrum series $A_1(\omega), A_2(\omega), \ldots, A_f(\omega)$ denoted by A_k, and their scattering by $\sigma(A_k)$. The same scattering can belong to an infinite number of spectrum series A_k, therefore the functions

$$E\{\Phi(A_k)_{\sigma=\mathrm{const}}\} = \frac{1}{n}\sum_{k=1}^{n}\Phi(A_k)_{\sigma=\mathrm{const}} \qquad (6)$$

and

$$\sigma^2\{\Phi(A_k)_{\sigma=\mathrm{const}}\} = \frac{1}{n}\sum_{k=1}^{n}[\Phi(A_k)_{\sigma=\mathrm{const}} - E\{\Phi(A_k)_{\sigma=\mathrm{const}}\}]^2 \qquad (7)$$

can be calculated, at a given place (t_0, d) for a given type of channels to be stacked.

The calculations and generating spectrum series of identical scattering take place in the same way as in the case of weight series.

As the result of calculations, the $E\{\Phi[\sigma(A), d]\}$ function calculated for the split-spread stacking channel type is shown by Fig. 5. — the $\sigma\{\Phi[\sigma(A), d]\}$ function

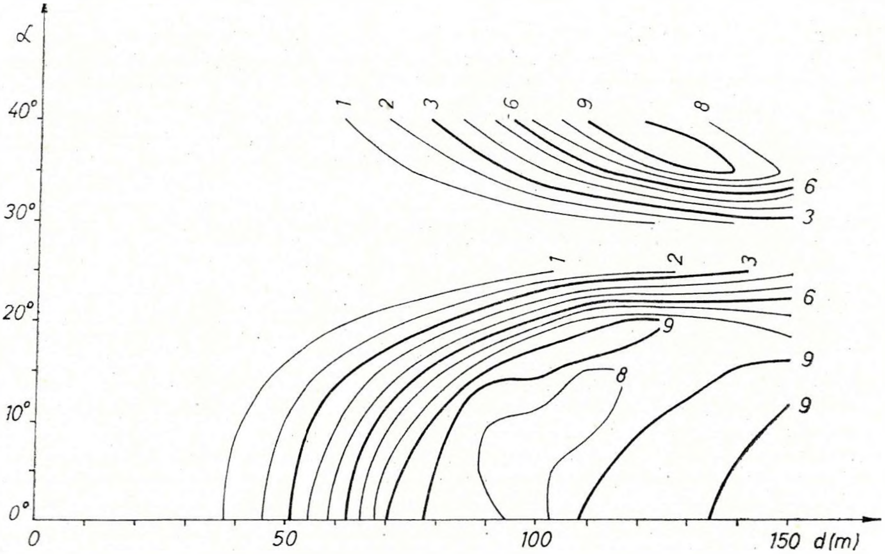

Fig. 5 The expected value of the multiple-attenuation of the stacking channel type (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) (in dB) as a function of the scattering of channel spectra $[\sigma(a_{\max})]$ and of seismometer spacing (d)

5. ábra. Az (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) összegcsatorna típus többszörös reflexió csillapításának várható értéke (dB-ben ábrázolva) a csatorna spektrumok szórásának $[\sigma(a_{\max})]$ és a geofontávolságnak (d) a függvényében

Рис. 5. Зависимость ожидаемой степени подавления кратных отражений (в дб) для суммотрассы типа (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) от разброса спектров каналов $[\sigma(a_{\max})]$ и от шага сейсмоприемников (d)

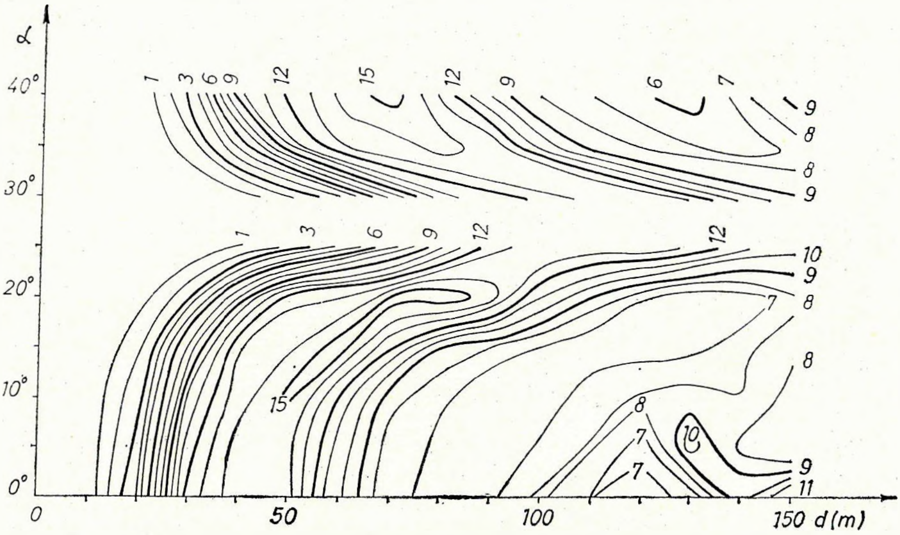

Fig. 6 The scattering of the multiple-attenuation of the stacking channel type (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) as a function of the scattering of channel spectra [$\sigma(a_{\max})$] and of seismometer spacing (*d*)

6. ábra. Az (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) összegcsatorna típus többszörös reflexió csillapításának szórása (dB-ben ábrázolva) a csatorna spektrumok szórásának [$\sigma(a_{\max})$] és a geofontávolságnak (*d*) a függvényében

Рис. 6. Зависимость разброса степени подавления кратных отражений (в дб) для суммотрассы типа (1,5; 2,5; 5,5; 6,5; 9,5; 10,5) от разброса спектров каналов [$\sigma(a_{\max})$] и от шага сейсмоприемников (*d*)

by Fig. 6. The same functions calculated for the offset-shotpoint spread system are illustrated by Figs. 7 and 8.

Examining the resulting eight figures, following conclusions can be drawn;

— The expected value of multiple-attenuating of stacked channels decrease slowly, practically insignificantly in case of an increasing scattering of the weights, resp. spectra of the channels, the place of maximum extinction being, however, independent of these quantities.

— The scattering of the multiple-attenuation of stacked channels is practically insignificant, slightly growing with increasing scattering of weights, resp. spectra of the arrivals to be stacked.

All in all, the multiple-attenuating effect of stacking is practically insensible even against comparatively coarse form-variations of multiple reflexions.

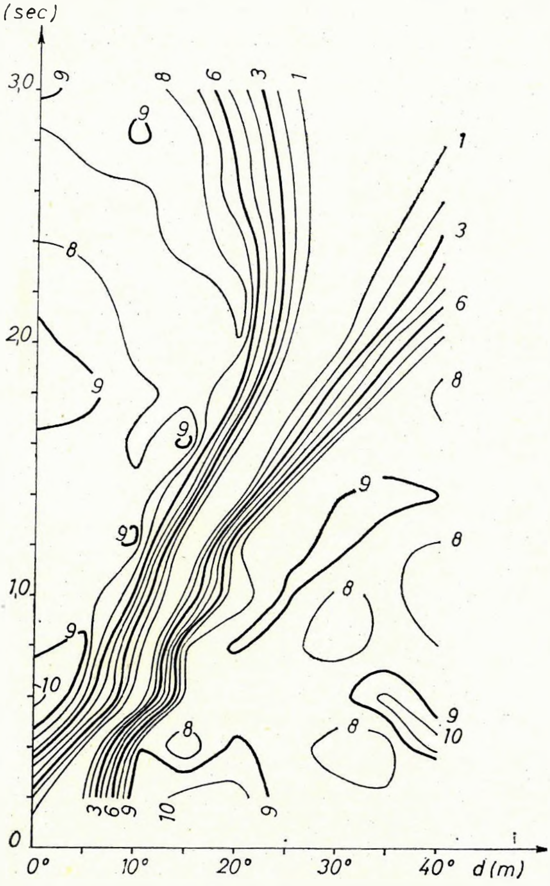

Fig. 7 The expected value of the multiple-attenuation of the stacking channel type (12, 16, 20, 24, 28, 32) (in dB) as a function of the scattering of channel spectra [$\sigma(a_{max})$] and of seismometer spacing (*d*)

7. ábra. A (12; 16; 20; 24; 28; 32) összeg-csatorna típus többszörös reflexió csillapításának várható értéke (dB-ben ábrázolva) a csatorna spektrumok szórásának [$\sigma(a_{max})$] és a geofontávolságnak (*d*) a függvényében

Рис. 7. Зависимость ожидаемой величины подавления кратных отражений (в дб) для суммотрассы типа (12, 16, 20, 24, 28, 32) от разброса спектров каналов [$\sigma(a_{max})$] и от шага сейсмоприемников (*d*)

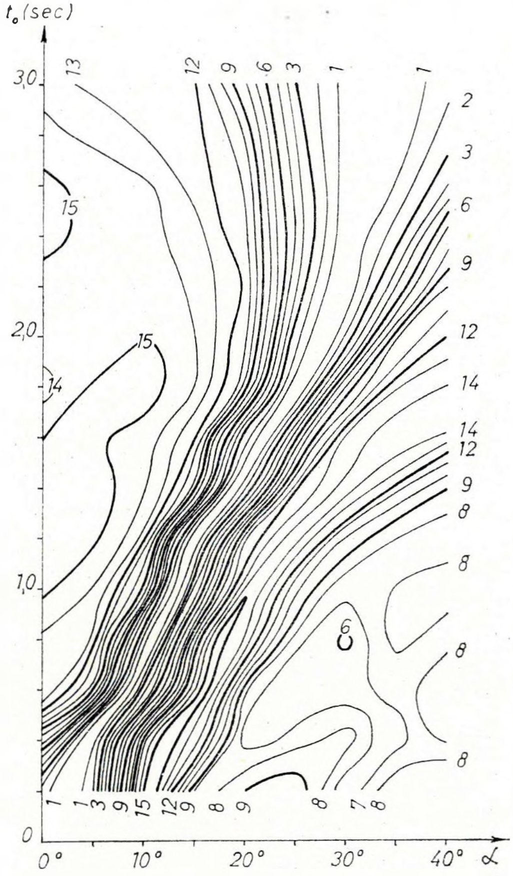

Fig. 8 The scattering of the multiple-attenuation of the stacking channel type (12; 16; 20; 24; 28; 32) (in dB) as a function of the scattering of channel spectra [$\sigma(a_{\max})$] and of seismometer spacing (*d*)

8. ábra. A (12; 16; 20; 24; 28; 32) összegcsatorna típus többszörös reflexió csillapításának szórása (dB-ben ábrázolva) a csatorna spektrumok szórásának [$\sigma(a_{\max})$] és a geofontávolságnak (*d*) a függvényében

Рис. 8. Зависимость разброса степени подавления кратных отражений (в дб) для суммотрассы типа (12, 16, 20, 24, 28, 32) от разброса спектров каналов [$\sigma(a_{\max})$] и от шага сейсмоприемников (*d*)

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BODOKY TAMÁS

A BEÉRKEZÉSEK ALAKVÁLTOZÁSAINAK HATÁSA A KÖZÖS MÉLYSÉGPONTOS ÖSSZEGEZÉSNÉL

A tanulmányban a szerző kiszámítja — a számításokat két összegcsatorna típusra végezte el — a közös mélységpontos összegezés többszörös csillapításának várható értékét és szórását az összegezendő többszörös beérkezések amplitúdószórásának függvényében. Ugyanezt a vizsgálatot a többszörös beérkezések csúcsfrekvencia-szórásának függvényében is elvégzi, a spektrumokat Ricker-féle wavelet spektrum alakúnak tételezve fel.

A számítások eredményeként a következő megállapításokat teszi:

1. az összegcsatornák többszörös csillapításának várható értéke lassan, gyakorlati szempontból jelentéktelen mértékben csökken a csatornák súlyainak, illetve spektrumainak növekvő szórásainál; a maximális kioltás helye azonban független ezen mennyiségektől.

2. Az összegcsatornák többszörös csillapításának szórása az összegezendő beérkezések súlyainak, illetve spektrumainak növekvő szórásánál gyakorlati szempontból csak enyhén növekszik.

Т. БОДОКИ

ВЛИЯНИЕ ИЗМЕНЕНИЙ ФОРМЫ ЗАПИСАННЫХ КОЛЕБАНИЙ ПРИ СУММИРОВАНИИ ПО МЕТОДУ ОГТ

В работе вычисляются ожидаемые степень и разброс подавления кратных отражений при суммировании по методу ОГТ для двух типов суммотрасс в зависимости от разброса амплитуд суммируемых кратных отражений. Подобный анализ проводится и в зависимости от разброса максимальной частоты кратных отражений, причем предполагается, что спектры имеют форму спектров волн Рикера.

В результате проведенных вычислений делаются следующие выводы:

1. Ожидаемая степень подавления кратных отражений на суммотрассах снижается медленно, в практически незначительной мере с увеличением разброса весов и спектров каналов. Однако, место максимального подавления кратных волн не зависит от этих величин.

2. Разброс степени подавления кратных отражений на суммотрассах увеличивается в практически незначительной мере с увеличением разброса весов и спектров суммируемых волн.