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COMPUTER PROCESSING AND REPRESENTATION OF MULTI-LAYER GEOELECTRIC SOUNDING CURVES

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The advantages offered by electronic computers for computation of families of theoretical curves had been recognized in the early sixties.

The first program for computation of two-layer $G(r)$ and $H(r)$ curves was set up for an IBM-628 computer. The external programming, small capacity and restricted number of operations of this machine had set limits to the preparation of any more general program.

The Polish computer UMC-1 could already tackle three-layer problems. The routine processing, however, due to its low operation speed was not economical.

In the meantime requirements were raised for computation of four-, or more than four-layer curve families. The facilities became more favourable as soon as the Soviet-made computer MINSK-2 started working.

The new machine required a completely new program. The idea necessarily arose: the new program for values $H(r)$ and $G(r)$ should apply to arbitrary n-s, and, on the other hand, its efficacity must be optimized — naturally within the computer's possibilities. In order to achieve this goal the program was written in machine language. The original algorithm for computing n-layer theoretical curves was prepared by Csókás (1969), certain modifications were later introduced because of computational ease.

The algorithm of the program is described in what follows.

In order to compute curves $G(r)$ and $H(r)$ one must give an algorithm for $q[n]$, figuring in the function

$$F(r) = 1 + 2\sum_{n=1}^{\infty} \frac{q[n]}{\left[1+\left(\frac{2n}{r}\right)^2\right]^{\alpha}}.$$

The computation of $q[n]$ depends on the number of layers, their relative resistivity and thickness according to the following recursive algorithm:

$$q[n] := 0 \text{ if } n \leq 0;$$

the value of $q[n]$ can be determined when all terms with smaller indices are known.

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Let the number of interfaces be denoted by N, the depths by $h(i)$ and the corresponding constants of resistivity by $k(i)$, where $1 \leq i \leq N-1$; $h(i)$ an integer: $k_i = \frac{\varrho_{i+1}-\varrho_i}{\varrho_{i+1}+\varrho_i}$, ϱ_i being the relative value of the electric resistivity of the i-th layer.

Let us arrange the pairs $h(i)$, $k(i)$ in a decreasing order with respect to the $k(i)$-s. Let us take further all binary numbers consisting of $N-1$ digits. Each binary number m will furnish a conditional term $F(m)$ and an absolute term $P(m)$. One has the following recursive formula for $q[n]$:

$$q[n] = \sum_{m=1}^{2^{N-1}} \{F(m) + P(m)\}.$$

The terms $F(m)$ and $P(m)$ are constructed as follows. We assign to each of the $N-1$ bits of the binary number m one of the pairs $h(i)$, $k(i)$ according to the order defined above. Now, we select only those values $h[i(m)]$, $k[i(m)]$ which correspond to 1-s in the binary representation of m and we form the product of the $k[i(m)]$ values selected; denote it by $K(m)$.

We compute $H(m)$, i.e. the sum with alternating signs of the $h[i(m)]$ values chosen, taking the greatest of them as positive. For example if m is the decimal number 11, its binary equivalent is 1011, i.e. in this case

$$K(m) = k_4\, k_2\, k_1$$

$$H(m) = h_4 - h_2 + h_1.$$

Having obtained $K(m)$ and $H(m)$ we count the number of ones in the binary form of m (in the above example this is 3) and check whether it is even or odd.

If even, then

$$F(m) = 0$$

$$P(m) = -K(m) \cdot q[N - H(m)].$$

If odd, then

$$F(m) = K(m), \text{ if } N = H(m)$$

$$P(m) = K(m) \cdot q[N - H(m)].$$

Care is taken by the program that the computation of the conditional term $F(m)$ for $n > h_{max}$ should cause no extra-work and that also the absolute terms be optimally computed.

The program has found practical applications for the computation of 3- and 4 layer curves and, in a few cases, also for 5- and 6 layer curves.

A further significant saving in machine time can be gained if one rearranges the infinite series in question.

Indeed, if in the expression $\varphi(x) = (1+x)^{-\alpha}$ one has $x \ll 1$, then the binomial series

$$\varphi(x) = \sum_{i=0}^{\infty} \binom{-\alpha}{i} x^i$$

has a rapid convergence. In our case the series

$$\varrho(r) = \varrho_1 \left(1 + 2 \sum_{n=1}^{\infty} \frac{q[n]}{\left(1 + \left(\frac{2n}{r}\right)^2\right)^{3/2}} \right)$$

can be re-written as

$$\varrho(r) = \varrho_1 \left(1 + \frac{r^3}{4} \sum_{n=1}^{\infty} \frac{q[n]}{n^3 \left(1 + \left(\frac{r}{2n} \right)^2 \right)^{3/2}} \right).$$

For a sufficiently large n, say $n = N$, we may suppose that $\frac{r}{2n} \ll 1$. In this case

$$\varrho(r) = \varrho_1 \left\{ 1 + \frac{r^3}{4} \left[\sum_{n=1}^{N} \frac{q[n]}{n^3 \left(1 + \left(\frac{r}{2n} \right)^2 \right)^{3/2}} + \right. \right.$$

$$\left. \left. + \sum_{n=N+1}^{\infty} \frac{q[n]}{n^3 \left(1 + \left(\frac{r}{2n} \right)^2 \right)^{3/2}} \right] \right\},$$

where the terms within the brackets may be further modified, making use of the general binomial relation mentioned above:

$$\sum_{n=N+1}^{\infty} \frac{q[n]}{n^3 \left(1 + \left(\frac{r}{2n} \right)^2 \right)^{3/2}} = \sum_{n=N+1}^{\infty} \frac{q[n]}{n^3} \sum_{i=0}^{\infty} \binom{-\frac{3}{2}}{i} \left(\frac{r}{2n} \right)^{2i}.$$

Changing the order of summations

$$\sum_{n=N+1}^{\infty} \frac{q[n]}{n^3 \left(1 + \left(\frac{r}{2n} \right)^2 \right)^{3/2}} = \sum_{i=1}^{\infty} \binom{-\frac{3}{2}}{i} \left(\frac{r}{2} \right)^{2i} \sum_{n=N+1}^{\infty} \frac{q[n]}{n^3 + 2i},$$

where the term

$$\sum_{n=N+1}^{\infty} \frac{q[n]}{n^3 + 2i}$$

is already independent of r, and for a given theoretical curve and for different values i can be computed in advance and stored in the memory for further applications. Since the development in general binomial series is applied only if $\frac{r}{2n} \ll 1$ holds, the convergence of the series is assured and is sufficiently rapid. The sum

$$\sum_{n=N+1}^{\infty} \frac{q[n]}{n^3 + 2i}$$

is to be determined only for a few values of i. For example, if $\frac{r}{2n} \leq 0.5$, then it is enough to go in the development of the series until $i = 7$, since in this case, the error is already of the order of 10^{-5}.

The computational method described makes it possible to avoid the repeated calculation of the $q[n]$-s and reduces the number of the time-consuming extraction of square roots.

The coordinates of the $\varrho(r)$ curves computed in this way must be prepared for automatic plotting. Curves, and families of curves, are plotted with a ZUSE Graphomat, in a form ready for printing, as a rule.

When writing the programs for automatic plotting we were faced with the following problem: the coordinates calculated furnished certain discrete points of the curves only, while field-work required completely continuous curves. The densification of the points is obviously not economical. On the other hand, the approximation of the curves by line segments through the points given, proved to be inaccurate. The continuity of the curves can only be assured by using a suitable polynomial interpolation relying on many points.

In the actual program third-order interpolating polynomials were used where, as a subcondition, the continuous matching of derivatives in the points of change had also been prescribed. The third-order polynomial sections obtained in this way possess the same tangent at the borders of sections, the curve used for interpolation is everywhere continuous and in each point differentiable. For practical purposes the curves obtained are of a continuous shape and approximate well enough the real situation. This piece-wise polynomial curve is then represented by the plotter by small line-segments so that the final result has the appearance of a continuous curve. The plotting of coordinate axes and scale is also incorporated in the program. The parameters of curves and other alphanumerical data can be, however, more conveniently displayed on the curves by a photo-compositor.

The complete processing is automatic: the computer calculates curve families, they are then represented by the graphomat (and the photo-compositor) with $\pm$ 0.1 mm accuracy in a form ready for printing.

Finally we show three illustrative examples, prepared for routine field-work by Mrs. E. Verő at the Geoelectric Mathematical Group.

Figure 1 displays families of three-layer curves of Q type ($\varrho_1 > \varrho_2 > \varrho_3$). Figure 2

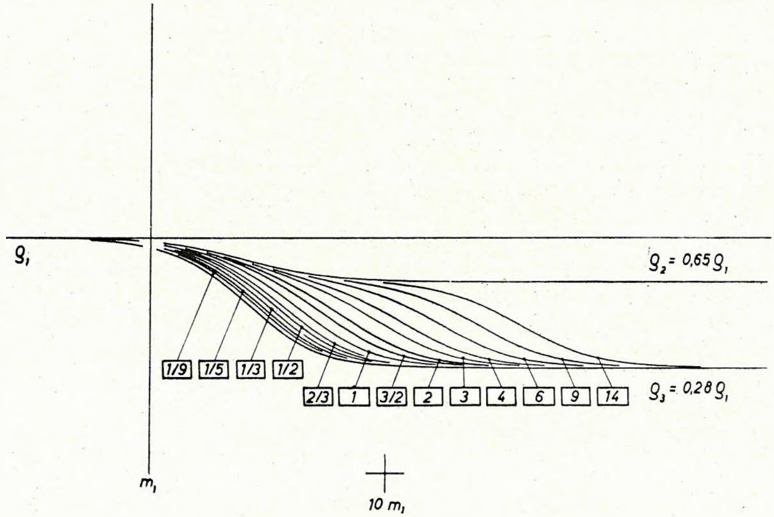

Fig. 1 Family of Q-type 3-layer curves
1. ábra Q típusú háromrétegeg görbesereg
Рис. 1. Семейство трехслойных кривых типа Q

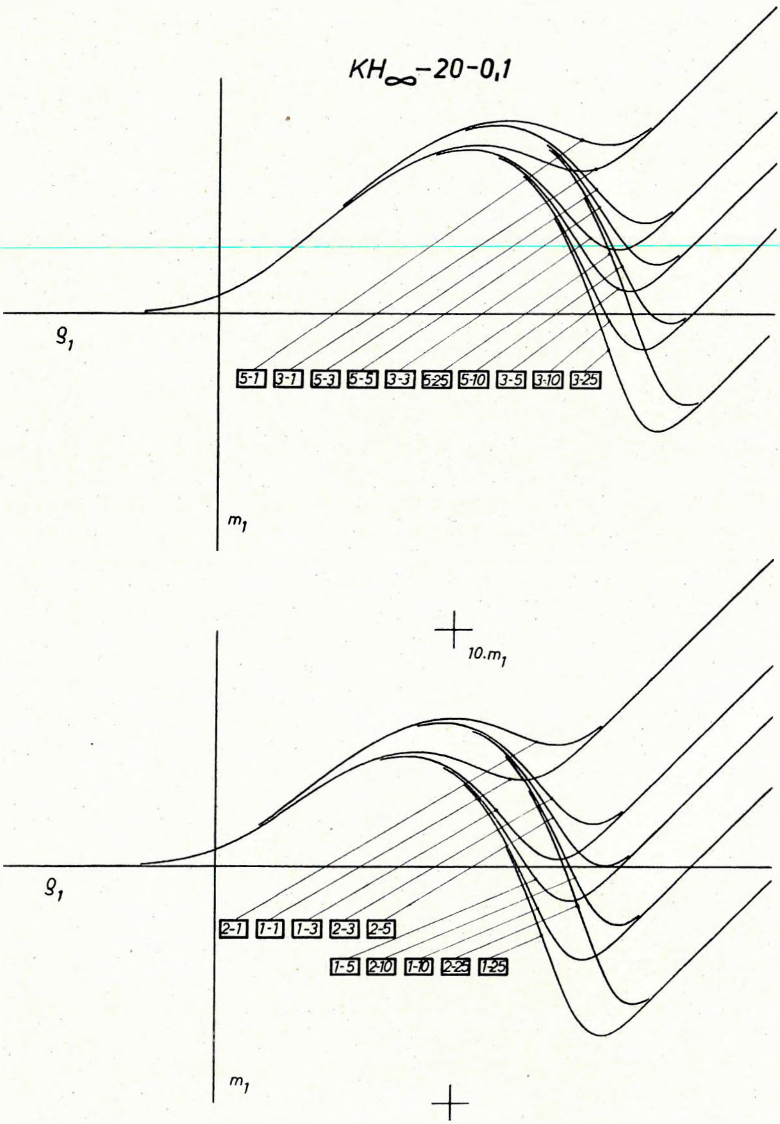

Fig. 2 Family of KH_∞ type 4-layer curves
2. ábra KH_∞ típusú négyréteges görbesereg
Рис. 2. Семейство четырехслойных кривых типа KH_∞

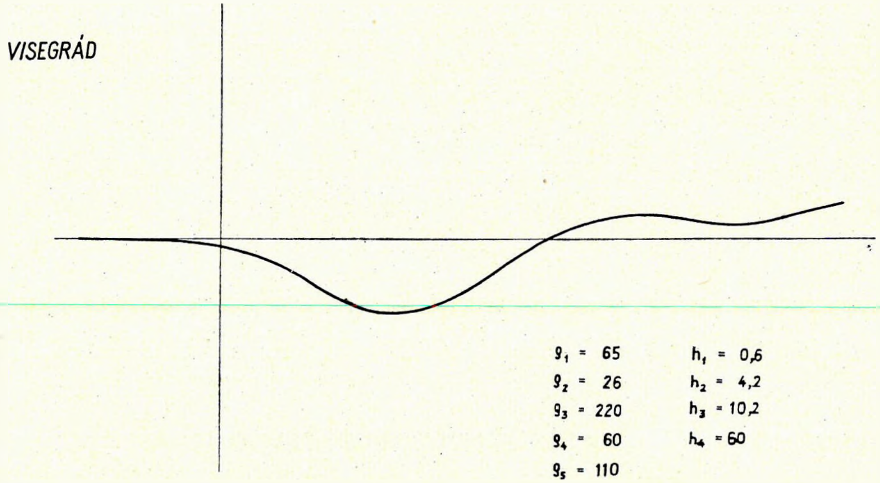

Fig. 3 5-layer sounding curves of the HKH type for direct interpretation of field-work

3. ábra Terepi kutatás közvetlen kiértékeléséhez számított *HKH* típusú ötréteges szondázási görbe

Рис. 3. Пятислойные кривые ВЭЗ типа *НКН*, вычисленные для непосредственной интерпретации данных полевых работ

gives curve families of the KH type ($\varrho_1 < \varrho_2 < \varrho_3 < \varrho_4$) for two variations (curve parameters: m_2 resp. m_2 and m_3; display: on commercial log-log paper of 6,25 mm mod).

Figure 3 is of special interest since it was used by the Hungarian party at the field-demonstration of the Hydrogeophysical Conference held by the UNESCO in Hungary, 1969 September. In Visegrád the subsurface geoelectric model was very complicated. Five-layered curves were prepared by computer in many variations and the curve illustrated by Fig. 3 gave the best approximation to the measured one. The participants of this demonstration were much impressed by an instant machine interpretation of the measurements. The results obtained were soon, actually within a couple of hours, verified by drilling.

REFERENCE

CSÓKÁS J., 1969: Use of computers in the development of the theory of geoelectrical sounding curves. Acta Geodaet., Geophys. et Montanist. Acad. Sci. Hung. 4., 1—2.

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SOKRÉTEGES ELEKTROMOS SZONDÁZÁSI GÖRBÉK GÉPI SZÁMÍTÁSA

A geoelektromos vertikális szondázási görbék pontjainak a számítása időigényes munka. A végtelen sor alakja, amely az elméleti függvényértékeket szolgáltatja, a rétegek számától és azok vastagságától függ.

Általános program készült az elméleti görbék pontjainak számítására, amely tetszőleges rétegszám és vastagságviszonyok között alkalmazható a gyakorlati követelmények határai közt. A gépi számítással nyert pontokat a ZUSE Graphomat ábrázolja.

A dolgozat részletesen ismerteti az elméleti görbék számítására talált rekurzív algoritmust és a számítás gyorsítására felhasznált módszereket.

A görbeseregek ábrázolásánál problémát jelentett, hogy a program csak bizonyos diszkrét pontokban adja meg a görbék koordinátáit, míg a terepi kiértékelés csak *teljesen folyamatos* görbéket tud használni. A számított pontok sűrítése gazdaságtalan, ehelyett egy különleges interpolációt használ a program. Az interpoláció harmadfokú parabolákkal történik, és a váltási pontokban a deriváltak folytonosságát is biztosítja.

A bemutatott ábrák között *Q* és *KH* típusú görbeseregek, valamint az UNESCO által 1969 szeptemberében megrendezett Hidrogeofizikai Konferencia terepi bemutatóján felhasznált görbék láthatók.

Л. ЗИЛАХИ-Ш., И. КЁРЁШ

ВЫЧИСЛЕНИЕ МНОГОСЛОЙНЫХ КРИВЫХ ЭЛЕКТРИЧЕСКОГО ЗОНДИРОВАНИЯ НА ЭВМ

Вычисление точек теоретических кривых вертикального электрического зондирования является трудоемкой работой. Форма бесконечного ряда, дающая значения теоретической функции, зависит от числа слоев и их мощности.

Разработана общая программа для вычисления точек теоретических кривых, которая может применяться для любого числа слоев и любых условий мощности в пределах практических требований. Точки, полученные в результате вычисления на ЭВМ, изображаются графопостроителем типа ZUSE Graphomat.

В работе подробно излагаются рекурзивный алгоритм, созданный для вычисления теоретических кривых, и способы, использованные для ускорения вычислений.

При изображении семейств кривых одна из проблем заключилась в том, что программа дает координаты кривых только в определенных дискретных точках, а полевая интерпретация требует *полностью непрерывных* кривых. Уплотнение расчетных точек является неэкономичным, вместо этого программа применяет специальную интерполяцию. Интерполяция осуществляется параболами третьей степени и обеспечивает также непрерывность производных в точках перегиба.

На прилагаемых рисунках показаны семейства кривых типов *Q* и *KH*, а также кривые, используемые при полевой демонстрации в связи с Гидрогеофизической конференцией, организованной ЮНЕСКО в сентябре 1969 г.