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SOME RECENT GRAVIMETRIC STUDIES ON THE ISOSTASY AND THE THICKNESS OF THE EARTH'S CRUST

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The gravimetric method was not born yesterday. In fact, it has already been used for almost half a century to explore new oil fields and other valuable deposits of the earth's crust, to study the fine structure and regional formations of the earth's interior, to investigate the isostatic equilibrium of the earth's crust, and to determine the figure of the earth, or the geoid. The first problem belongs to the oil- and mining industry, the second and third to the geophysics, the fourth to geodesy. Therefore, we speak about exploration, geophysical and geodetic applications of the gravimetric method. The last one is also called the physical geodesy.

The gravimetric method measures only the size and direction of the gravity, tries to find the size and the location of the disturbing mass layers — geophysical gravimetry — and the effect of these disturbing masses on the figure of the earth — geodetic gravimetry. The gravity, g, at any point of the earth's surface is the sum of the attraction of the earth's mass and of the centrifugal force caused by the diurnal rotation of the earth around its axis. The rotation itself, however, brings about also the flattening, α, of the earth. The gravity g_{90} at the pole, is 5,3 mgal larger than the gravity g_0 at the equator, and the polar radius of the earth is about 21,5 km shorter than the equatorial radius. The only quantities which we use in the gravimetry are the gravity anomalies Δg.

As we know, the gravity anomaly Δg is the difference between the observed — and to sea level reduced — gravity g_0 and the theoretical gravity γ obtained from the gravity formula; $\Delta g = g_0 - \gamma$. It is good to keep in mind that g_0 refers to the geoid surface, γ to the spheroid surface of the more or less smooth earth. We have several spheroid surfaces depending on what formula we use for the increasing of the density of the earth with the depth and for some other characteristic parameters of the earth. All of these spheroids differ from one another very little. The International gravity formula has been derived so that the corresponding spheroid is also ellipsoid. In general, the distances N between the spheroid and ellipsoid are rather small — seldom exceeding $\pm$ 50 m.

In fact, it is strange that we compare two gravity values with one another, which refer to different surfaces. The important Bruns term will make this comparison consistent.

The famous formula of Helmert (4), derived already during the last part of the 19[th] century

$$D = 2\left\{\frac{2}{3}\frac{R}{\gamma}\,\Delta g + N\right\}$$

gives the connection between the gravity anomalies Δg, the geoid distances N, and the thickness D of the disturbing layer condensed to the sea level (Ideelle störende Schicht) so that its density is half of the mean density ϱ_m of the earth. So if we like to compute the thickness D of the disturbing layer at any point, we have to know not only Δg but the undulation N of the geoid as well.

The gravity anomalies depend mostly on the density differences of the earth's crust, on the reduction method used to reduce the gravity to sea level, but partly also on the theoretical gravity γ. If, for instance, γ is 5 mgal too small, the Δg is 5 mgal too large and vice versa.

The theory of the rotating earth, as well as the gravity observations carried out at different latitudes, show that the normal gravity reduced to sea level increases with the square of sine of the latitude φ. If we also consider the small longitude term of the gravity, the gravity formula is this:

$$\gamma = \gamma_E[1 + \beta \sin^2 \varphi - \varepsilon \sin^2 2\varphi + r \cos^2 \varphi \cos 2(\lambda - \lambda_0)],$$

in which β is the coefficient of the important $\sin^2 \varphi$-term, E is a small term computed theoretically and r the amplitude of the longitude term and λ_0 the direction of the long equatorial axis. During the last years there has been quite a bit of discussion concerning the longitude term.

Now when huge gravity material exists from different parts of the world and when we can partly use also the satellite geodesy to determine the shape of the earth, we can derive also higher order harmonics to the expansion of the gravity anomalies and of the geoid undulations N.

I give here some gravity formulas—without longitude term as well as with it.

Some Gravity Formulas

Helmert, 1901, $\gamma = 978{,}030\ [1 + 0{,}005302 \sin^2 \varphi - 0{,}000007 \sin^2 2\varphi]$; $\alpha = 1:298{,}2$

Heiskanen, 1928, $\gamma = 978{,}049\ [1 + 0{,}005289 \sin^2 \varphi - 0{,}000007 \sin^2 2\varphi]$; $\alpha = 1:297{,}06$

International, 1930, $\gamma = 978{,}049\ [1 + 0{,}0052884 \sin^2 \varphi - 0{,}0000059 \sin^2 2\varphi]$; $\alpha = 1:297{,}0$

Heiskanen, 1938, $\gamma = 978{,}045\ [1 + 0{,}0050326 \sin^2 \varphi - 0{,}0000059 \sin^2 2\varphi]$; $\alpha = 1:298{,}2$

Heiskanen, 1928, $\gamma = 978{,}049\ [1 + 0{,}005293 \sin^2 \varphi - 0{,}0000070 \sin^2 2\varphi + 0{,}000019 \cos^2 \varphi \cos 2(\lambda - 0^\circ)]$; $\alpha = 1:297{,}0$

Niskanen, 1945, $\gamma = 978{,}0468\ [1 + 0{,}0052978\ \sin^2 2\varphi - 0{,}0000059\ \sin^2 2\varphi + 0{,}0000230\ \cos^2 \varphi\ \cos 2(\lambda + 4^\circ)]$; $\alpha = 1 : 297{,}8$

The International gravity formula, accepted in 1930 (1), is, in fact international. The last term is derived by E. Wiechert (a German), by H. A. Darwin (an Englishman), and by G. Cassinis (an Italian); the second term by J. F. Hayford (an American); and the first term by me (Finn).

The gravimetric studies of last years and the satellite geodesy have given for the equatorial gravity γ_E of the gravity formula about 978,042 cm/sec², for the coefficient $\beta = 0{,}005302$, and correspondingly for the flattening value $\alpha = 1 : 298{,}2$, as obtained already by Helmert.

Isostasy

We know that there exist quite a few methods to reduce the gravity to sea surface. We know also that mostly used methods are the free air reduction with the condensation method of Helmert, the Bouguer reduction, and the isostatic reductions. The free air reduction is easiest to compute, but will not give representative gravity values.The Bouguer reduction changes the geoid too much — even hundreds of meters — to suit the geodetic purposes. So it is best to use the isostatic reduction which gives representative gravity anomalies and changes the geoid only some few meters.

The gravimetric and the seismic studies of last decades have shown quite clearly that the isostatic equilibrium prevails in broad lines. Also it is almost sure that the floating type of the equilibrium, presented first by G. B. Airy more than one hundred years ago, seems to correspond to the real facts in the earth's interior. Figure 1. shows schematically the meaning of the isostatic equilibrium. The mountains are not absolute mass surplus areas, but they will, so to say, float in the heavier underlayer of the crust. They have roots similar as an iceberg has in the ocean, only located deeper. The thickness of the *root formation* is a linear function of the mountain elevation and inversely proportional to the density differences $\Delta\varrho$ between the underlayer and the crust. This root of light crustal material compensates the effect of the mass surplus of the mountains. In the ocean areas the equilibrium is brought about by the *anti-root* of the ocean basin. The mass deficiency of the ocean will be compensated by the heavy antiroot. Again the thickness of the anti-root is linear function of the depths of the ocean and is inversely proportional to the density difference $\Delta\varrho$. The equilibrium prevails, i. l., the surface unit, if it is not too small, will be under the same pressure regardless of whether it is under the mountains, under the level land, or under the oceans.

Since I have realized that there exists now and then confusion concerning the real meaning of the free air, Bouguer, and isostatic reductions, I try to explain them.

In the free air reduction we simply forget the mass between the sea level and the physical earth's surface. In other words, we think the mountain mass to be compensated at the sea level; consequently, the free air reduction is a modification of an isostatic reduction. It corresponds to the thickness of the earth's crust, T = 0. In the Bouguer reduction we think the mountain mass to be absolute mass surplus and simply substract the effect of it from the observed gravity. If the topography is irregular, we have still to consider the effect of it; or the terrain corrections (Geländereduktion). I particularly emphasize the significance of the terrain correction. Fortunately, it is almost negligible except in the rugged mountains, where it is always positive regardless of whether the station is at the mountain top or in the valley, and can be +20, +30, in some cases even more than +50 mgal; at Mont Blanc not less than +123 mgal. Consequently, the Bouguer anomalies in mountain areas without the terrain correction have not much practical value. They only mislead both the geodesists and the geophysicists.

When we add to the observed gravity the positive free air reduction and the negative Bouguer plate reduction with the positive terrain correction we get the Bouguer anomalies which should be close to zero. Unfortunately it is not so. The Bouguer anomalies in the mountains are the more negative the higher the mountain and at the oceans the more positive the deeper the ocean, they are, in a way, the exaggerated picture of the topography. This fact is brought about by the isostatic equilibrium.

Therefore, we most consider also the effect of this compensation, and that happens in the isostatic reduction. In the Bouguer reduction we, so to say, "carry" the mountain mass into infinity, which is not right. Therefore in the isostatic reduction we carry the same mass from infinity to the root formation of the mountain. In the ocean areas we in Bouguer reduction fill the ocean by a mass with the density of ($\varrho = 1{,}03$), which again is wrong —1,03 is the density of the ocean water. Instead, to carry the mass to infinity we simply take mass surplus of the antiroot and transfer it to the ocean basin. This is the meaning of the isostatic reduction.

If the rigidity of the earth were infinite, then the earth's crust would not yield at all and Bouguer anomalies would be close to zero. But since the earth's crust can yield, the root- and anti-root formations are possible, the isostatic equilibrium prevails and the isostatic anomalies are close to zero.

It has been frequently claimed that because the free air anomalies are, not only in the continents but also at the oceans, rather small, the isostatic equilibrium will not prevail. In fact, however, the small free air anomalies are good evidence of the isostatic equilibrium, because, as I already mentioned, the free air reduction is an isostatic reduction, with the depth of compensation zero. The only drawback of this reduction is that we condense the topographic masses to a wrong level. In the proper isostatic reductions we transfer the topographic masses to the existing root formation where they have, so to say, a ready made „bed" to lay down.

The scientists who for some reasons do not like the isostasy, claim that this reduction is unnecessary because the average value of the free air anomalies and the isostatic anomalies is almost the same. They have, however, forgotten the important fact that all kinds of gravity anomalies are in flat lands, and even on high plateaus, almost the same. The attraction effect of an infinite slab of density ϱ and elevation h, depends only on the product $\varrho \cdot h$, but not on each of them separately. So we can in case of the infinite broad plateau condense the topographic masses either to the sea level — free air reduction — to a root formation — the isostatic reduction — or distribute them in the infinitely thick layer — Bouguer reduction.

If we try without any prejudices to study the isostasy, we have to use the gravity material of the rugged mountains and ocean coastal

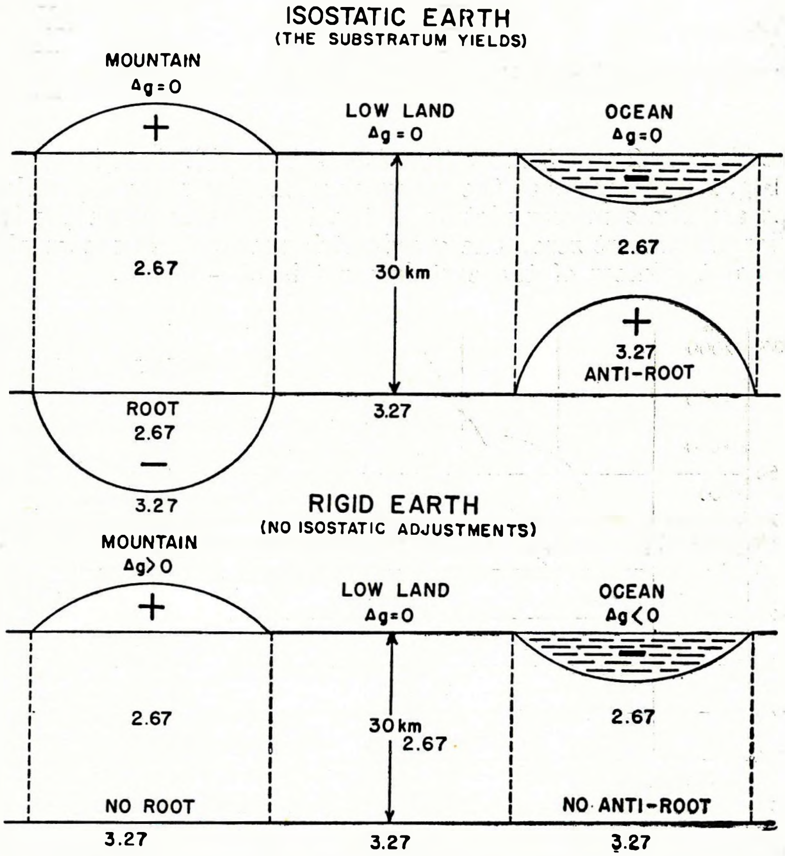

Fig. 1. Difference between the isostatic and the rigid earth. Figure shows that the mountain roots of light material and the heavy antiroots of the oceans compensate the effect of the topographic masses. The isostatic gravity anomalies are everywhere relatively small. In the rigid earth the gravity anomalies would be strongly positive in the mountain areas and strongly negative in the ocean areas.

areas, or, in general, of the regions where the topography is as rugged as possible. There exists hardly any larger mountain or ocean area where the Bouguer anomalies are closer to zero than the isostatic anomalies. Table I shows the average Bouguer and isostatic anomalies, T = 30 km — International gravity formula used — in some mountainous and ocean regions.

Region	Mean Bouguer Anomaly	Mean Isostatic Anomaly T = 30 km
USA	—108 mgal	—8 mgal
Canada	—135 mgal	+5 mgal
India	—115 gmal	+7 mgal
East Africa	—146 mgal	—10 mgal
The Alps	—115 mgal	—7 mgal
The Oceans	about +68 × h mgal (h, depth in unit 1 km)	—5 mgal

The Figures 2 and 3 show that isostatic equilibrium in broad lines exists. Fig. 2 indicates that the Bouguer anomalies along a profile across the Alps are like a mirror picture of the topography while the isostatic anomalies are almost zero. The gravimetric method has shown also that the normal thickness of the earth's crust is 30—35 km.

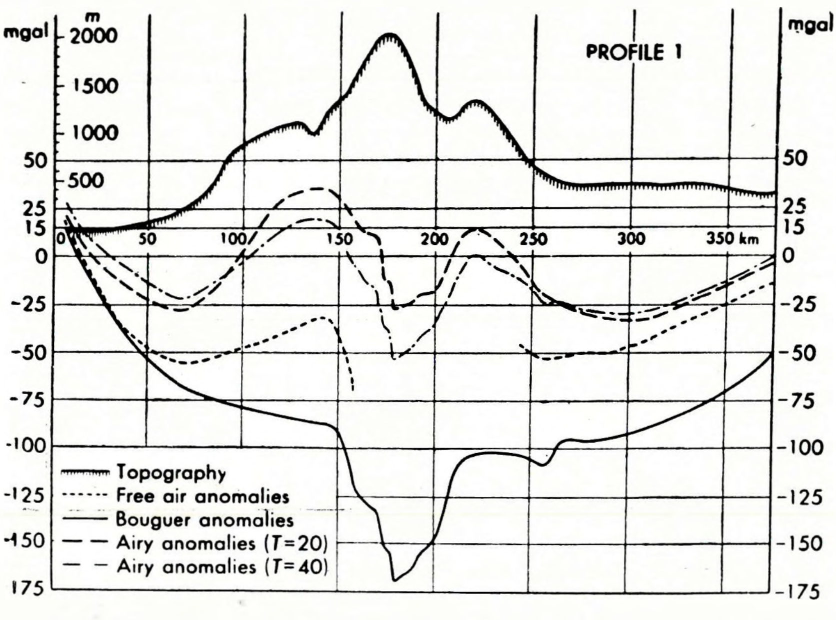

Fig. 2. Profile of Perano—Landau across the Alps shows that the Bouguer anomalies are mirrored pictures of the topography while isostatic anomalies are much less. (Publ. Isos. Inst., IAG, No. 16, 1947.)

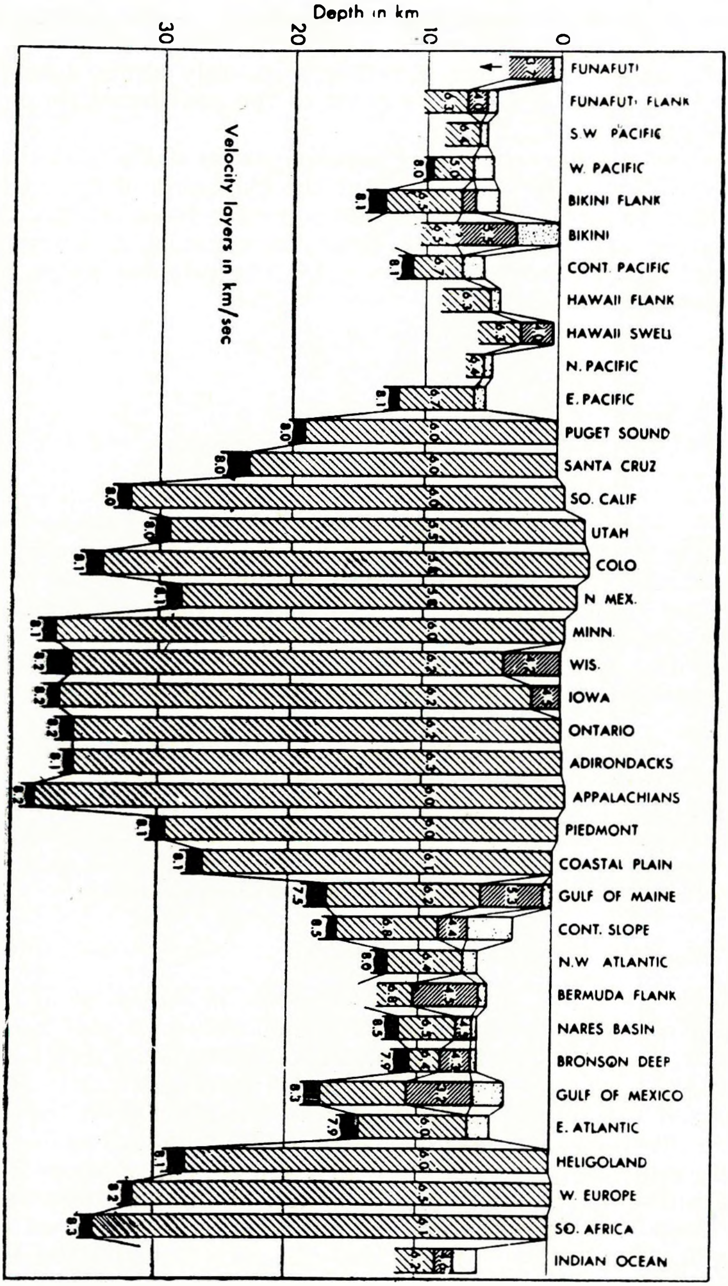

Fig. 3. Seismological evidence for the Airy–Heiskanen isostatic theory. According to the material analyzed by Woollard, the earth's is under Europe and North America about 25–38 km thick, but under the Atlantic and Pacific only about 4–8 km thick (3, p. 208).

Figure 3 gives — according to Woollard — the seismic evidence from around the world. We realize easily that the earth's crust is under the Atlantic and Pacific Oceans rather thin, only about 4 to 7 km (7), while the thickness of the earth's crust of the continents for zero elevation — is between 25—30 km.

These and innumerable other similar graphs show that the floating isostatic equilibrium prevails and that the thickness of the earth's crust corresponding to zero elevation is not very far from 30 km. As to the root formation and the antiroots, their thickness is of course inversely proportional to the density difference $\Delta\varrho$. The total thickness, T_C, under the continents is

$$T_C = T + \frac{\varrho}{\Delta\varrho} h + h$$

and the thickness T_0 under the oceans

$$T_0 = T - \left(\frac{\varrho}{\Delta\varrho} h' + h'\right)$$

where ϱ is the density of the earth's crust; $\Delta\varrho$ the compensation density; h the elevation of the mountain; and h' the depths of the ocean. For the value $\Delta\varrho = 0{,}6$ the thickness of the root is 4,5 h km, and the thickness of the anti-root is 2,73 h, where h is in the first case the elevation of the mountain and in the second case the depths of the ocean. If the density difference is 0,3, the corresponding thicknesses are 9,0 h and 5,5 h km.

In order not to be misunderstood I would like to emphasize that although the isostasy is a proven fact, the thickness of the earth's crust can and will be different in different parts of the world, depending on the geological structure, tectonic phenomena, and other factors which in one way or another are working against the general trend of the earth's interior to reach the isostatic equilibrium.

The Big Isostatic Experiment of Nature

Perhaps the most striking phenomena in favor of the isostatic equilibrium are the big experiments which nature is just now making before the eyes of the scientists. I mean the post-glacial uplift of Fennoscandia and the ice caps of Greenland and Antarctic which in many cases are immersed even more than thousand meters below the sea level.

As we all know, the land uplift of Fennoscandia has been studied extensively geologically and geodetically. The ice cap of about 2500 meter thickness pushed the earth's crust about 700 meters downward, obviously so deep that the isostatic equilibrium in broad lines prevailed. When the ice cap began to melt, the load decreased and the land started gradually to uplift, in the beginning slowly, but at the time when the whole ice cap was melted, very fast, even about 13 cm/100 years. With the postglacial centuries the uplift, of course, slowed down.

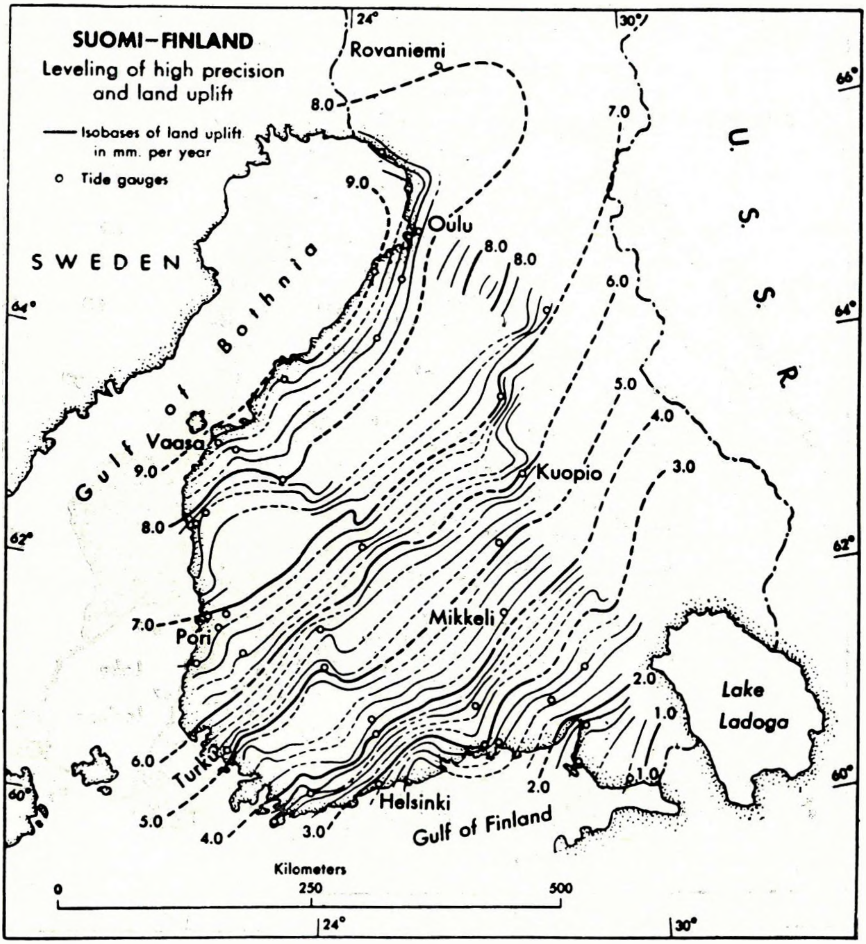

Fig. 4. Land uplift in Finland according to E. Kääriäinen. The upheaval is at the end of the Gulf of Finland almost zero but in the area of Gulf of Bothnia about 9 cm/100 years. Result obtained by two precise levellings at the interval of about 50 years.

At the end of the glacial period the land had uplifted already about 250 m.

During the postglacial period this uplift has continued so that now the maximum postglacial uplift is about 270 m. When we add to this the 250 m uplift occuring before the end of the glacial period, the whole amount is about 520 m. Fig. 4 shows the curves of equal speed of the uplift in Finland according to the results of two precise levellings carried out in Finland 1892—1910 and 1935—1955 (5). We see that the zero line of the upheaval is close to Leningrad area, from where it increases

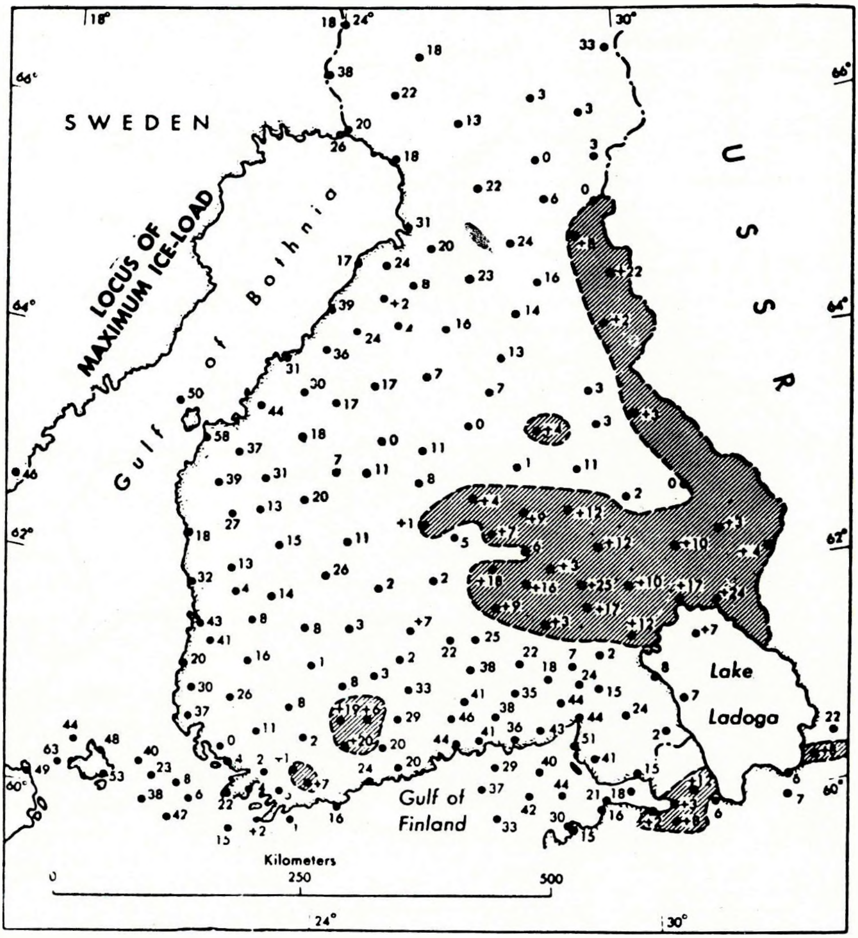

Fig. 5. Correlation between the land uplift and the gravity anomalies (according to R. A. Daly). The more negative the gravity anomalies, the larger the land uplift.

almost linearly in northwest direction reaching the maximum more than 90 cm/100 years in Gulf of Bothnia and at the east coast of Sweden.

Fig. 5 shows that the gravity anomalies are in land uplift area systematically negative. This means that the isostatic equilibrium does not yet completely prevail. On basis of the negative gravity anomalies, E. Niskanen (6) has computed that the earth's crust has to uplift about 200 meters until the complete isostatic equilibrium prevails. The resistance of the earth's crust is, however, so great that obviously complete isostatic equilibrium will never be reached.

One of the most interesting results of the studies of the IGY was the seismic discovery that in a large part of the Antarctic the ice cap reach deep under the present ocean level. The under boundary of the ice cap is like a shallow bowl. In some cases the depths of the bowl is even of the order of 1500 m. Similar behavior of the ice cap has been found earlier in Greenland. What does this mean? One can hardly imagine that the ice cap is „born" in the ocean. There must have been a continent on which the ice began to grow. With the extra load of the ice cap the earth's crust began to sink. Obviously the glacial period there has lasted so long time that the earth's crust has reached almost complete isostatic equilibrium.

So we have before our eyes two different phases of the isostatic experiment of nature. In Fennoscandia the experiment is almost in the final phase. The time elapsed from the glacial period is so long that the essential part of the land uplift has already occurred. In Antarctic and Greenland is the second phase happening just now. The ice cap has been there sufficiently long time and has pushed the earth's crust down so that the bowl-like under boundary of the ice cap was the result.

The continuous observations from decade to decade perhaps can discover whether these continents are in the static phase or whether also there the ice sheet is becoming thinner and the land is uplifting. Last alternative almost surely happens.

Before I finish I mention the interesting experiments that they have planned in America to *drill through the earth's crust* to the M-discontinuity. The drilling of course will be made in the parts of the ocean, like in the West Indies and close to the west coast of Central America where the M-discontinuity is only about 4 km under the sea bottom. We do, of course, not know when this drilling will be done, but we know that the significance of it will be enormous. It will be the first time when we empirically can measure the thickness of the earth's crust, get the crust samples and the density values from different depths. Also it is possible to check how suddenly the density at the M-discontinuity changes from, say 2,8 to 3,2. Needless to say, all geophysicists and geodesists are waiting impatiently this new information.

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