Magyar Állami Eötvös Loránd Geofizikai Intézet
GEOFIZIKAI KÖZLEMÉNYEK
XX. kötet, 3–4. szám

APPROXIMATION OF THE OPTIMUM-FILTERS USED IN SEISMIC DATA PROCESSING

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Introduction

It is well known that optimum filter theory has found a wide application in many stages of seismic data processing ($S/S+N$ filters, deconvolution filters and, in special problems, *OVS* or *OHS* filters). In the actual design of filters various approximations must be made and in most cases the necessary filter parameters are estimated from the recorded data (by means, e.g., of correlation analysis). The filter is only optimum if all approximations made are justified and even then only with respect to the parameters used. The filter finally applied to a given channel is, even in the best case, a good approximation.

In the usual derivation of $S/S+N$ filters, for example, it is assumed that signal and noise are non-correlated and both are realizations of stationary stochastic processes. Power spectra are estimated from the autocorrelation functions. It is well-known, however, that the frequency content of the seismic signal changes during propagation, i.e. stationarity which is assumed may hold only within reasonable short time gates. The coefficients figuring in the correlation functions are random variables and if one estimates them from a small number of samples, the corresponding confidence intervals become rather wide. Consequently, the estimation of correlation functions becomes the less reliable the narrower is the time gate. If we strive at the fulfilment of one of the criteria (stationarity), the indeterminacy of the estimation of the parameters will be increased and vice versa.

Similar difficulties are encountered in case of other problems of optimum filtering. These problems do not make the computation of optimum filters superfluous since their application results in much clearer seismic sections. It is, however, reasonable to try to design sub-optimum filters if these lead to an ease in the computations or a substantial saving in computer time.

The methods of sub-optimum filter design can be classified in two large groups. In the first of them the design model is chosen in such a way that the determination of the transfer function or weight function of the filter be simple. Investigations of this kind were reported by Foster and Sengbush (1965). But, in reality, also the *OHS* and *OVS* filters—although they are termed as optimum by the authors (Schneider et al. 1964, 1965)—were designed under such assumptions on the input channels which were meant to make the filters suitable for computation.

Manuscript received 30, 6, 1971.
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Another type of sub-optimum filters is obtained if we approximate in the frequency domain some filter which had been designed according to the theory of optimum filters. The approximating function (i.e. the transfer function of the sub-optimum filter) has only a few parameters. Its form assures that the corresponding weight-function be computable with a closed formula from the parameters given. The actual values of the parameters can be determined by fitting the sub-optimum transfer function to the optimum one.

The computation of sub-optimum filters is justified, among others, by the convenient properties of the weight functions: smoothness, shortness. Consequently, the otherwise necessary smoothing and truncation may be omitted.

This paper will treat a special group of sub-optimum filters of the second type. After a brief discussion of the algorithm of design, a concrete application will be dealt with.

Determination of the transfer function of the sub-optimum filter

Let us denote by $S_0(f)$ the transfer function of some optimum filter and by $S(f, \alpha_1, \alpha_2, \ldots, \alpha_n)$ that of the suboptimum filter, where $\alpha_1, \alpha_2, \ldots, \alpha_n$ are parameters which are to be determined.

In order to fit the transfer function we determine the series of parameters for which

$$\frac{1}{2F}\int_{-F}^{F} |S_0(f) - S(f, \alpha_1, \alpha_2, \ldots, \alpha_n)|^2 \, df = \text{minimum}. \tag{1}$$

In practical cases instead of $S_0(f)$ its sampled version

$$S_0(if_0) \qquad (i = 0, \pm 1, \pm 2, \ldots \pm N)$$

is given and instead of integral (1) the sum

$$I = \frac{1}{2N+1}\sum_{i=-N}^{N} |S_0(if_0) - S(if_0, \alpha_1, \alpha_2, \ldots, \alpha_n)|^2 \tag{2}$$

must be minimum. If the transfer functions of the optimum filter and of the approximating sub-optimum filters are real, the modulus sign can be omitted. In what follows, we shall be concerned with this simpler case.

By virtue of the condition of optimality the partial derivatives with respect to the parameters α_k are zero:

$$\frac{\partial I}{\partial \alpha_k} = 0; \qquad (k = 1, 2, \ldots, n). \tag{3}$$

After performing differentiation we obtain a system of equations for the unknown parameters $\alpha_1, \alpha_2, \ldots, \alpha_n$:

$$\sum_{i=-N}^{N} [S_0(if_0) - S(if_0, \alpha_1, \alpha_2, \ldots, \alpha_n)] \frac{\partial S}{\partial \alpha_k} = 0. \tag{4}$$

This system of equations is generally non-linear. We shall attempt to solve it after linearization, by an iterative procedure. Let us introduce for sake of simplicity the notations:

$$S(if_0, \alpha_1, \ldots, \alpha_n) = S(if_0, \alpha),$$

$$\operatorname{grad}_\alpha S = \left(\frac{\partial S}{\partial \alpha_1}, \ldots, \frac{\partial S}{\partial \alpha_n} \right).$$

The order of approximations in the course of the iteration will be shown by the upper index of the parameters. Let the initial n-tuple of parameters be

$$\alpha_1^{(0)}, \alpha_2^{(0)}, \ldots, \alpha_n^{(0)},$$

while in the j-th step

$$\alpha_1^{(j)}, \alpha_2^{(j)}, \ldots, \alpha_n^{(j)}.$$

The $(j+1)$-st approximation can be constructed from the j-th one as follows. Let us introduce the notation $\Delta_k^{(j)}$ for the differences (corrections) between successive approximations, i.e.

$$\alpha_k^{(j+1)} = \alpha_k^{(j)} + \Delta_k^{(j)}. \tag{5}$$

Developing the function $S(f, \alpha_1^{(j+1)}, \alpha_2^{(j+1)}, \ldots, \alpha_n^{(j+1)})$ in a Taylor-series at the neighbourhood of $\alpha_k^{(j)}$:

$$S(if_0, \alpha_k^{(j+1)}) = S(if_0, \alpha_k^{(j)}) + (\bar{\Delta}^{(j)}, \operatorname{grad}_\alpha S) + O[(\Delta_k^{(j)})^2] \tag{6}$$

—where the components of vector $\bar{\Delta}^{(j)}$ are the values defined by (5);
the arguments of vector $\operatorname{grad}_\alpha S$ are the parameters of the j-th approximation.

Assuming that the terms which are of the second order in the corrections $\Delta_k^{(j)}$ are sufficiently small:

$$S(if_0, \alpha_k^{(j+1)}) = S(if_0, \alpha_k^{(j)}) + (\bar{\Delta}^{(j)}, \operatorname{grad}_\alpha S) \tag{7}$$

Substituting this to Eq. (2) and differentiating with respect to the corrections $\Delta_k^{(j)}$ we obtain that

$$\sum_{i=-N}^{N} [S_0(if_0) - S(if_0, \alpha_k^{(j)}) - (\bar{\Delta}, \operatorname{grad}_\alpha S|_{\alpha_k = \alpha_k^{(j)}})] = \left. \frac{\partial S}{\partial \alpha_k} \right|_{\alpha_k = \alpha_k^{(j)}}. \tag{8}$$

This last system of equations is already linear in the corrections $\Delta_k^{(j)}$. After solving it, the $(j+1)$-st approximation of the parameters are obtained by means of Eq. (5).

We have not established the convergence of the procedure theoretically. It was found in practice, however, that for an appropriate initial n-tuple of parameters, the corrections $\Delta_k^{(j)}$ become small after a few iterative steps. The choice of initial parameters has a crucial role since it has been always assumed that corrections are small.

The brief description of the above iterative method was thought appropriate since this algorithm had been used throughout our investigations. Analyses of other methods and a detailed investigation of the convergence problems involved will be subject matters of our further research work.

An application example

We shall proceed to approximate the transfer function illustrated by Fig. 1a The corresponding weight function is also given, Fig. 1b.

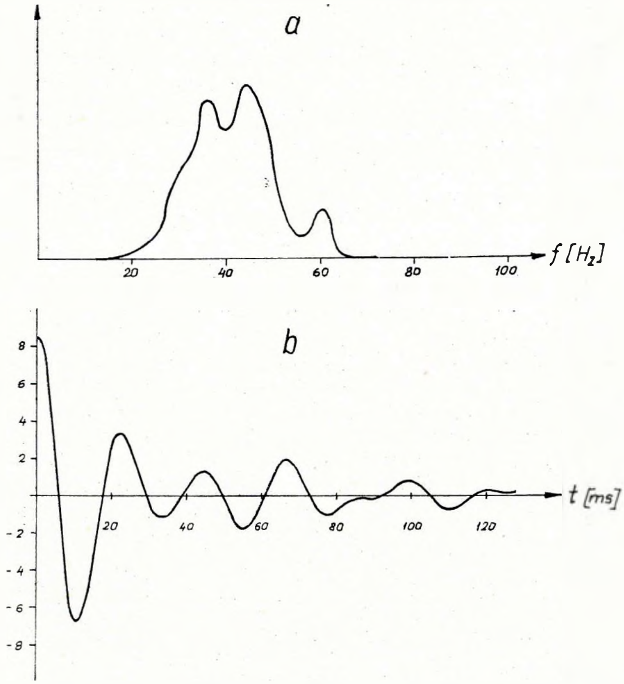

Fig. 1 Transfer function *(a)* and weight function *(b)* of the optimum filter

1. ábra. A közelítendő optimumszűrő átviteli függvénye *(a)* és súlyfüggvénye *(b)*

Рис. 1. Характеристика (*а*) и весовая функция (*д*) аппроксимируемого оптимального фильтра

Considering the shape of the transfer function which is to be approximated it seems appropriate to seek for an approximating function of the form

$$S(f, \alpha_k) = \alpha_1 [e^{-\alpha_2 (f-\alpha_3)^2} + e^{-\alpha_2 (f+\alpha_3)^2}], \tag{9}$$

that is, the transfer function of the sub-optimum filter consists of two Gaussian curves, placed symmetrically to the origin of the co-ordinate system. Parameter α_1 determines the amplitude, and α_2 the distance of the centres from the origin. Parameter α_3 is in connection with the slope of cut-off. (The transfer function of the optimum filters has the same properties. The imaginary part is identically zero).

The inverse Fourier transform of (9) is given by a closed formula:

$$S(t, \alpha_k) = \alpha_1 \sqrt{\frac{\pi}{\alpha_2}}\, e^{-\frac{\pi^2}{\alpha_2}} t^2 \cos 2\pi\alpha_3 t. \tag{10}$$

It is evident that the weight-function is smooth and automatically truncated. The initial choice of parameters for the iterative procedure described above was

$$\alpha_k^{(0)} = (0.7;\ 0.005;\ 10.0).$$

The change of parameters in course of the iterative procedure and the mean square deviation between the transfer functions of the optimum and sub-optimum filters are shown in Table I. The values of the parameters remain practically unchanged after the first iterative step. The mean square deviation decreases with a jump in the first step and attains nearly the same value afterwards.

Table I.

Change of the values of the parameters and of the mean square error in course of the iterations

Iteration	α_1	α_2	α_3	Mean square deviation
0	0.7	0.00500	10,00000	169.201
1	0.87057	0.00635	10.29375	0.121
2	0.87755	0.00631	10.21597	0.119
3	0.88065	0.00642	10.20840	0.119
4	0.87827	0.00633	10.21633	0.119
5	0.87947	0.00637	10.21249	0.119
6	0.87829	0.00633	10.21648	0.119
7	0.87829	0.00633	10.21659	0.119

Table II.

Values of optimum filter $S_0(if_0)$ and of the sub-optimum filter ($f_0 = 4$ cps)

i	$S_0(if_0)$	$S(if_0)$	i	$S_0(if_0)$	$S(if_0)$
0	0.00001	0.00004			
1	0.00000	0.00016	11	0.92376	0.82540
2	0.00000	0.00095	12	0.73508	0.63654
3	0.00005	0.00451	13	0.28234	0.40095
4	0.01496	0.01757	14	0.12135	0.20626
5	0.04185	0.05590			
6	0.09131	0.14524	16	0.02846	0.02974
7	0.34375	0.30820	17	0.00019	0.00834
8	0.53016	0.53415	18	0.00008	0.00191
9	0.85302	0.75609	19	0.00015	0.00036
10	0.68332	0.87413	20	0.00000	0.00005
			21	0.00000	0.00001

The values of the original (optimum) and of the approximating (sub-optimum) transfer function are given in Table II. Figure 2a illustrates the transfer function of the sub-optimum filter while Fig. 2b the corresponding weight function. The transfer function and weight function approximated are plotted in these same Figures by

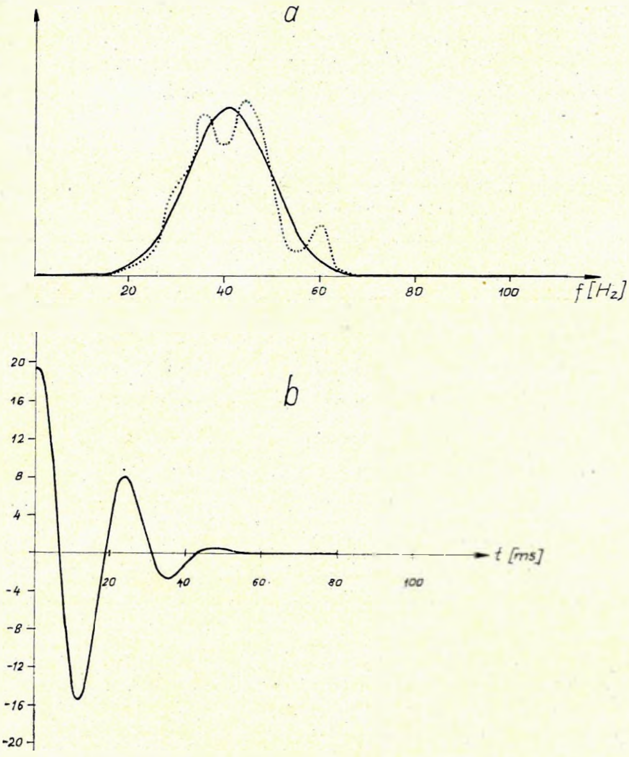

Fig. 2 Transfer function *(a)* and weight function *(b)* of the sub-optimum filter. Curves plotted with dotted lines are the transfer resp. weight functions of the optimum filter

2. ábra. A szuboptimumszűrő átviteli függvénye *(a)* és súlyfüggvénye *(b)*. A szaggatott vonallal rajzolt görbék az optimumszűrő átvileti függvényét illetve súlyfüggvényét ábrázolják

Рис. 2. Характеристика (*a*) и весовая функция (*д*) субоптимального фильтра. Кривые, проведенные пунктиром, соответствуют характеристике и весовой функции оптимального фильтра

dotted lines. We emphasize again the fact that the approximating weight function is automatically truncated due to the proper choice of the form of approximation (9); the weight-function of the sub-optimum filter is some two and a half times shorter than the original one.

Acknowledgement

This paper was prepared at the Geophysical Exploration Co. of the National Oil and Gas Trust. The permission of the management of the Exploration Co. to publish this paper is gratefully acknowledged.

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A DIGITÁLIS SZEIZMIKUS ADATFELDOLGOZÁSBAN ALKALMAZOTT OPTIMUMSZŰRŐK KÖZELÍTÉSÉRŐL

Az optimumszűrők tervezésének korlátai (az alkalmazott elhanyagolások és paraméterbecslések hibái) indokolttá teszik közelítések alkalmazását. A szuboptimumszűrő használatának további előnye, hogy eltávolítja az eredeti átviteli függvényből a stochasztikus ingadozásokat. A dolgozat az optimumszűrő átviteli függvényéhez néhány paraméteres szuboptimumszűrő illesztését javasolja. A szuboptimumszűrőt úgy választjuk, hogy a hozzá tartozó súlyfüggvény zárt alakban előállítható, simított és csonkított legyen.

Az eredeti, és a szuboptimumszűrő átviteli függvénye közötti átlagnégyzetes különbséget írjuk fel. Az átlagnégyzetes eltérés minimalizálásával a szuboptimumszűrő meghatározandó paramétereire nem-lineáris egyenletrendszert kapunk. Az egyenletrendszert iterációs módszerrel oldjuk meg. A javasolt eljárás szerint a szűrőtervezés az illesztésben alkalmazott iterációs algoritmussal bővül; a numerikus inverz Fourier transzformáció-számítás és csonkítás elmarad.

Az eljárás alkalmazását szuboptimális simítószűrő tervezésével illusztráljuk.

А. МЕШКО—П. ЖЕЛЛЕР

ОБ АППРОКСИМАЦИИ ОПТИМАЛЬНЫХ ФИЛЬТРОВ, ПРИМЕНЯЕМЫХ ПРИ ЦИФРОВОЙ ОБРАБОТКЕ СЕЙСМИЧЕСКИХ ДАННЫХ

В связи с ограничениями, характерными для разработки оптимальных фильтров (погрешности применяемых пренебрежений и оценки параметров) обосновано применять аппроксимации. Дополнительное достоинство субоптимальных фильтров заключается в возможности исключения стохастических колебаний из первоначальной переходной характеристики. В работе предлагается разработать субоптимальный фильтр с несколькими параметрами для переходной характеристики оптимального фильтра. Субоптимальный фильтр выбирается с таким расчетом, чтобы соответствующая весовая функция была получена в замкнутой, выравненной и усеченной форме.

Записывается среднеквадратичная разница между первоначальной переходной характеристикой и переходной характеристикой субоптимального фильтра. Посредством минимализации среднеквадратичного расхождения получается система нелинейных уравнений для определения параметров субоптимального фильтра. Система уравнений решается итерационным методом. По предлагаемому способу разработка фильтра расширяется итерационным алгоритмом, применяемым для согласования; отпадают выравнивание и усечение численной обратной трансформацией Фурье.

Применение метода иллюстрируется на примере разработки субоптимального выравнивающего фильтра.