Discrete Kalman Filter Invariant to Perturbations

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Abstract: Fault detection problems in dynamic objects and their localization are a very critical and rather challenging tasks for many practical applications. The Kalman-filter technology is used for these purposes most often. The correct operation indicator of the specified filter is the innovation process to be represented as a normal uncorrelated stochastic process with zero mean value and a priori calculated covariation matrix, except the specified conditions, are violated in case of unforeseen perturbations. The aim of the presented work is to develop a method allowing to restore the normal performance of the Kalman filter in the presence of uncertain disturbances. This aim is attained by applying a special one-to-one transformation of the output equation of the testing system, as a result of it, the disturbance component is modified by the extrapolation equation of the state vector dynamic system. This feature will be used in the sequel when modified Kalman filter is applied to the transformed system. The properties of the obtained filter concerning the stability of estimation errors, their convergence, and optimality are discussed. The efficiency of the method has been verified by the method of statistical modeling on a test example of a third-order dynamic system.

Keywords: states estimation; linear dynamic systems; Kalman filter; uncertain structure perturbations

1 Introduction

The fault detection problems in dynamic objects and their localization are relevant and rather difficult tasks for many practical applications [1-9]. IFAC SAFEPROCESS (Symposium on Fault Detection, Supervision and Safety for Technical Processes) defines a fault, at least, as the inadmissible deviation of one feature or parameter from its rated value to have been regulated by the standard norms [10-11]. A performance impairment can happen in separate modules of a control subject, in the regulator subsystems, switching equipment, or in observations channels, etc. The FDIR system (Fault Detection, Isolation, and Reconfiguration) is defined as a design strategy of control systems to be capable to ensure continuously functional safety or operating capacity of a control subject at beginning of a fault by its timely detection and isolation (FDI) with a possibility of the subsequent reconfiguration of the control unit in response to fault influence.

Usually, problems of fault detection and their localization are solved in two stages. On the first of them, it is necessary to make the binary decision from two mutually excluding alternative hypotheses "a system it is operational" - "the system is faulty". This stage is imperative for any functional diagnostics system. At the second stage, the place of fault emergence and its possible reason is defined. This stage, as a rule, is desirable, but isn’t regulated strictly [12]. In general, this design strategy is geared to the introduction of the redundancy concept, both by a physical layer and an analytical level.

The procedure of comparison of the duplicated signals created by the various hardware is the basis for the concept of physical redundancy introduction. For example, the same signal is observed by means of several sensors operating at different modes of operation. The standard practice implementing the hardware redundancy consists in the application of cross procedures of measuring channels cross-check, difference signals forming on the basis of a parity relations method, and further processing of the received signals by the corresponding methods, for example, using of Wavelet [41] or TP [42] transformations.

Conversely, the concept of analytical redundancy actively uses a mathematical model of a system in aggregate with the special methods of estimation considering features of the FDI systems. This concept doesn’t assume installations of the additional hardware, and in this sense is preferable in comparison with the concept of introduction of hardware redundancy. Distinctly, the maximum effect can be gained by a combination of both concepts in the uniform integrated system. However, methods of introduction of analytical redundancy are more difficult as it is necessary to guarantee stability in relation to the operating noise, unknown perturbations and incomplete information about parameters of a mathematical model.

So far the problem subject of FDI can be considered almost created. It found the reflection in the conventional classification of the existing methods, the published books, and periodical review articles. For example, the methods of analytical redundancy introduction can be separated into two major sub-classes. In the first of them the methods oriented to the application of quantitative models in an explicit form and based on are used: concepts of the parity relations [13-16]; full order observers or unknown input observers [17-20]; properties of the updated process created by a Kalman filter [21-24]; procedures of joint states estimation and unknown parameters [25-26]; stochastic algorithms [27, 29]; optimization methods [28, 30]. The general property of the above-mentioned methods is the use of specified sections of the modern control theory for the purposes to form special signals in the FDI systems. The missing information, at the same time, is generated from the results of observations.

In the second case, qualitative models on the basis of artificial intelligence methods using a mathematical apparatus of fuzzy logic [32] are applied; qualitative methods; the methods using knowledge bases; linguistic methods. For the analysis of fuzzy logic methods, we will consider a problem of difference signal forming. The difference signal, even in nominal conditions, is never equal to zero in accuracy. There is a lot of reasons for that: incomplete separation, nonlinearity, perturbations, noise, etc. Therefore, the main problem becomes to make a correct decision in the conditions of inadequate or incomplete information. As opposed to classical logic, fuzzy logic allows for making justified decisions, based on fuzzy knowledge, heuristic logic, and their combinations. Conceptually, signals processing by means of fuzzy logic consists of three stages. At the first stage, the difference signals are compared by means of special membership function. In most cases it has the triangular format. At the second stage, the smaller exit from two previous is selected. At the third stage, the procedure of center balance finding or another averaging method is used. It allows to resolve uncertainties and to lead to the probable correct decision. However, in this case, the major problem preventing of perspective technology implementation in practical applications is caused by the complexity of the training process. So, for example in [33, 34] used the basic principles of fuzzy logic for the solution of a difference signal estimation problem. The procedure of the weighed summing was made use of there instead of the categorical procedure of type "yes" - "no". In this area, it is possible to find out more about the latest advances in works [35-37] and also in the recent review publications [3, 38] related to application aspects of FDI in the context of chemical and technology objects based on AI technology.

In as much as formulation of the correct mathematical model of a control system is time-consuming method and complicated problem, many attempts to construct an acceptable qualitative diagnostic model on the basis of declarative knowledge of a system, for example, the pole analyzing of the variable, trend like "increase or decrease", a variable or a constant, etc. were made in due time. These concepts are the baseline of the qualitative method, and with their help, entirely possible, to construct the diagnostic system steadily in a sense. Moreover, comprehensive diagnostics of faults demands of, as a rule, different levels of prior information beginning from quantitative, analytical, heuristic, and finishing by expert level. It can be carried out on the basis of the expert systems functional diagnostics [39, 40] by using the complex integrated solution.

The submitted paper belongs to a subclass of the functional diagnostic methods be actively using in an explicit form quantitative mathematical model of a controlled system and relies on characteristics of the innovation process created by a Kalman filter. In [43] the possibility of faults diagnostics, using well-known statistical methods of the likelihood ratio or the generalized likelihood ratio for testing of a difference signal for "whiteness", its mean square value and an error covariation matrix of prediction was attempted for the first time. A little later, in [44, 45] was offered the adaptive estimation algorithm constructed on the basis of model conditional Kalman filter bank (MMAE-a method). The difference signal characteristics of the MMAE method were in detail studied in work [46]. Applications of these methods to FDI problems in the flight control systems are known in [47, 48].

By today, in this direction, two concepts of estimation problems with present faults and perturbations were formulated. The first of them is based on the conception of state vector expansion at the cost of connecting to it the additional unknown input associated with the active faults and perturbations influence. At the same time, it is supposed that the mathematical model of an unknown input dynamics is a priori available, and the optimal solution of an estimation problem is guaranteed by an expanded Kalman filter (EKF). However, at a large number of the considered faults and perturbations, the dimensions of the filter will be much more the control system dimensions. In [49] it was offered, by the introduction of special UV-of transformation, the procedure of EKF separation into smaller dimension constituent parts working in parallel and independently. Further, the basic idea [47] was adapted to stochastic type of faults and perturbations [51, 52]. The main efforts of researchers in this direction are made for the search of the EKF approximation methods to combine acceptable estimation accuracy with the restrictions not too complicated in terms of practical applications [53, 54].

The basis of the alternative concept is the assumption of total absence of the prior information in regard to dynamic properties of unknown inputs. In [55] it was for the first time solved this problem for the purpose of the derivation of linear unbiased estimations with the minimum generalized variance by the introduction of the certain restrictions imposed on structure of the analyzed system. In [56-57] the results [55] were generalized having applied a parametrical approach to deduce of optimum estimations. Later in [59] the optimum filter with the minimum generalized variance considering a problem of degradation characteristics inherent in the filter [55] was offered. In [60, 61] solved a fault detection problem of fault detection and their localization by means of geometrical approach, creating at the same time difference signals with the directed properties.

A specific feature above the proposed solutions is the complexity of the applied mathematical apparatus connected with the use of function spaces transformations of finite dimensions. It is to a certain extent by exposing to difficulties the practicing engineers as these sections of mathematics, often, remain outside to the standard training programs of the engineering profile specialists. Therefore, it is desirable to obtain a rather simple theoretical justification of the difference signal separation from the influence of uncertain disturbances, applying at the same time a mathematical apparatus of minimum acceptable level complexity. Unlike a traditional way of the filter structure adaptation to the mathematical model structure the authors used a reduction way of the mathematical model to the equivalent form where the disturbance component is absent in an explicit form. It allowed for it to be limited to the application of the well-known (standard) form of Kalman filter guaranteeing the derivation of the estimate state convergence in more usual terms "an bounded input – an bounded exit".

The following structure of the article is assumed: in Section 2 – the problem statement is formulated in the mathematical sense; in Section 3 – discusses a one-to-one mathematical transformation of the output of the original system, designed to absorb disturbances in the output; the results of applying the Kalman method to the transformed system are discussed in Section 4 and the main properties of the offered filter in Section 5. In Section 6 the illustrative example of the third dynamic system order for the operability purpose demonstration of the offered method is given. Subsequent sections present the results of modeling, summarize the research results, and list the literature used.

2 Problem Definition

Let's consider a linear discrete stochastic system, a mathematical model that can be described in terms of state variables

$$\mathbf{s}(k+1) = \mathbf{W}(k)\mathbf{s}(k) + \mathbf{G}(k)\mathbf{u}(k) + \mathbf{D}_s(k)\mathbf{d}(k) + \mathbf{n}_s(k); \tag{1}$$

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{n}_y(k), \tag{2}$$

where $\mathbf{s}(k) \in \mathbb{R}^n$ – the current system state, $\mathbf{y}(k) \in \mathbb{R}^m$ – the output vector, $\mathbf{u}(k) \in \mathbb{R}^p$ – exactly known control influence, $\mathbf{d}(k) \in \mathbb{R}^q$ – the indefinite structure perturbation, $\mathbf{n}_s(k) \in \mathbb{R}^n$ – the noise of a state variables, $\mathbf{n}_y(k) \in \mathbb{R}^m$ – the system output noise, $\mathbf{W}(k)$, $\mathbf{G}(k)$, $\mathbf{D}_s(k)$, $\mathbf{H}(k)$ –the known system matrices of the corresponding dimensions. The initial state $\mathbf{s}(0) \in \mathbb{R}^n$ represents a Gaussian random vector with mean value $E\{\mathbf{s}(0)\}$ and a positive definite covariation matrix $\mathbf{P}(0)$. Random sequences $\mathbf{n}_s(k)$, $\mathbf{n}_y(k)$ are independent white Gaussian noise uncorrelated with $\mathbf{s}(0)$, have zero mean values and limited covariation matrixes $E\{\mathbf{n}_s(k),\ \mathbf{n}_s^T(k)\} = \mathbf{Q}(k)$ and $E\{\mathbf{n}_y(k),\ \mathbf{n}_y^T(k)\} = \mathbf{R}(k)$, respectively. The listed assumptions coincide with those that are usually accepted in the classical theory of linear filtration without taking into account the matrix $\mathbf{D}_s(k)$ which is missing there. It is in the case under consideration supposed that perturbations $\mathbf{d}(k)$ have neither the probabilistic description nor even property of limitation from above. Otherwise stated, it is absolutely indefinite function. However, for the solvability of the problem, the following additional assumptions are introduced:

– the sequence of matrixes $\mathbf{H}(k + 1)\ \mathbf{D}_s(k)$ should be limited;

– $q \le m$ i.e. the number of perturbations are no more than number of output sensors;

– for all $k \in \square_0$, the smallest singular values of the matrix product $\mathbf{H}(k+1)\,\mathbf{D}_s(k)$ not less γ, where γ – is the set of positive numbers. The last two restrictions essentially mean that the matrix product $\mathbf{H}(k+1)\,\mathbf{D}_s(k)$ has a full rank in the columns, and they are necessary for the perturbation absorption procedure. The task is to develop a simplified method for estimating the state vector $\mathbf{s}(k)$, free from the influence of disturbances $\mathbf{d}(k)$, based on the availability of observation results $\mathbf{y}(k)$, a sequence of precisely known control actions $\mathbf{u}(k)$ and system matrixes $\mathbf{W}(k)$, $\mathbf{G}(k)$, $\mathbf{D}_s(k)$, $\mathbf{H}(k)$. In the theory of optimal linear filtration, the stability of the Kalman filter is guaranteed by the introduction of assumptions about the controllability and observability of the system under study [3]. Similar conditions will be formulated for the case under consideration after the procedure for absorbing the disturbances has been carried out.

3 One-to-One Transformation System Exit

In this section, a local goal is pursued, namely, the justification of the procedure for absorption of the component $\mathbf{H}(k+1)\mathbf{D}_s(k)$ in the equation of state of the system by introducing a supplementary transformation of the output equation so that later it becomes possible to use the standard Kalman filter. It, in turn, will promote the forming of states estimation errors free from the influence of perturbations, subject to the transformed exit. For the first time, the problem of a difference signal separation from influence of unknown perturbations was considered in [55], where the structure of a linear filter was determined by the equation

$$\hat{\mathbf{s}}\left({}^{k+1}\!/_{k+1}\right) = \mathbf{W}(k)\hat{\mathbf{s}}\left({}^{k}\!/_{k}\right) + \mathbf{L}(k)\left[\mathbf{y}(k+1) - \mathbf{H}(k+1)\mathbf{W}(k)\hat{\mathbf{s}}\left({}^{k}\!/_{k}\right)\right]. \tag{3}$$

It is worthy of note that in this equation the component $\mathbf{G}(k)\mathbf{u}(k)$ was not taken into account, since it is a precisely known quantity and is insignificant for the problem of optimal linear filter synthesis. For this case, the state vector estimation is equivalent to vector error estimation of filtering. The transfer matrix of the filter $\mathbf{L}(k+1)$ was defined by minimization of an error covariation estimation matrix on condition that introduced restriction to be correct.

$$\mathbf{L}(k+1)\mathbf{H}(k+1)\mathbf{D}_s(k) - \mathbf{D}_s(k) = 0\,. \tag{4}$$

This restriction guaranteed the lack of influence of a component $\mathbf{D}_s(k)\mathbf{d}(k)$ on a state estimation error. The solution of the local optimization problem taking into account (4) led to a significant complication of the process of calculating the transfer matrix $\mathbf{L}(k+1)$ compared to the classical Kalman filter, and it was not entirely obvious how to analyze the stability of the synthesized filter. Therefore, in this article, the main attention is paid to the issue of disturbance absorption even before the filter design process and at the second stage, the modified Kalman filter option is applied to the transformed state equation (1) where the perturbation component $\mathbf{D}_s(k)\mathbf{d}(k)$ are missing.

Suppose there is some bounded matrix sequence $\mathbf{M}(k) \in \mathbb{R}^{n\times m}$ the specific type of which will be determined a little later. Then, relation (2) immediately implies

$$\mathbf{M}(k+1)\left[\mathbf{y}(k+1) - \mathbf{H}(k+1)\mathbf{s}(k+1) - \mathbf{n}_y(k+1)\right] = 0\,. \tag{5}$$

Further we will add to each party of the equation (5) the equation (1)

$$\begin{aligned} &\mathbf{s}(k+1) = \mathbf{W}(k)\mathbf{s}(k) + \mathbf{G}(k)\mathbf{u}(k) + \mathbf{D}_s(k)\mathbf{d}(k) + \mathbf{n}_s(k) + \\ &+ \mathbf{M}(k+1)\left[\mathbf{y}(k+1) - \mathbf{H}(k+1)\mathbf{s}(k+1) - \mathbf{n}_y(k+1)\right]. \end{aligned} \tag{6}$$

After reducing such terms, we get an expression in which the equation for the output of participation no longer takes:

$$\begin{aligned} &\mathbf{s}(k+1) = \mathbf{Z}(k+1)[\mathbf{W}(k+1)\mathbf{s}(k) + \mathbf{G}(k)\mathbf{u}(k) + \mathbf{D}_s(k)\mathbf{d}(k)] + \\ &+ \mathbf{M}(k+1)\mathbf{y}(k+1) + \mathbf{Z}(k+1)\mathbf{n}_s(k) - \mathbf{M}(k+1)\mathbf{n}_y(k+1), \end{aligned} \tag{7}$$

where $\mathbf{Z}(k+1) \triangleq [\mathbf{I}_n - \mathbf{M}(k+1)\mathbf{H}(k+1)]$.

If the matrix sequence $\mathbf{M}(k + 1)$ is chosen so that in each instant of k restriction $\mathbf{Z}(k + 1)\ \mathbf{D}_s(k) = 0$ is carried out, then the equation (7) will take a form:

$$\mathbf{s}(k+1) = \mathbf{W}1(k)\mathbf{s}(k) + \mathbf{G}1(k)\mathbf{u}(k) + \mathbf{M}(k+1)\mathbf{y}(k+1) + \mathbf{w}(k), \tag{8}$$

where

$\mathbf{W}1(k) \triangleq \mathbf{Z}(k+1)\mathbf{W}(k);\ \mathbf{G}1(k) \triangleq \mathbf{Z}(k+1)\mathbf{G}(k);$

$\mathbf{w}(k) \triangleq \mathbf{Z}(k+1)\mathbf{n}_s(k) - \mathbf{M}(k+1)\mathbf{n}_y(k+1)$.

In this case, the covariance matrix of the transformed state noise $\mathbf{w}(k)$ should be calculated on each computation cycle by the formula

$$\mathbf{Q}1(k) \triangleq \mathbf{Z}(k+1)\mathbf{Q}(k)\mathbf{Z}^T(k+1)^T + \mathbf{M}(k+1)\mathbf{R}(k)\mathbf{M}^T(k+1). \tag{9}$$

Turning to equation (8) it is easy to notice that now the disturbing effect $\mathbf{D}_s(k)\ \mathbf{d}(k)$ is excluded from further transformations in an explicit form. At this stage, it is important that the perturbation is absorbed at the moment $k + 1$ instead of k an instant is important. The condition that the transmission matrix of the filter under consideration must satisfy $\mathbf{M}(k + 1)$ similar to that introduced in [12], namely $\mathbf{Z}(k + 1)\mathbf{D}_s(k) = [\mathbf{I}_n - \mathbf{M}(k + 1)\mathbf{H}(k + 1)]\mathbf{D}_s(k) = 0$. However, there is a significant difference here. Expression in square brackets provides more degrees of freedom in choosing the value of the matrix transmission coefficient $\mathbf{M}(k + 1)$ since it is not related to solving the minimization problem. This matrix can be determined by solving the matrix equation $\mathbf{M}(k + 1)\mathbf{H}(k + 1)\mathbf{D}_s(k) = \mathbf{D}_s(k)$. Since it is assumed that the assumption is valid that in the matrix product $\mathbf{H}(k + 1)\mathbf{D}_s(k)$ the number of columns does not exceed the number of rows, this means that the solution for $\mathbf{M}(k + 1)$ must exist. In most practical applications, this inequality is satisfied. Taking this remark into account, we obtain [5]

$$\mathbf{M}(k+1) = \mathbf{D}_s(k)\left[\mathbf{H}(k+1)\mathbf{D}_s(k)\right]^*, \tag{10}$$

where the symbol $[\cdot]^*$ the pseudoinverse Moore-Penrose matrix is denoted [63].

4 Result of the Kalman Filter Method Application

Returning to the transformed state model (8), it is easy to see that state vectors $\mathbf{s}(k)$, an entrance $\mathbf{u}(k)$, and an exit $\mathbf{y}(k)$ remained the same. Thus, the original model (1) and the modified (8) are essentially equivalent, since they describe the same system, and the component $\mathbf{M}(k + 1)\ \mathbf{y}(k + 1)$ can be interpreted as a new known input. In this case, there are no formal obstacles to the application of the standard estimation procedure by the Kalman method, since the disturbance component $\mathbf{D}_s(k)\ \mathbf{d}(k)$ is not present. Then, the application of the classical Kalman filter to the transformed model (8)

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{n}_y(k),$$

$$\mathbf{s}(k+1) = \mathbf{W}1(k)\mathbf{s}(k) + \mathbf{G}1(k)\mathbf{u}(k) + \mathbf{M}(k+1)\mathbf{y}(k+1) + \mathbf{w}(k),$$

will generate in a recurrent form the following estimates of the states $\mathbf{s}^*(k/k)$ and their covariation matrixes $\mathbf{P}(k/k)$ for all $k \in \mathbb{N}_0$,

$$\mathbf{P}\left(k+1/k\right) = \mathbf{W}1(k)\mathbf{P}\left(k/k\right)\mathbf{W}1^T(k) + \mathbf{Q}1(k); \tag{11}$$

$$\mathbf{K}(k+1) = \mathbf{P}\left(k+1/k\right)\mathbf{H}^T(k+1)\mathbf{P}^{-1}_{\mathbf{r}}(k+1); \tag{12}$$

$$\mathbf{P}_{\mathbf{r}}(k+1) \triangleq E\left\{\mathbf{r}(k+1)\mathbf{r}^T(k+1)\right\} = \mathbf{H}(k+1)\mathbf{P}\left(k/k\right)\mathbf{H}^T(k+1) + \mathbf{R}(k+1); \tag{13}$$

$$\mathbf{P}\left(k+1/k+1\right) = \left[\mathbf{I}_n - \mathbf{K}(k+1)\mathbf{H}(k+1)\right]\mathbf{P}\left(k+1/k\right); \tag{14}$$

$$\mathbf{r}(k+1) = \mathbf{y}(k+1) - \mathbf{H}(k+1)\mathbf{s}^*\left(k+1/k\right); \tag{15}$$

$$\mathbf{s}^*\left(k+1/k\right) = \mathbf{W}1(k)\mathbf{s}^*\left(k/k\right) + \mathbf{G}1(k)\mathbf{u}(k) + \mathbf{M}(k+1)\mathbf{y}(k); \tag{16}$$

$$\mathbf{s}^*\left(k+1/k+1\right) = \mathbf{s}^*\left(k+1/k\right) + \mathbf{K}(k+1)\mathbf{r}(k+1). \tag{17}$$

$$\begin{aligned} &\mathbf{s}^*\left({}^{0}/_{0}\right) = E\left\{\mathbf{s}(0)\right\}, \\ &\mathbf{P}\left({}^{0}/_{0}\right) = \mathbf{P}(0). \end{aligned} \tag{18}$$

where matrixes $\mathbf{Z}(k + 1)$, $\mathbf{M}(k + 1)$, $\mathbf{W}1(k)$, $\mathbf{G}1(k)$ have to be previously calculated according to formulas (7), (10), (8), respectively, with initial conditions (18).

At the same time, it is necessary to emphasize that the offered filter differs from a normal Kalman filter a little. First, the distribution matrix of again entered control input is calculated (10) using a pseudoinverse concept, and secondly, more significantly, the covariation matrix of the generalized perturbation, being non-stationary, has to be updated in each computing cycle. In these repeated calculations there is the absorption perturbations procedure with indefinite structure in the implicit form. Besides, the seeming simplicity of the offered filter presents the additional problem in the form of generalized perturbation correlation components. Usually in practice, for simplicity of the estimation procedure neglect this correlation often. At the same time, the filter becomes quasi-optimal filter.

5 Analysis of the Modified Kalman Filter Properties

In order for the modified Kalman filter to be guaranteed to be stable, it is necessary to formulate two more constraints, in addition to those already given in the second section. Their essence is reduced to uniform full observability of the pair $[\mathbf{W}1(k), \mathbf{H}(k)]$ and uniform full controllability of the pair $\{\mathbf{W}1(k)[\mathbf{I}_n - \mathbf{K}(k)\mathbf{H}(k)]\mathbf{Q}^{1/2}(k)\}$, defined using the Gram matrix. They differ somewhat from those accepted in the theory of linear Kalman filtration since this theory cannot be directly applied to the specific problem under consideration. Moving on to the discussion of the properties of the synthesized filter, the following should be noted:

1) Data limitation. If the above-introduced assumptions are valid, then the recursively calculated matrices $\mathbf{P}(k + 1/k + 1)$ and $\mathbf{P}(k + 1/k)$, so and $\mathbf{P}_r(k + 1)$, $\mathbf{K}(k)$ are limited. In other words, this property guarantees the boundedness of all recurrent computations, excluding the estimates of the state vector. Due to the presence of white Gaussian noise, the state estimation errors, in principle, cannot be limited (11), (15) but the second point is limited – the covariance matrix of filtering errors. This property is important for applications where all calculations must be performed in real-time.

2) Stability. At the made assumptions, dynamic errors of state estimates (17), (18) will only be exponentially stable, since in this case one of the main conditions of the Kalman filter theory is violated – the reversibility of the transition matrix of states $\mathbf{W}1(k)$. In fact, this matrix according to expression (8) is always singular.

However, if we introduce the notation for the estimation error of the state vector $\mathbf{s}^{\sim}(k/k) \triangleq \mathbf{s}^{\sim}(k) - \mathbf{s}^{*}(k/k)$ then the dynamics of estimation errors can be represented by the equation

$$\begin{aligned}\tilde{\mathbf{s}}\left(\tfrac{k+1}{k+1}\right) = \left[\mathbf{I}_n - \mathbf{K}(k+1)\mathbf{H}(k+1)\right]\mathbf{W}1(k)\tilde{\mathbf{s}}\left(\tfrac{k}{k}\right) + \\ + \left[\mathbf{I}_n - \mathbf{K}(k+1)\mathbf{H}(k+1)\right]\mathbf{w}(k) - \mathbf{K}(k+1)\mathbf{n}_y(k+1).\end{aligned} \tag{19}$$

It is common knowledge that the convergence of estimation errors is defined only by the determined member of equation (19) which is characterized by the expression $\mathbf{W}1(k)[\mathbf{I}_n - \mathbf{K}(k+1)\mathbf{H}(k+1)]$. As calculations $\mathbf{K}(k+1)$ are determined, then the stability of the dynamics of filtering errors is provided only by the assumptions made with respect to $\mathbf{W}1(k)$, $\mathbf{H}(k+1)$, $\mathbf{Q}1(k)$. Therefore, these properties don't affect the correlation processes $\mathbf{w}(k)$ and $\mathbf{n}_y(k)$. More complete proof of the estimations convergence can be deduced using of Lyapunov functions or (and) having investigated at the same time stability of the Riccati solutions at a singular transfer matrix [60]. However, it goes beyond the objects set in this work.

3) Optimality. It should be noted here that ignoring the correlation between processes $\mathbf{w}(k)$, $\mathbf{n}_y(k)$ leads to the loss of optimality of the modified filter. However, as shown by further modeling, these losses are relatively small. Besides, it is not entirely obvious how one should take into account the correlation caused by the introduced transformation of the equation of the system's output, and this can be the subject of further research.

6 Results of the Method Feasibility Testing

As the illustrative example, untied any specific application, let's consider the continuous third-order dynamic system with transfer function [64]

$$F(p) \triangleq \frac{U_{out}(p)}{U_{in}(p)} = \frac{1}{\left(T_1^2 p^2 + 2\xi T_1 p + 1\right)\left(T_2 p + 1\right)}$$

where matrixes T_1, T_2 – the time constants of the oscillatory and aperiodic links connected consistently, ξ – the damping factor ($1 < \xi < 0$), $p = (\sigma + j\omega)$ – the complex variable. As numerical values of the specified parameters we will choose $T_1 = 1$, $T_2 = 4$, $\xi = 0{,}5$. Let's convert this system in terms of a state variable. For this purpose we will use the method of direct programming [65] and will perform the following transformations:

$$U_{out}(p) = F(p)U_{in}(p) = \frac{1}{\left(T_1^2 p^2 + 2\xi T_1 p + 1\right)\left(T_2 p + 1\right)} U_{in}(p) =$$

$$=\frac{p^{-3}U_{in}(p)}{p^{-3}+(2\xi T_1+T_2)p^{-2}+\left(T_1^2+2\xi T_1T_2\right)p^{-1}+T_1^2T_2}.$$

Let's enter new denotation

$$E(p)\triangleq\frac{U_{in}(p)}{p^{-3}+(2\xi T_1+T_2)p^{-2}+\left(T_1^2+2\xi T_1T_2\right)p^{-1}+T_1^2T_2}.$$

We will solve the obtained relation relatively $E(p)$

$$E(p)\triangleq-\frac{1}{T_1^2T_2}E(p)p^{-3}-\frac{(2\xi T_1+T_2)}{T_1^2T_2}E(p)p^{-2}-\frac{\left(T_1^2+2\xi T_1T_2\right)}{T_1^2T_2}E(p)p^{-1}+\frac{1}{T_1^2T}U_{in}(p).$$

As state variables we will choose the integrators outputs and will make a functional diagram having the next parameters:

$$\frac{\left(T_1^2+2\xi T_1T_2\right)}{T_1^2T_2}\triangleq a;\quad \frac{(2\xi T_1+T_2)}{T_1^2T_2}\triangleq b;\quad \frac{1}{T_1^2T_2}\triangleq c.$$

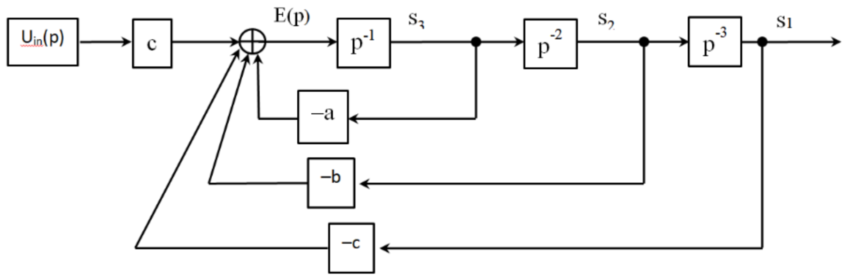


Figure 1

The function circuit of an illustrative system in terms of the state variables

On the basis of the function circuit we make the equation system

$$\begin{cases}\dot{s}_1(t)=\ \ 0\cdot s_1(t)+1\cdot s_2(t)+0\cdot s_3(t)+0\cdot u_{вх}(t);\\ \dot{s}_2(t)=\ \ 0\cdot s_1(t)+0\cdot s_2(t)+1\cdot s_3(t)+0\cdot u_{вх}(t);\\ \dot{s}_3(t)=-c\cdot s_1(t)-b\cdot s_2(t)-a\cdot s_3(t)+c\cdot u_{вх}(t).\end{cases}$$

Therefore, system matrixes in the absence of perturbations will be such:

$$\mathbf{s}(t)=\begin{bmatrix}s_1(t)\\ s_2(t)\\ s_3(t)\end{bmatrix};\quad \mathbf{W}(t)=\begin{bmatrix}0&1&0\\0&0&1\\-c&-b&-a\end{bmatrix};\quad \mathbf{G}(t)=\begin{bmatrix}0\\0\\c\end{bmatrix};\quad \mathbf{H}(t)=\begin{bmatrix}1&0&0\\0&0&1\end{bmatrix}.$$

Under determining an observation matrix it was supposed that the structure of measuring means allows a possibility of states measurements $\mathbf{s}_1(t)$ and $\mathbf{s}_3(t)$.

The matrix of perturbations distribution was defined from the following conditions [66]. It was supposed that a perturbations source are changes of parameters *a, b, c*, and changes of matrixes $\mathbf{W}(t)$, $\mathbf{G}(t)$ we will provide linear approximations $\Delta\mathbf{W}(t)$, $\Delta\mathbf{G}(t)$, i.e

$$\mathbf{D}_s(t)\mathbf{d}(t) = \mathbf{D}_s(t)\left\{\left[\Delta a(t)\ \Delta b(t)\ \Delta c(t)\right]\mathbf{s}(t) + \Delta c(t)u_{in}(t)\right\};$$

- when determining a matrix $\mathbf{D}^T_s(t)=[1\ 0\ 0]^T$ we will be limited to a case when changes of parameters influence a component $\mathbf{s}_1(t)$.

The system discrete equivalent was calculated by formulas [63]:

$$\mathbf{W}(k) = \mathbf{e}^{\mathbf{W}\Delta T};\quad \mathbf{G}(k) = \mathbf{W}^{-1}\left[\mathbf{W}(k) - \mathbf{I}_3\right]\mathbf{G};\quad \mathbf{D}_s(k) = \mathbf{W}^{-1}\left[\mathbf{W}(k) - \mathbf{I}_3\right]\mathbf{D}_s$$

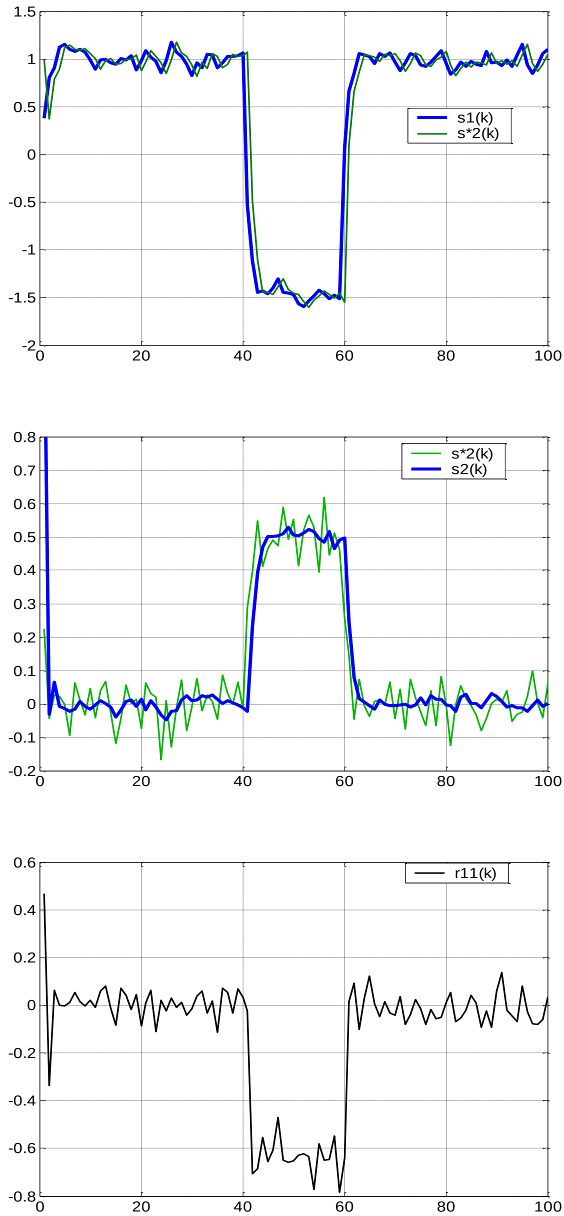
At above preset values T_1, T_2, ξ and ΔT = 4 system matrixes accept values

$$\mathbf{W}(k) = \begin{bmatrix} 0.4729 & 0.5765 & 0.6260 \\ -0.1565 & -0.3096 & -0.2060 \\ 0.0515 & 0.1010 & -0.0521 \end{bmatrix};\ \mathbf{G}(k) = \begin{bmatrix} 0.5271 \\ 0.1565 \\ -0.0515 \end{bmatrix};\ \mathbf{D}_s(k) = \begin{bmatrix} 3.2121 \\ -0.5271 \\ -0.1565] \end{bmatrix}$$

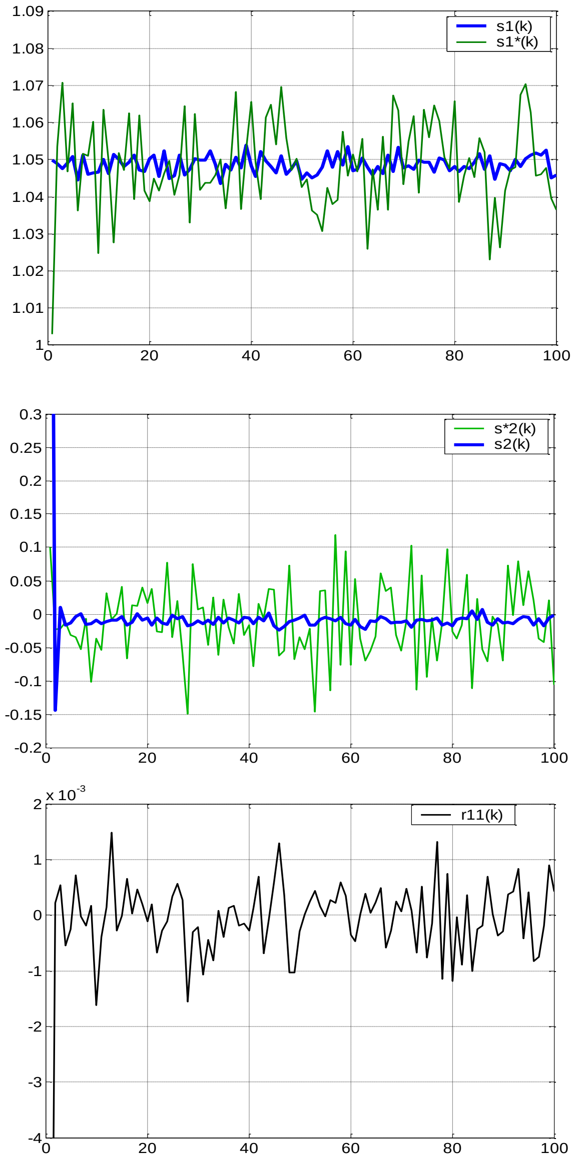
Covariance matrices of noise processes $\mathbf{n}_s(k)$, $\mathbf{n}_y(k)$, took the values $\mathbf{Q} = 0.0025\mathbf{I}_{(3)}$; $\mathbf{R} = 0.0001\mathbf{I}_{(2)}$, respectively, and the mean values were assumed to be zero. The modeling process was carried out in the computing MATLAB environment according to formulas (11)-(18). The perturbations process was imitated by expression:

$$d(k) = \begin{cases} k \le 40, & d(k) = 0; \\ k > 40, & d(k) = -0.2 + 0.025 * randn; \\ k \ge 60, & d(k) = 0. \end{cases}$$

The following values were taken as the initial conditions: $\mathbf{s}^T(0) = [1; 0; -1]^T$ $\mathbf{u}(k) = 1$; $\mathbf{s}^{T*}(0/0) = [0; 0; 0]^T$; $\mathbf{P}(0/0) = \mathbf{I}_{(3)}$. Results of estimating the first two components of the state vector $\mathbf{s}_1^*(k/k)$, $\mathbf{s}_2^*(k/k)$ for system transient response and differential signal $\mathbf{r}(k)$ are shown in Fig. 2 a, b.



a)



b)

Figure 2

Modeling results of states estimation process: a) standard Kalman filter; b) modified Kalman filter

If the Kalman filter works correctly, then the difference process, called the updating process, is an uncorrelated Gaussian random process with a zero mean value and a covariance matrix recursively calculated by the formula [46]:

$$\mathbf{P}_{\mathbf{r}}\left(k+1\right) \triangleq E\left\{\mathbf{r}\left(k+1\right)\mathbf{r}^{T}\left(k+1\right)\right\} = \mathbf{H}\left(k+1\right)\mathbf{P}\left({}^{k}\!/\!_{k}\right)\mathbf{H}^{T}\left(k+1\right)+\mathbf{R}\left(k+1\right). \qquad (20)$$

Matching charts $\mathbf{r}(k)$ located in the bottom line of Fig. 2, confirms the correct operation of only the modified Kalman filter, the estimates error are not affected by the appeared disturbance. Consequently, the problem of decoupling (decomposition) of the estimation process from disturbances with an indefinite structure is satisfactorily solved.

Conclusions

The problem of obtaining estimates of the states of linear dynamic systems, free from the influence of disturbances, the structure of which is not defined, is relevant for many of the applied research, including in the field of functional diagnostics.

Unlike the traditional way based on an adaptation of the filter structure to the model set structure of, authors solved a problem by modification of the model set model to an equivalent form where the perturbations component in an explicit form is absent. It allowed for being limited to the application of one of Kalman filter forms guaranteeing convergence of states estimations.

In comparison with other known design methods, the proposed method is natural and simple, and the conditions for the existence of a solution are easily verified. Considerations are presented regarding the guarantee of the properties of the obtained solution: stability, boundedness and optimality. Since the issues under consideration have not yet found sufficient coverage in the periodicals, there is reason to believe that the presented results introduce an element of novelty into the research topics related to obtaining estimates of the dynamical systems states that are indifferent to disturbances of an indefinite structure.

References

[1] V. Venkatasubramanian, R. Rengaswamy, and S. N. Kavuri, "A review of process fault detection and diagnosis: Part I: Quantitative model-based methods," Computers & Chemical Engineering, Vol. 27, No. 3, pp. 293-311, Mar. 2003, doi: 10.1016/S0098-1354(02)00160-6

[2] V. Venkatasubramanian, R. Rengaswamy, and S. N. Kavuri, "A review of process fault detection and diagnosis: Part II: Qualitative models and search strategies," Computers & Chemical Engineering, Vol. 27, No. 3, pp. 313-326, Mar. 2003, doi: 10.1016/s0098-1354(02)00161-8

[3] V. Venkatasubramanian, R. Rengaswamy, S. N. Kavuri, and K. Yin, "A review of process fault detection and diagnosis: Part III: Process history based methods," Computers & Chemical Engineering, Vol. 27, No. 3, pp. 327-346, Mar. 2003, doi: 10.1016/s0098-1354(02)00162-x

[4] H. Long and X. M. Wang, "Aircraft Fuel System Diagnostic Fault Detection Through Expert System", 2008 7th World Congress on Intelligent

Control and Automation, 2008, pp. 7104-7107, doi: 10.1109/WCICA.2008.4594020

[5] K. S. Gaeid and H. A. F. Mohamed, "Diagnosis and Fault Tolerant Control of the Induction Motors Techniques a Review", Australian Journal of Basic and Applied Sciences, 4(2), 2010, pp. 227-246

[6] J. L. Aravena and F. N. Chowdhury, "Fault detection of flight critical systems," 20th DASC. 20th Digital Avionics Systems Conference (Cat. No.01CH37219), 2001, pp. 1C6/1-1C6/10 Vol. 1, doi: 10.1109/DASC.2001.963315

[7] C. Hajiyev, F. Caliskan, "Fault diagnosis and reconfiguration in flight control systems", Springer Science & Business Media 2003, 342 p. doi: 10.1007/978-1-4419-9166-9

[8] F. Kimmich and R. Isermann, "Model based fault detection for the injection, combustion and engine-transmission," IFAC Proceedings Volumes, Vol. 35, No. 1, pp. 203-208, 2002, doi: 10.3182/20020721-6-es-1901.00765

[9] L. Salvatore, B. Pires, M. C. M Campos and M. B. A. De Souza Jr M. B., "A Hybrid Approach to Fault Detection and Diagnosis in a Diesel Fuel Hydrotreatment Process", Proceedings of world academy of science, engineering and technology, Vol. 7 August, 2005, pp. 379-384

[10] D. van Schrick, "Remarks on Terminology in the Field of Supervision, Fault Detection and Diagnosis," IFAC Proceedings Volumes, Vol. 30, No. 18, pp. 959-964, Aug. 1997, doi: 10.1016/s1474-6670(17)42524-9

[11] R. Isermann, Fault-Diagnosis Systems. Springer Berlin Heidelberg, 2006, doi: 10.1007/3-540-30368-5

[12] M. Blanke, M. Kinnaert, J. Lunze, and M. Staroswiecki, Diagnosis and Fault-Tolerant Control. Springer Berlin Heidelberg, 2016, doi: 10.1007/978-3-662-47943-8

[13] R. Isermann, "Model-based fault-detection and diagnosis – status and applications," Annual Reviews in Control, Vol. 29, No. 1, pp. 71-85, Jan. 2005, doi: 10.1016/j.arcontrol.2004.12.002

[14] J. J. Gertler and R. Monajemy, "Generating directional residuals with dynamic parity relations," Automatica, Vol. 31, No. 4, pp. 627-635, Apr. 1995, doi: 10.1016/0005-1098(95)98494-q

[15] J. J. Gertler, X. Fang, and Q. Luo, "Detection and Diagnosis of Plant Failures: The Orthogonal Parity Equation Approach," in Control and Dynamic Systems, Elsevier, 1990, pp. 159-216, doi: 10.1016/b978-0-12-012737-5.50010-4

[16] R. J. Patton and J. Chen, "A Review of Parity Space Approaches to Fault Diagnosis," IFAC Proceedings Volumes, Vol. 24, No. 6, pp. 65-81, Sep. 1991, doi: 10.1016/s1474-6670(17)51124-6

[17] Frank P. M., "On-line fault detection in uncertain nonlinear systems using diagnostic observers": a survey. Int. J. Systems Sci., Vol. 25/12, pp. 2129-2154, 1994

[18] V. Krishnaswami and G. Rizzoni, "A Survey of Observer Based Residual Generation for FDI," IFAC Proceedings Volumes, Vol. 27, No. 5, pp. 35-40, Jun. 1994, doi: 10.1016/s1474-6670(17)48000-1

[19] J. Chen, H. Zhang, "Robust detection of faulty actuators via unknown input observers", Proceedings of International Journal of Systems Science, Vol. 22, Iss. 10, 1991, pp. 1829-1839, doi: 10.1080/00207729108910753

[20] A. Volovyk, V. Kychak, D. Kudriavtsev, D. Havrilov, A. Yarovyi and L. Krylik, "Simultaneous Estimation in Linear Dynamic Systems with the Indeterminate Structure Disturbances", 2020 IEEE 40th International Conference on Electronics and Nanotechnology (ELNANO), Kyiv, Ukraine, 2020, pp. 651-655, doi: 10.1109/ELNANO50318.2020.9088884

[21] C. Yang, J. Zheng, X. Ren, W. Yang, H. Shi and L. Shi, "Multi-Sensor Kalman Filtering with Intermittent Measurements" in IEEE Transactions on Automatic Control, Vol. 63, No. 3, pp. 797-804, March 2018, doi: 10.1109/TAC.2017.2734643

[22] P. Bania and J. Baranowski, "Bayesian estimator of a faulty state: Logarithmic odds approach," 2017 22nd International Conference on Methods and Models in Automation and Robotics (MMAR), 2017, pp. 253-257, doi: 10.1109/MMAR.2017.8046834

[23] J. Keller and D. D. J. Sauter, "Kalman Filter for Discrete-Time Stochastic Linear Systems Subject to Intermittent Unknown Inputs," in IEEE Transactions on Automatic Control, Vol. 58, No. 7, pp. 1882-1887, July 2013, doi: 10.1109/TAC.2013.2264739

[24] S. Huang, K. K. Tan and T. H. Lee, "Fault Diagnosis and Fault-Tolerant Control in Linear Drives Using the Kalman Filter," in IEEE Transactions on Industrial Electronics, Vol. 59, No. 11, pp. 4285-4292, Nov. 2012, doi: 10.1109/TIE.2012.2185011

[25] S. Simani, C. Fantuzzi, and R. J. Patton, "Model-based fault diagnosis in dynamic systems using identification techniques", Advances in Industrial Control. Springer-Verlag London, 2003, p. 282

[26] R. Isermann, "Supervision, fault-detection and fault-diagnosis methods — An introduction," Control Engineering Practice, Vol. 5, No. 5, pp. 639-652, May 1997, doi: 10.1016/s0967-0661(97)00046-4

[27] H. Wang and W. Lin, “Applying observer based FDI techniques to detect faults in dynamic and bounded stochastic distributions,” International Journal of Control, Vol. 73, No. 15, pp. 1424-1436, Jan. 2000, doi: 10.1080/002071700445433

[28] Zall R., Kangavari M., "On the construction of multi-relational classifier based on canonical correlation analysis ", International Journal of Artificial Intelligence, Vol. 17, No. 2, pp. 23-43, 2019

[29] A. Y. Volovik, L. V. Krylik, I. M. Kobylyanska, A. Kotyra, S. Amirgaliyeva, “Methods of stochastic diagnostic type observers”, Proc. SPIE 10808, Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments 2018, 108082X (1 October 2018), doi: 10.1117/12.2501693

[30] J.-Y. Keller, “A Fault Detection Filter Including an Adaptive Noise Cancellation Strategy,” European Journal of Control, Vol. 13, No. 6, pp. 627-638, Jan. 2007, doi: 10.3166/ejc.13.627-638

[31] J. Stoustrup and H. H. Niemann, “Fault estimation – a standard problem approach,” Int. J. Robust Nonlinear Control, Vol. 12, No. 8, pp. 649-673, 2002, doi: 10.1002/rnc.716

[32] L. A. Zadeh, K. S. Fu, K. Tanaka, M. Shimura, and C. V. Negoita, “Fuzzy Sets and Their Applications to Cognition and Decision Processes,” IEEE Trans. Syst., Man, Cybern., Vol. 7, No. 2, Feb. 1977, doi: 10.1109/tsmc.1977.4309670

[33] P. M. Frank and N. Kiupel, “Fuzzy supervision and application to lean production†,” International Journal of Systems Science, Vol. 24, No. 10, pp. 1935-1944, Oct. 1993, doi: 10.1080/00207729308949605

[34] P. M. Frank, “Application of Fuzzy Logic to Process Supervision and Fault Diagnosis,” IFAC Proceedings Volumes, Vol. 27, No. 5, pp. 507-514, Jun. 1994, doi: 10.1016/s1474-6670(17)48077-3

[35] W. Cheng and C. Juang, "A Fuzzy Model With Online Incremental SVM and Margin-Selective Gradient Descent Learning for Classification Problems," in IEEE Transactions on Fuzzy Systems, Vol. 22, No. 2, pp. 324-337, April 2014, doi: 10.1109/TFUZZ.2013.2254492

[36] C.-F. Juang, Y.-Y. Lin, and R.-B. Huang, “Dynamic system modeling using a recurrent interval-valued fuzzy neural network and its hardware implementation,” Fuzzy Sets and Systems, Vol. 179, No. 1, pp. 83-99, Sep. 2011, doi: 10.1016/j.fss.2011.05.015

[37] R. Precup, T. Teban, A. Albu, A. Borlea, I. A. Zamfirache and E. M. Petriu, "Evolving Fuzzy Models for Prosthetic Hand Myoelectric-Based Control," in IEEE Transactions on Instrumentation and Measurement, Vol. 69, No. 7, pp. 4625-4636, July 2020, doi: 10.1109/TIM.2020.2983531

[38] R. J. Patton, F. J. Uppal, and C. J. Lopez-toribio, “Soft Computing Approaches to Fault Diagnosis for Dynamic Systems: A Survey,” IFAC Proceedings Volumes, Vol. 33, No. 11, pp. 303-315, Jun. 2000, doi: 10.1016/s1474-6670(17)37377-9

[39] R. Isermann, “Integration of Fault Detection and Diagnosis Methods,” IFAC Proceedings Volumes, Vol. 27, No. 5, pp. 575-590, Jun. 1994, doi: 10.1016/s1474-6670(17)48088-8

[40] Claudiu Pozna, Radu-Emil Precup, Applications of signatures to expert systems modeling. Acta Polytechnica Hungarica 11(2): January 2014, p. 21-39

[41] Merry R. J. E.: Wavelet Theory and Applications. Eindhoven, 2005, p. 49

[42] E. Hedrea, R. Precup, R. Roman, and E. M. Petriu, “Tensor product‐ based model transformation approach to tower crane systems modeling,” Asian J Control, Vol. 23, No. 3, pp. 1313-1323, Mar. 2021, doi: 10.1002/asjc.2494

[43] R. K. Mehra and J. Peschon, "An innovations approach to fault detection and disgnosis in dynamic systems". Automatica, Vol. 7, pp. 637-640, 1971

[44] D. Magill, "Optimal adaptive estimation of sampled stochastic processes," in IEEE Transactions on Automatic Control, Vol. 10, No. 4, pp. 434-439, October 1965, doi: 10.1109/TAC.1965.1098191

[45] D. Lainiotis, "Optimal adaptive estimation: Structure and parameter adaption," in IEEE Transactions on Automatic Control, Vol. 16, No. 2, pp. 160-170, April 1971, doi: 10.1109/TAC.1971.1099684

[46] P. D. Hanlon and P. S. Maybeck, "Characterization of Kalman filter residuals in the presence of mismodeling," in IEEE Transactions on Aerospace and Electronic Systems, Vol. 36, No. 1, pp. 114-131, Jan. 2000, doi: 10.1109/7.826316

[47] Hajiyev Ch. M., "Fault detection in multidimensional systems based on statistical analysis of Kalman filter". IFAC Symposium on Fault Detection Supervision and Safety for Technical Processes, SAFERPROCESS – Helsinki, 1994, Vol. 1, pp. 45-49

[48] C. Hajiyev and F. Caliskan, “Sensor/actuator fault diagnosis based on statistical analysis of innovation sequence and Robust Kalman Filtering,” Aerospace Science and Technology, Vol. 4, No. 6, pp. 415-422, Sep. 2000, doi: 10.1016/s1270-9638(00)00143-7

[49] B. Friedland, "Treatment of bias in recursive filtering," in IEEE Transactions on Automatic Control, Vol. 14, No. 4, pp. 359-367, August 1969, doi: 10.1109/TAC.1969.1099223

[50] J. Chen and R. J. Patton, “Optimal filtering and robust fault diagnosis of stochastic systems with unknown disturbances,” IEE Proceedings - Control

Theory and Applications, Vol. 143, No. 1, pp. 31-36, Jan. 1996, doi: 10.1049/ip-cta:19960059

[51] M. Ignagni, "Optimal and suboptimal separate-bias Kalman estimators for a stochastic bias," in IEEE Transactions on Automatic Control, Vol. 45, No. 3, pp. 547-551, March 2000, doi: 10.1109/9.847741

[52] Fu-Chuang Chen and Chien-Shu Hsieh, "Optimal multistage Kalman estimators," in IEEE Transactions on Automatic Control, Vol. 45, No. 11, pp. 2182-2188, Nov. 2000, doi: 10.1109/9.887678

[53] Chien-Shu Hsieh, "Extension of the robust two-stage Kalman filtering for systems with unknown inputs," TENCON 2007 - 2007 IEEE Region 10 Conference, 2007, pp. 1-4, doi: 10.1109/TENCON.2007.4429133

[54] C. Hsieh, "A unified extension of the robust two-stage Kalman filter and its application to functional filtering," 2009 American Control Conference, 2009, pp. 3824-3829, doi: 10.1109/ACC.2009.5159992

[55] P. K. Kitanidis, “Unbiased minimum-variance linear state estimation,” Automatica, Vol. 23, No. 6, pp. 775-778, Nov. 1987, doi: 10.1016/0005-1098(87)90037-9

[56] J. Y. Keller and M. Darouach, “Two-stage Kalman estimator with unknown exogenous inputs,” Automatica, Vol. 35, No. 2, pp. 339-342, Feb. 1999, doi: 10.1016/s0005-1098(98)00194-0

[57] A. Volovyk, V. M. Kychak, “Detection Filter Method in Diagnostic Problems for Linear Dynamic Systems”, Visnyk NTUU KPI Seriia – Radiotekhnika Radioaparatobuduvannia, 2021, Iss. 84, pp. 30-39, DOI: https://doi.org/10.20535/RADAP.2021.84

[58] S. Gillijns and B. De Moor, “Unbiased minimum-variance input and state estimation for linear discrete-time systems with direct feedthrough,” Automatica, Vol. 43, No. 5, pp. 934-937, May 2007, doi: 10.1016/j.automatica.2006.11.016

[59] Chien-Shu Hsieh, “Optimal Minimum-Variance Filtering for Systems with Unknown Inputs,” presented at the 2006 6th World Congress on Intelligent Control and Automation, 2006, doi: 10.1109/wcica.2006.1712679

[60] M. A. Massoumnia, "A geometric approach to the synthesis of failure detection filters," in IEEE Transactions on Automatic Control, Vol. 31, No. 9, pp. 839-846, September 1986, doi: 10.1109/TAC.1986.1104419

[61] J. White and J. Speyer, "Detection filter design: Spectral theory and algorithms," in IEEE Transactions on Automatic Control, Vol. 32, No. 7, pp. 593-603, July 1987, doi: 10.1109/TAC.1987.1104682

[62] C. de Souza, M. Gevers and G. Goodwin, "Riccati equations in optimal filtering of nonstabilizable systems having singular state transition

matrices," in IEEE Transactions on Automatic Control, Vol. 31, No. 9, pp. 831-838, September 1986, doi: 10.1109/TAC.1986.1104415

[63] Arthur Albert, Regression and the Moore-Penrose pseudoinverse, New York : Academic Press, 180 p., 1972

[64] Makarov I. M., Mensky B. M. Linear automatic systems (elements of theory, calculation methods and reference material), Mashinostroenie, 1982, 505 p.

[65] Julius T. Tou, Modern Control Theory, NY, McGraw-Hill, 1964, 427 p.

[66] A. Varga, Solving Fault Diagnosis Problems. Springer International Publishing. Vol. 84. 2017, p. 394, doi: 10.1007/978-3-319-51559-5