Analytical Description of the Steady-State Creep of Metals in the Presence of Direct Current

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Abstract: This paper is aimed to develop a model for the analytical description of the effect of direct current (DC) upon the steady-state creep of metals. For the mathematical apparatus, the synthetic theory of irrecoverable deformation is taken. As a result, relationships between creep rate, stress, temperature, and current intensity have been derived. For this purpose, a term, taking into account the passing of DC, is entered into the constitutive equation of the theory. The model results fit well experimental data. The analysis of loading surface in steady-state creep for the ordinary loading and that coupled with DC is provided.

Keywords: steady-state creep rate; direct current; synthetic theory

1 Introduction

The electrical-mechanical behavior of materials has been extensively studied analytically and numerically for the potential applications in micro electromechanical systems and field-assisted sintering process.

A number of investigators have clearly established that mechanical properties such as flow stress, creep, and stress relaxation undergo significant changes under the influence of electrical field. Whatever the detailed mechanism for the effect of electric current on deformation behavior, the effect is most likely associated with the interaction of the electric field with the structural defects, notably dislocations. When current is imposed on the deformation, it will cause not only heating and thus expansion of material, but also will weaken the binding forces between dislocations and obstacles impeding dislocation motion that otherwise cannot be overcome by thermal activation alone. As a result, the nucleation and multiplication of dislocations will be alleviated that in turn will increase the density of mobile dislocations. Therefore, electron wind frees more and more dislocation from the pinning defects and enhances the mobility of dislocations that will be reflected as an increased rate of deformation.

Researches [1-7] report an increase steady-state creep rate due to the passage of DC, see Fig. 2, which is suggested to be caused by the following:

(i) DC-induced Joule heating causing a change in local temperature and resulting in time-dependent plastic deformation [8-10].

(ii) Themomentumexchangebetweenmovingelectronsandlatticeatomsreducesthee nergybarrierandincreasesthemigration velocity of atoms [11-13].

(iii) The intensification of the current field assisted sliding rate and diffusional creep [14-16]

Such metals and their alloys as copper, nickel, aluminum, tin, etc. [1-3, 6] show the increase in their steady-state creep due to the passage of direct current (DC). It must be noted that, so far, experimental results dominates over analytical studies about the influence DC on the steady-state creep of metals. Here can be mentioned work [2] proposing a linear relation between the creep rate and squared current intensity.

This paper continues the investigations presented in [17], where, in terms of the synthetic theory of irrecoverable deformation, formulae $\dot{\varepsilon} = \dot{\varepsilon}(\sigma, J)$ at a given temperature have been proposed (J is current intensity). At the same time, a deeper analysis of the results obtained shows unsatisfactory results for variable temperature. Consequently, the goal of this paper is to model the effect of DC upon the steady-state creep of metals encompassing all the three parameters affecting the creep, i.e. $\dot{\varepsilon} = \dot{\varepsilon}(\sigma, T, J)$.

2 Synthetic Theory

The synthetic theory incorporates the Batdorf-Budiansky slip concept [18] and the Sanders flow theory [19]. The theory models small irrecoverable (plastic/creep) deformations of hardening materials. While works [20-22] provide comprehensive information on the basic notions and relationships of the synthetic theory, here we utilize only its key formulae.

The modeling of irrecoverable deformation takes place in the three-dimensional subspace of the Ilyushin five-dimensional space of stress deviators [23]. The loading process is expressed by a stress vector, $\mathbf{S}$, whose components are converted from the stress deviator tensor components – S_{ij} ($i, j = x, y, z$) – as follows [20, 21]:

$$\mathbf{S}\left[\sqrt{3/2}S_{xx}, S_{xx}/\sqrt{2} + \sqrt{2}S_{yy}, \sqrt{2}S_{xz}\right]. \tag{1}$$

Since the synthetic theory is of two-level nature, the macrodeformation (strain vector **e**) is calculated a sum of slips occurring at the microlevel of material (strain intensity φ_N)

$$\mathbf{e} = \int\limits_{\alpha}\int\limits_{\beta}\int\limits_{\lambda} \varphi_N \mathbf{N} dV , \quad dV = \cos\beta d\alpha d\beta d\lambda \tag{2}$$

where angles α, β and λ give the orientation of slip system and the slip direction.

The strain vector components are related to the strain-deviator tensor components, e_{ij} ($i, j = x, y, z$), in the following way

$$e_1 = \sqrt{3/2} e_{xx} , \quad e_2 = e_{xx} \big/ \sqrt{2} + \sqrt{2} e_{yy} , \quad e_3 = \sqrt{2} e_{xz} . \tag{3}$$

For the case of steady-state creep ($\mathbf{S}(t) = const$), the strain intensity takes the form as [20, 21]

$$\dot{\varphi}_N = \frac{K}{r}\left[\left(\mathbf{S}\cdot\mathbf{N}\right)^2 - 2/3\sigma_P^2 \right], \tag{4}$$

$$K = K_1(T) K_2(\sigma_{eff}) , \; K_1(T) = \exp\left(-\frac{Q}{RT}\right), \; K_2(\sigma_{eff}) = \frac{9\sqrt{3}cr}{2\sqrt{2}\pi}\sigma_{eff}^{k-2} \tag{5}$$

where r, c and k are model constants; σ_P the creep limit of material in uniaxial tension, Q creep activation energy. According to Schimd's law, the driving force of plastic slip within a slip system is resolved shear stress, which is determined by the product $\mathbf{S}\cdot\mathbf{N}$, where unit vector $\mathbf{N}$ gives the orientation of slip system.

In Eq. (5), We define the function K_2 via the Bailey-Norton law (power law creep). In terms of the synthetic theory, the effect of power index is expressed by constant k .

In uniaxial tension ($S_1 = \sqrt{2/3}\sigma$, $N_1 = \cos\alpha\cos\beta\cos\lambda$ [19, 20]), Eq. (2) gets

$$\dot{e}_1 = \frac{2K}{3r}\int\limits_{-\alpha_1}^{\alpha_1}\int\limits_{-\beta_1}^{\beta_1}\int\limits_{0}^{\lambda_1}\left[\left(\sigma\cos\alpha\cos\beta\cos\lambda\right)^2 - \sigma_P^2\right]\cos\alpha\cos^2\beta\cos\lambda d\alpha d\beta d\lambda . \tag{6}$$

The boundaries of integration are [20, 21]

$$\cos\lambda_1 = \frac{\sigma_P}{\sigma\cos\alpha\cos\beta}, \quad \cos\alpha_1 = \frac{\sigma_P}{\sigma\cos\beta}, \quad \cos\beta_1 = \frac{\sigma_P}{\sigma} . \tag{7}$$

By integrating in (6), we obtain

$$\dot{e}_1 = AF(b),\ A = \frac{\sqrt{3}c\sigma_P^2}{2\sqrt{2}}\sigma^{k-2}K_1(\Theta), \quad (8)$$

$$F(b) = \frac{1}{b^2}\left(2\sqrt{1-b^2} - 5b^2\sqrt{1-b^2} + 3b^4 \ln\frac{1+\sqrt{1-b^2}}{b}\right),\ b = \cos\beta_1. \quad (9)$$

In [20-22] it is shown that $F(b)$ increases with the decrease in b. On the other hand, the last formula in Eq. (7) shows that b decreases as the stress σ grows. Therefore, the final result is that the function F is an increasing function of the acting stress. For the sake of further simplification, we approximate function F from (9) as

$$F \approx \left(\frac{1}{b} - 1\right)^2,\quad F(1) = F'(1) = 0. \quad (10)$$

3 Relationships of the Synthetic Theory in the Case of Creep Accompanied by Current

To model the action of DC which it exerts on the processes occurring in a slip system during steady-state creep, we enter into Eq. (4) terms containing current intensity, J (kA/cm^2).

1) To take into account the current passage induced temperature increase (Joule heating), we write down the function $K_1(T)$ from (5) as

$$K_1(T) = \exp\left(-\frac{Q}{R\left(T + 5.23J^2\right)}\right), \quad (11)$$

where the factor 5.23 is taken from [2].

2) A new term, C, is inserted into formula (4) for φ_N,

$$\varphi_N = \frac{K}{r}\left[(\mathbf{S}\cdot\mathbf{N})^2\left(1 + C^2\right) - 2/3\sigma_P^2\right]. \quad (12)$$

The presence of C in (12) symbolizes the increase in deformation within a slip system caused by electric field. We propose to define the function C as a product of two functions,

$$C = U(J)\cdot W(T), \quad (13)$$

both related to their arguments as power functions

$$U = u_1 J^{u_2} , \tag{14}$$

$$W = \left(w_1 T - w_2 \right)^{w_3} + w_4 , \tag{15}$$

where u_i and w_j model constants to be determined to best fit experiments.

The choice to adopt the term standing for the electric field in the form of C^2 in (12) is motivated by the proposition that the creep rate can be taken as a linear function of J^2, $\dot{\varepsilon} = \dot{\varepsilon}_0 + aJ^2$ [2]. Therefore, we follow this phenomenological fashion, at least, at the microlevel of material. The further complication of the function C, which can be obtained from the observation on the microlevel of material to understand what effect the current has on the dynamics of slides, is strongly needed.

By defining the function C as the product $U(J) \cdot W(T)$, we try to follow the tendency from formula (5) when defining the K. i.e. the effect from DC is decomposed on two components: (a) the force interaction between the electron wind and the material lattice $U(J)$ and (b) temperature-dependent effect. The temperature increase (11) alone is not enough to reach a good agreement between experimental and model results.

To be in line with experimental results [2], stating that DC does not strongly affect the slope of $\log\dot{\varepsilon} \sim \log\sigma$ lines, we enter function C into formula (12) as a factor multiplying σ. This means a parallel alignment of the $\log\dot{\varepsilon} \sim \log\sigma$ plots for different current intensities at a given stress. At the same time, the change in the slope must be inspected due to formula (11).

Long-term study of the behavior of $\dot{\varepsilon} = \dot{\varepsilon}(J,\sigma)\big|_T$ and $\dot{\varepsilon} = \dot{\varepsilon}(J,T)\big|_\sigma$ dependencies has been shown that these are the relationships (14) and (15) that leads to the results fitting experimental data.

To calculate the value of the first stress inducing creep deformation in the presence of DC (σ^C), we let $\varphi_N = 0$ and $\alpha, \beta, \lambda = 0$ in Eq. (12):

$$\sigma^C = \sigma_P \Big/ \sqrt{1 + C^2} . \tag{16}$$

As one can see, Eq. (16) expresses the fact that the passage of current decreases the stress needed to initiate the development of creep deformation ($\sigma^C < \sigma_P$).

Now, taking into account (12)-(15), Eq. (2) takes the following form:

$$\dot{e}_1 = \frac{2K}{3r} \times \int_{-\alpha_{1C}}^{\alpha_{1C}} \int_{-\beta_{1C}}^{\beta_{1C}} \int_{0}^{\lambda_{1C}} \left[\left(\sigma \cos\alpha \cos\beta \cos\lambda \right)^2 \left(1 + C^2 \right) - \sigma_P^2 \right] \cos\alpha \cos^2\beta \cos\lambda \, d\alpha \, d\beta \, d\lambda \tag{17}$$

where

$$\cos\lambda_{1C} = \frac{\sigma_P}{\sigma\sqrt{1+C^2}\cos\alpha\cos\beta}, \quad \cos\alpha_{1C} = \frac{\sigma_P}{\sigma\sqrt{1+C^2}\cos\beta},$$

$$\cos\beta_{1C} = \frac{\sigma_P}{\sigma\sqrt{1+C^2}} < \cos\beta_1 \Rightarrow \beta_1 < \beta_{1C}. \tag{18}$$

The secondary creep rate in uniaxial tension under the action of DC, $\dot{e}_1^C$, is calculated from Eqs. (17) and (18) as

$$\dot{e}_1^C = AF\left(b_C\right), \; b_C = \cos\beta_{1C}. \tag{19}$$

The following conclusions can be drawn from Eqs. (17) and (18):

1) formula (17) shows that the action of DC intensify the slips, (factor $1+C^2$),

2) formula (18) means the enlarging of integration domain, comparing to the case of ordinary creep; in other words, the number of slip systems contributing to creep strain increases under the action of DC.

Consider results obtained in terms of the synthetic theory for the steady-state creep rate of tin in uniaxial tension (melting point 505.08 K). The following four diagrams are plotted to inspect their fits with the following experiments [2]:

(i) Fig. 1a. Steady-state creep of tin $\dot{\varepsilon}$ as a function of tensile stress σ without current ($J = 0$); experiments were conducted at different temperatures (323,348,373,398,423 K).

(ii) Fig. 1b. Steady-state creep of tin $\dot{\varepsilon}$ as a function of temperature T without current ($J = 0$); experiments were conducted under tensile stress $\sigma = 3.09$ MPa .

(iii) Fig. 2a. Steady-state creep of tin $\dot{\varepsilon}$ as a function of tensile stress σ under the action of current of different intensities – $J = 0,\ 1.26,\ 1.89,\ 2.52,\ 2.835,\ 3.15\ \mathrm{kA/cm^2}$; experiments were conducted at $T = 323$ K .

(iv) Fig. 2b. Steady-state creep of tin $\dot{\varepsilon}$ as a function of current intensity squared J^2; experiments were conducted under tensile stress $\sigma = 3.09$ MPa at different temperatures (see (i)).

To complete the tasks announced, first, the model constants c and k must be chosen for constructing $\dot{\varepsilon}\sim\sigma$ and $\dot{\varepsilon}\sim T$ graphs at $J=0$. Plots in Fig. 1 are obtained by Eqs. (8)-(10) at $k=6$ and $c=26$ (activation energy here and further throughout is $Q=7.0\cdot10^4$ J/mole [2]). The experimental data for Fig. 1b are read from Fig. 2b at $J=0$.

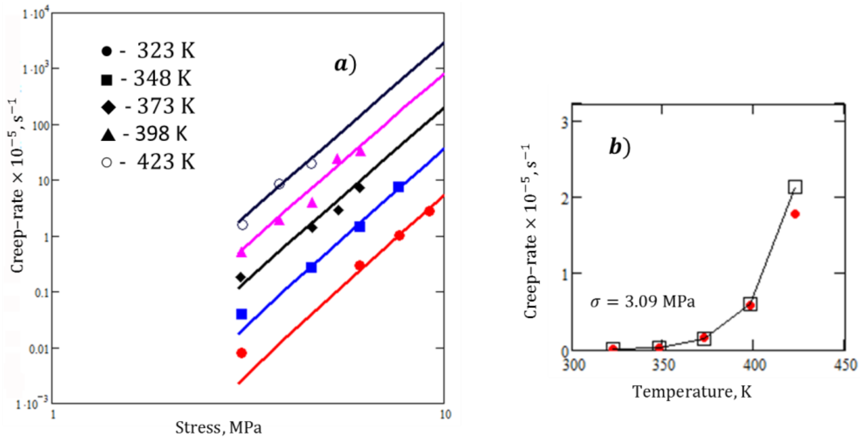

Figure 1

log$\dot{\varepsilon}\sim$logσ (a) and $\dot{\varepsilon}\sim T$ (b) diagrams of tin ($J=0$) (points – experiment [2], lines– model)

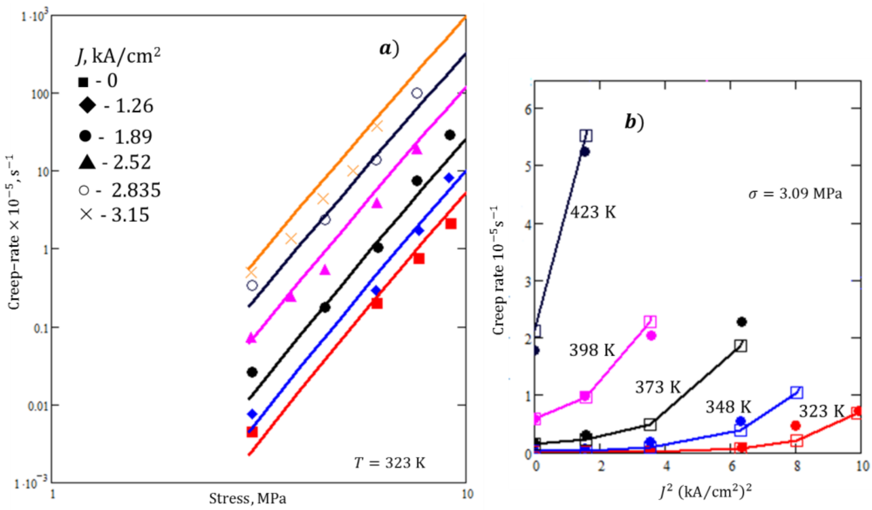

Figure 2

log$\dot{\varepsilon}\sim$logσ (a) and $\dot{\varepsilon}\sim J^2$ (b) diagrams of tin ($J=0$) (points– experiment [2], lines– model)

The next step is $\dot{\varepsilon}=\dot{\varepsilon}(J,\sigma)\big|_T$ and $\dot{\varepsilon}=\dot{\varepsilon}(J,T)\big|_\sigma$ diagrams of tin subjected to DC. Results obtained by Eqs. (18) and (19), together with (13-15), calculated at

$u_1 = 0.5\ \left(\mathrm{cm}^2/\mathrm{kA}\right)^{u_2}$, $u_2 = 3.0$, $w_1 = 0.0121/K$, $w_2 = 4.102$, $w_3 = 6$, $w_4 = 0.12$, are shown in Fig. 2 (the values of c and k, naturally, are the same as above). Fig. 2bdemonstrates that the synthetic theory follows Zhao's proposition [2] to express $\dot{\varepsilon}{\sim}J$ dependency as $\dot{\varepsilon} = \dot{\varepsilon}_0 + aJ^2$ (the model $\dot{\varepsilon}{\sim}J^2$ curve has a great radius of curvature.

The change in the slope of $\dot{\varepsilon}{\sim}\sigma$ lines in Fig. 2a, caused by formula (11), is only of 3.57%, which is in accordance with experiments.

As one can see, all the four analytical results give good agreement with the experimental data. It must be stressed once more that the analytical curves in Figs. 1 and 2 are obtained via formulae of the synthetic theories using one and the same set of constants: k, c, u_i, w_j .

4 The Analysis of Loading Surface in Steady-State Creep Coupled with DC

The essential characteristics of the plastic/creep constitutive models are:

(i) The yield criterion that defines the material state at the transition from elastic to elastic-plastic behavior. Yield function actually describes the surface in the stress space that demarks the boundary between the elastic and plastic/creep behavior of materials.

(ii) The flow rule that determines the increment in plastic strain from the increment in load.

(iii) The hardening rule that gives the evolution in the yield criterion during plastic deformation. In other words, the hardening rule describes how the yield surface changes (size, center, shape) as the result of permanent deformation. Development of yielding criteria is hence pivotal in predicting whether or not a material will begin to yield under a given stress state.

The synthetic theory utilizes the Von-Mises yield criterion, which results in the sphere in three-dimensional stress-deviator space [21, 24]:

$$S_1^2 + S_2^2 + S_3^2 = S_P^2, \tag{20}$$

where $S_P = \sqrt{2/3}\sigma_P$, and σ_P is the creep limit of material.

The flow rule is defined by Eq. (2) for one slip system at the micro-level of material, and the macro deformation is calculated via Eq. (2) (for the case of uniaxial tension we obtain formula (6)).

The evolution in yield criterion (i.e. the behavior of subsequent yield surface, or loading surface) obeys the principle proposed by Sanders [19]. The yield surface (20) is treated as the inner envelope of the planes tangential to the sphere, i.e. we have the continuous set of equidistant planes. During loading, stress vector ($\mathbf{S}$) translates at its endpoint some set of planes to which it reaches. The displacement of plane at the endpoint of the $\mathbf{S}$ symbolizes the development of permanent defamation within appropriate slip system.

In the case of steady-state creep the equations for the plane distances under the action of stress vector $\mathbf{S}$ (H_N, index N stands for the normal vector $\mathbf{N}$ that gives the orientation of the plane) are

$$H_N = \begin{cases} \mathbf{S} \cdot \mathbf{N} & \text{The stress vector displaces the plane} \\ \sqrt{2/3}\sigma_P & \text{Stationary planes} \end{cases} \tag{21}$$

The loading surface for uniaxial tension, which is the inner envelope of the planes whose distances are determined by the formulae above, is shown in Fig. 3, surface Ω. It consists of two portions:

(i) the sphere formed by stationary planes, i.e. those which are not reached by the stress vector,

(ii) the cone whose generator is made up of boundary planes reached by the stress vector (their orientation is given by angle β_1).

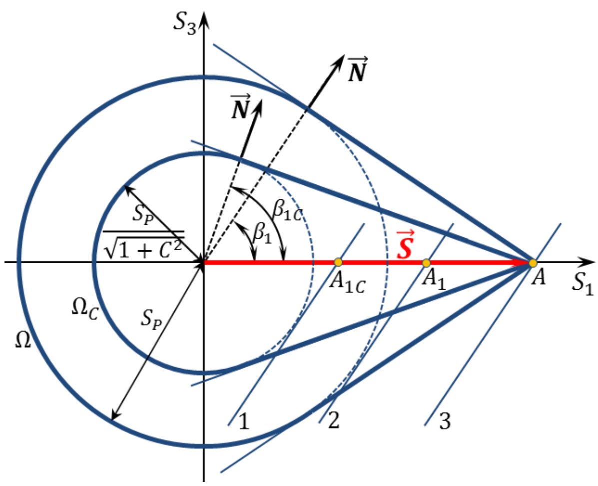

Figure 3

Loading surfaces in S_1-S_3 coordinate plane for uniaxial tension under the condition of stready-state creep; Ω – ordinary load, Ω_C – creep with DC

As one can see, the synthetic theory gives rise to a singular point on the loading surface, loading point A.

When considering the creep accompanied by direct current, formulae (16)-(18) lead to the following transformations in the loading surface.

Formula (16) results in the decreasing of the initial radius of yield surface (the virgin state of material), namely $S_P \big/ \sqrt{1+C^2}$ instead of S_P (Fig. 3, surface Ω_C). This fact implies that the stress needed to induce plastic shift within a slip system is less for the case of simultaneous action of loading and the passage of current. Really, while, in the case of ordinary loading, the stress vector reaches plane 2 from Fig. 3 at point A_1, the plane with the same orientation (plane 1) is achieved by the stress vector at point A_{1C} (creep with DC).

Another result is that, for a given stress vector, the distance covered by a plane increases due to the action of DC. Indeed, both planes from Fig. 3, plane 1 and 2, are translated by the stress vector into position 3, but it is clear that the plane 1 travels the greater distance than the plane 2 does. This result is in accordance with the conclusion obtained from formula (17), namely, the passage of current intensifies the slip within every slip system.

In addition, Fig. 3 demonstrates the final result, the passage of current increases the number of slip systems where plastic shifts occur ($\beta_1 < \beta_{1C}$), obtained by Eq. (18).

Conclusions

The model describing the steady-state creep of metals under the action of direct current has been developed in terms of the synthetic theory of irrecoverable deformation. To catch the effect the current exerts upon the creep rate, we extend the constitutive relationships of the synthetic theory, which govern the hardening of material, by a term containing the current intensity. As a result, we have derived relationships for the steady-state creep rate coupled with DC. Results obtained in terms of our model show good agreement with experimental data. Understanding the evolution of loading surface, i.e. the onset and development of irrecoverable deformation (plastic or creep), is critical in modeling any type of deformation. For this reason the analytical results obtained are accompanied by the analysis of loading surface for the case of ordinary steady-state creep and that in electrical field.

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