Multidisciplinary Optimization of Journal Bearings, using a RVA Evolutionary Type Optimization Algorithm

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Abstract: In this paper, the optimum geometry of a journal bearing is calculated for minimum friction coefficient and for maximum load carrying capacity. The optimized versions can be compared, which makes it possible to draw important conclusions concerning the necessary constructional changes in journal bearings if we want to increase the load carrying capacity or to decrease the energy loss due to friction. It is also interesting to see the differences in the load carrying capacity when the friction coefficient is minimal or in the friction coefficient when the load carrying capacity is maximal. During the investigations the basic equation of the THD (Thermo-Hydrodynamic) state of hydrodynamic journal bearings is solved by using the finite difference technique, while for the optimization the RVA (Random Virus Algorithm) is used. As the result of the optimization process, the load carrying capacity can be increased by more than 28% or the friction coefficient in the oil film can be decreased by 29% compared to the starting design.

Keywords: friction coefficient; journal bearing; load bearing capacity; optimization; RVA

1 Introduction

Hydrodynamic sliding and journal bearings are commonly used in many fields of mechanical and energy engineering [1]. The efficiency and performance of such bearings are determined by their load carrying capacity and frictional coefficient, or friction force. Decreasing the friction force in the bearings makes it easier to maintain the motion, which will decrease the energy (fuel) consumption, resulting in the possibility of significant cuts in operational costs and environmental pollution.

Finding the maximum load carrying capacity or the minimum frictional coefficient needs optimization techniques, while the presence of the lubricant (most often oil) and the effects of the temperature will enlarge the analysis process into a multi-physics or multi-disciplinary analysis process. Therefore the whole optimization process will be an example of Multi-physics Optimization or Multidisciplinary Optimization (MDO) [2]. The disciplines involved in this complex process are: fluid flow, heat transfer, solid mechanics, elasticity and tribology. The complexity of these analysis processes makes it necessary to use several numerical methods (finite difference, finite element), which can sometimes be time consuming and takes a large amount of computing capacity. Therefore very efficient and quick optimization algorithms are needed for the Multidisciplinary Optimization of hydrodynamic bearings, in order to avoid overwhelming calculations and excessively long calculation times.

Over the last 2-3 decades, evolutionary type optimization algorithms have provided the best ways to solve MDO problems, because of their efficiency, robustness and quick convergence. The basic idea of these algorithms came from the study of the behavior and reproduction of several natural systems [3] such us genetic engineering (Genetic Algorithm GA [4]), evolution of biological populations (Evolutionary Programming EP [5], or Evolutionary Strategies ES [6]), Reproduction of Bacteria (Bacterial Foraging Algorithm, BFA [7]), behavior of natural swarms (Particle Swarm Optimization, PSO [8], or Virus-Evolutionary Particle Swarm Optimization, VEPSO [9]), behavior of animal colonies (Ant Colony Algorithm, ACA [10]), or behavior and reproduction of viruses (Random Virus Algorithm, RVA [11]).

In this paper, the Random Virus Algorithm (RVA) is used for the optimization of hydrodynamic journal bearings. For the numerical analysis of the hydrodynamic bearings in each step the finite difference technique is used. The temperature dependence of the lubricant characteristics (density, viscosity) is taken into consideration by iterative steps during the numerical solution of the governing partial differential equation. Two optimized geometries are compared: in first case, the geometry of the bearing is optimized for maximum load bearing capacity. In the second case, the bearing is optimized for minimum frictional coefficient in the lubricant film. Both optimization processes start from the same starting design and are compared each to another and to the original design. On the basis of the comparisons interesting conclusions can be drawn concerning necessary constructional and geometrical changes in order to increase the load carrying capacity of the bearing or to decrease the friction coefficient.

This paper is organized as follows: Section 2 describes the finite difference calculation used for determining the pressure distribution in the lubricant film. Section 3 shows the details of the optimization problems and RVA optimization algorithm. Section 4 gives the results of the optimization procedures and Section 5 contains conclusions.

2 Pressure Distribution in the Lubricant Film

The applied numerical method is applicable to any problem that can be described by linear partial differential equations [12]; in this work it is used for solving the pressure distribution *p(x,z)* in the fluid film of hydrodynamic journal bearings, for a given gap shape function *h(x,z)*. The governing equation of this problem is the Reynolds equation:

$$\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)-6\eta U\frac{\partial h}{\partial x}-12\eta\frac{\partial h}{\partial t}=0 \tag{1}$$

In Equation (1) the relative velocity of the sliding surfaces is denoted by U, and η means the absolute viscosity of the lubricant. Geometry of the bearing is shown in Fig. 1.

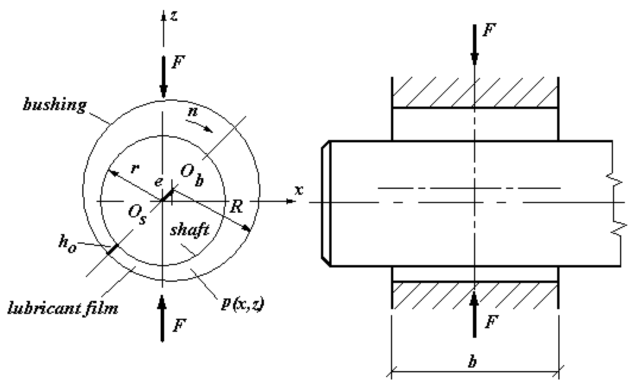

Figure 1

The geometry and dimensions of a hydrodynamic journal bearing

Equation (1) can be written into matrix form, after the discretization of the fluid film domain between the sliding surfaces, by using finite differences, as shown in equation (2). The vector **p** collects the nodal values of the pressure function, and elements of matrix **K** depend on the nodal values of the gap shape function: During the finite difference solution of the equation (1), $h_{i,j}$ represent the nodal values of the gap function and $p_{i,j}$ are the nodal values of the pressure function in the fluid film. By using this notation, the Reynolds- equation can be written in nodal points marked by i,j [15].

In case of a finite difference mesh having *u X v* nodes, the matrix **K** will have a bandwidth of *2v – 3*, after the applications of the boundary conditions.

$$\mathbf{Kp} + \mathbf{g} = \mathbf{0} \tag{2}$$

The nodal form of the Reynolds equation:

$$A_{i,j}p_{i+1,j} + B_{i,j}p_{i-1,j} + C_{i,j}p_{i,j+1} + D_{i,j}p_{i,j-1} + E_{i,j}p_{i,j} + G_{i,j} = 0$$

where

$$A_{i,j} = \frac{1}{4dx^2}\left(h_{i+1,j} - h_{i-1,j}\right) + \frac{h_{i,j}}{3dx^2} = -B_{i,j}$$

$$C_{i,j} = \frac{1}{4dz^2}\left(h_{i,j+1} - h_{i,j-1}\right) + \frac{h_{i,j}}{3dz^2} = -D_{i,j}$$

$$E_{i,j} = -\frac{2h_{i,j}}{3}\left(\frac{1}{dx^2} + \frac{1}{dz^2}\right) \quad ; \quad G_{i,j} = \frac{\eta U}{dx}\left(\frac{1}{h_{i+1,j}} + \frac{1}{h_{i-1,j}}\right)$$

The density and the viscosity of the lubricant is the function of the operating temperature of the bearing. This is taken into account by an iteration during this numerical solution. At the beginning, an approximate temperature is supposed and the equation is solved with characteristics calculated for this temperature. On the basis of the results, new and more accurate temperature value can be determined. The whole calculation will be repeated with lubricant characteristics calculated with this new temperature value. Several trial calculations and experiences show that after three or four iteration cycles, the difference between the temperature values before and after a calculation step will be smaller than 1°C, which is enough accurate for the further calculations. The elastic deformation of the shaft and housing could be checked by finite element model after the solution (quasi-TEHD state), this could be effective if these deformations are small comparing to the gap size (for example in case of steel shaft and steel bushing).

Once we have the solution of this process for the nodal values of the pressure function, the load carrying capacity of the surface pairs F_n can be calculated by numerical integration, using the characteristic sizes (r, R, b, h_o, e) of the bearing, according to Fig. 1.

$$F_1 = \int_{\varphi=-(\beta-\varphi_1)}^{\varphi_1} \int_{z=-\frac{b}{2}}^{b/2} p r d\varphi dz \cos\varphi \quad ; \quad F_2 = \int_{\varphi=-(\beta-\varphi_1)}^{\varphi_1} \int_{z=-\frac{b}{2}}^{b/2} p r d\varphi dz \sin\varphi \quad ;$$

$$F_n = \sqrt{F_1^2 + F_2^2} \quad . \qquad (3)$$

The friction force, which is the force needed for the relative motion between the shaft and bushing can be determined as follows:

$$F_f = \int_{x=-r(\beta-\varphi_1)}^{r\varphi_1} \int_{z=-\frac{b}{2}}^{b/2} \left(\frac{1}{2}\frac{\partial p}{\partial x} h - \eta \frac{r\omega}{h}\right) dx dz \qquad (4)$$

In equation (4) the angular velocity $\omega = 2 \Pi n$, if the unit of the angular velocity is radians per seconds and the n rotation speed is in rotations per seconds.
The frictional coefficient μ can be calculated as $\mu = F_f / F_n$. Lubrication angle β is shown in Fig. 2 together with the angle φ marking a general position of the gap function $h(\varphi)$. In general position the thickness of the lubricant film (gap function) can be calculated as:

$$h(\varphi) = R - r - e\cos\varphi \tag{5}$$

This calculation method has been verified and compared to the analytical solutions for infinite width bearings given by Szota and Döbröczöni [12], optimized for maximum load carrying capacity, and good agreement was found between the theoretical and numerical results [15]. Another verification of the method was in the case of finite sliding bearings [2] where the results of this finite difference based code were compared with those calculated by the ANSYS-FLUENT [13] program system and once again good agreement was detected.

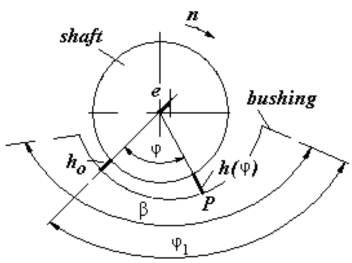

Figure 2
Characteristic angles of the bearing in general case

On the basis of these comparisons it can be concluded that the finite difference based calculation method proposed here can be applied for further investigations of THD state of hydrodynamic sliding surface pairs with finite or infinite width and for hydrodynamic sliding bearings and journal bearings. This calculation method will be integrated in this work with the RVA optimization algorithm for the optimization of a finite width hydrodynamic journal bearing. Two optimization processes will be compared: in the first case the bearing is optimized for maximum load carrying capacity (the objective function is F_n), in the second case, the same bearing will be optimized for minimum friction coefficient (the objective function is μ) all the other parameters of the investigations will be the same. Optimal geometrical parameters are compared for this two objective functions in order to draw some useful conclusions for the manufacturers, designers or users of this type of bearings about efficient ways to increase the load carrying capacity or decrease the friction resistance of the bearing by modifying only the geometrical sizes.

3 Description of the Optimization Problem and the Random Virus Algorithm

As a starting design of the bearing optimization, a hydrodynamic bearing of an electric generator, is selected. The input data of the bearing are the following: input power: *P = 1300 kW*, the minimum required load bearing capacity: *F = 31400 N*, rotational speed: *n = 1000 rpm*, width to diameter ratio: *b/d = 1.3*, environmental temperature: 20°C, lubrication angle*:* $\beta = 180^o$, maximum permissible operational temperature: $T_{max} = 80\ ^oC$, material of the shaft: structural steel with yield stress $R_{eH} = 275\ MPa$, material of the bushing: structural steel, with white alloy lining, maximum permissible value of the average pressure in the fluid film: 1 MPa and average surface roughness value: on the shaft - 0.16 μm, on the bushing - 0.32 μm.

The design variables are the nodal coordinates of the finite difference mesh keypoints. For the meshing 40 key nodes are used with variable coordinates (these are the optimization variables) and remaining nodes are placed depending on the keypoints in order to make higher density mesh. For the first optimization problem the objective function is the load carrying capacity of the bearing F_n, which is to be maximized. In the second optimization problem the friction factor μ is minimized. For both optimization problems the generator bearing is used with the given input data as the starting design.

Size constraints: 0 [mm] < *r* < 500 [mm] , 0 [mm] < *R* < 500 [mm] , 0 [mm] < *e* < 10 [mm]. Implicit constraints: the pressure function should fulfill the Reynolds equation (1) of hydrodynamic surface pairs; the shaft diameter should be higher than the minimum required diameter given in equation (6); the average pressure in the fluid film should be smaller than the maximum permissible average pressure as it is shown in equation (6); and the minimum gap distance h_o should be considerably higher than the sum of the maximum roughness of the surfaces. Maximum permissible operation temperature of the bearing is 80 °C.

$$r \geq 0.5\sqrt{\frac{F}{\bar{p}_{adm} b/d}}, \qquad h_0 \geq 4.5(R_{a1} + R_{a2}), \qquad \bar{p} \leq \bar{p}_{adm} \tag{6}$$

According to the logic of the RVA optimization algorithm, the first step is to create the first (or starting) population of the possible solutions fulfilling the constraints (Fig. 3). Once the starting population has been generated, each member of the population will reproduce, creating three new members each. This process is stronger than a nuclear explosion, so in the remaining part of the optimization the selection of the best members and elimination of members without good enough objective function values will be very important. At least 60% of the new and of the total members should be eliminated after each population in order to avoid overwhelming calculations. The members that survive this strict selection procedure will give the second population. The programming of the RVA algorithm is very simple, easy to carry out in any programming language or in macro languages of finite element program systems, if available.

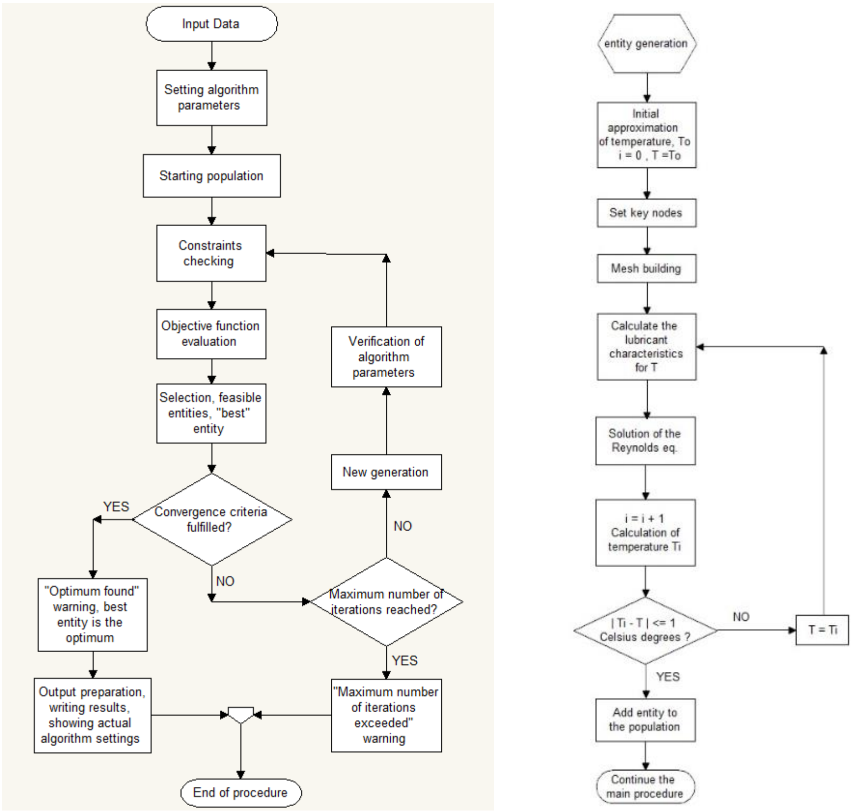

Figure 3

Flow chart of the RVA optimization algorithm and entity generation

This procedure will continue until the pre-defined optimum conditions are fulfilled. Several benchmark problem runs and numerical experiments have shown that the algorithm is very efficient: in the optimization problem investigated in this work 6 populations were enough to find the optimum. The total computation time required for a complete run was 35 minutes on an Intel core i5 desktop computer. The j^{th} population: $P_j = \{x_i\}_j$; the reproduction formula:

$$y_k = x_k + R_k q^* (up_i - lw_i) \qquad (7)$$

Where y_k means the k^{th} variable value of the new member, q* is the spreading parameter, and R_k is a random number between 0 and 1, simulating the possibility of random mutations. Setting the spreading parameter properly is also very important, because it can have an important effect on the efficiency of the algorithm. This needs a great deal of experimentation and unique fine-tuning work for each optimization problem. For this optimization process the best value for the spreading parameter was 0.8 in the first three populations and 0.25 afterwards. If the maximum number of iterations is reached without fulfilling the convergence criteria, it means that the search procedure needs more iterations and so the optimization is stopped, but during the results display a warning will say that there is a danger of a local optimum and possibly a new run will be necessary with other parameters or with a higher maximum number of iterations permitted.

4 Results of the Optimizations

As numerical example a hydrodynamic journal bearing of an electric generator [14] has been optimized by using the multi-disciplinary optimization (MDO) procedure described in the section 2 and 3. Two calculations have been made: in the first one the bearing is optimized for minimum friction factor in the lubricant film, which gives minimal force necessary to maintain the relative motion (turning) of the shaft. In the second study the same starting design of the bearing was optimized for the maximum load carrying capacity. The two resulting optimized version can be compared in order to draw conclusions for the further design, fabrication and operation of the bearings. Table I shows all the important parameters of the bearing, using the optimum results of the design variables for the calculation of the geometrical dimensions of the bearing. In the table it can be seen that important achievements were made as results of the optimizations: The load carrying capacity of the bearing was increased by more than 28%, while the friction factor was decreased by 29%.

Optimization results show that the increase of the load carrying capacity was realized by changing the shaft radius from 80 mm to 95 mm and changing the bushing radius from 80.13 mm to 95.16 mm. The eccentricity was increased from 79.86 μm to 101.72 μm. As the result of these changes the minimum gap h_o increased from 50.54 μm to 59.77 μm. In the case of maximum load carrying capacity the average value of the pressure in the lubricant was decreased comparing to the starting design from 0.9435 MPa to 0.8630 MPa, but the friction factor remains the same, at 0.003. The temperature of the lubricant T is the active constraint, 79.68^{o}C while the permissible temperature is 80^{o}C. The joint quality of the bearing remains unchanged.

Regarding the optimum results, for minimum friction factor, it can be seen in the Table I that compared to the starting design the shaft diameter remains the same, the bushing radius decreased from 80.13 mm to 80.104 mm and the eccentricity decreased from 79.86 μm to 68.90 μm. As the result of these changes, the minimum gap decreased from 50.54 μm to 35.10 μm.

Table I

Optimization results for two different objective functions

Parameters	Starting	Min μ	Max F_n
r [mm]	80	80	95
R [mm]	80.130	80.104	95.161
e [mm]	0.0799	0.069	0.1017
μ	0.00305	0.002163	0.00305
F_n[N]	31400	31400	40500
T [^{o}C]	74.95	58.88	79.68
$\bar{p}$ **[MPa]**	0.9435	0.9435	0.8630
Decrease in μ [%]	-	- 29.24	0
Increase in F_n [%]	-	0	+ 28.98
h_o [μm]	50.54	35.10	59.77
Joint (ISO)	H7/a9	H7/b8	H7/a9

In the case of the minimum friction factor the load carrying capacity (31400 N) and average pressure in the lubricant (0.9435 MPa) remain the same. The temperature of the lubricant decreased to 58.88^{o}C from the original 74.95^{o}C. In this case the active constraint is the average pressure. The tolerance of the shaft is stricter (narrower) than for the starting design. The relative position of the shaft diameters and the diameters of the bushing can be compared in Fig. 4. The figure shows that in order to increase the load carrying capacity of the bearing it is necessary to increase the shaft diameter and the bushing diameter comparing to the starting design, and the eccentricity and the minimum gap size should be also increased.

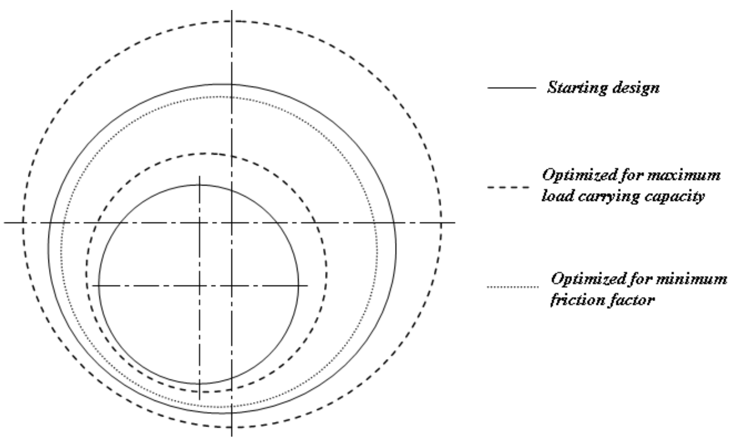

Figure 4

Schematic position of the most important diameters, comparing optimized versions for different objective functions

It can be seen from Fig. 4 that in order to decrease the friction factor, one should decrease the bushing diameter, the shaft diameter should remain unchanged, and the eccentricity and minimum gap size should be decreased compared to the starting design. Fig. 4 shows only the relative position of the diameters (it is possible to see only which one to increase, which one to decrease, and not the real dimensions, because of very small differences).

Conclusions

In this paper, a cylindrical hydrodynamic journal bearing (THD state) has been optimized for two different objective functions, all other parameters and constraints are the same. The starting design for the optimizations is the hydrodynamic journal bearing of an electric generator. During the first optimization, the objective function is the load carrying capacity and its maximum is determined. The second optimization study, is the minimization of the friction factor in the bearing.

The pressure distribution is determined by numerical solution of the Reynolds equation, using a finite difference computational code and the algorithm of the optimization is the RVA algorithm. During both of the optimizations the design variables are the nodal coordinates of the keypoints used for the finite difference mesh. The implicit constraints are:

- The pressure field in the lubricant film should fulfill the Reynolds equation
- Shaft diameter should be higher than the minimum necessary shaft diameter
- Maximum admissible value of the average pressure is 1MPa in the lubricant film
- Minimum gap distance between the shaft and the bushing should be higher than the sum of the maximum roughness of the surfaces
- Maximum permissible operation temperature in the lubricant is $80^{o}C$
- Temperature dependence of the lubricant characteristics is taken into account by an iterative process during entity generation

Final results of the optimizations show significant achievements: a 29% decrease in the friction coefficient, and a 28% increase in the load carrying capacity. The decrease in the friction coefficient can be very encouraging in terms of operation costs (since a smaller amount of energy is needed for the motion, this allows a large amount of fuel to be saved), and environmental protection (a smaller amount of fuel leads to lower levels of pollution). Higher load carrying capacity can be of interest to designers and/or manufacturers, as this can improve the market position of the factory or decrease the manufacturing costs.

The final optimal results are collected in table (Table I), showing all the important parameters of the starting design, the optimal version for the minimum friction coefficient and the optimal version for maximum load carrying capacity. The schematic position of the most imported sizes (shaft radius, bushing radius, eccentricity) can be seen in one figure (Fig. 4) together, in order to compare more easily the positions and relations of these sizes regarding all three versions (starting design, minimum friction coefficient and maximum load carrying capacity).

Comparison of the numerical results of the optimizations leads to the following conclusions:

- Increasing the load carrying capacity requires increasing all the principal sizes. As a result of these changes, the minimum gap distance will also increase. In this case the active constraint is the temperature of the lubricant, while the friction factor remains the same as it was in case of the starting design. The average value of the pressure in the lubricant decreased by approximately 10%.
- Decreasing the friction factor, requires decreasing the bushing radius and the eccentricity, while the shaft radius remains the same. The minimum gap distance also decreases. In this case the active constraint is the maximum permissible average pressure in the lubricant, and the temperature decreases by approximately 20%. The load carrying capacity remains the same as it was in the starting design.
- The changes arising from optimizations will have an effect the ISO quality of the joint between the shaft and the bushing. In the case of maximum load carrying capacity, the joint can be the same as it was in the starting design (H7/a9), but for the minimum friction coefficient it should meet higher standard: H7/b8, which will need a finer surface for the shaft.
- Regarding the manufacturing costs for the changes needed, the optimal versions, the necessary modifications for the minimum friction coefficient optimization seem to be easier and cheaper, because in this case the shaft diameter does not need to be altered, although finer surface treatment will be necessary, and the bushing diameter can be decreased slightly by methods such as the application of some coatings. The maximum load carrying capacity alternations may be more expensive, because a higher shaft diameter will be necessary (this can be realized by changing the shaft or applying a sleeve on the shaft) and a higher bushing diameter will be necessary, which may require a cutting process and could have further costs.
- It is interesting to see that the two different objective functions need changes, to the starting design, which are totally in contrast: maximum load carrying capacity requires increasing the sizes, while the minimum friction coefficient needs these parameters to be decreased. Therefore, it is advised, to consider carefully, the selection, as the objective function, in a real case.

In further investigations, more parameters could be included as design variables or objective functions (oil viscosity, surface roughness of the shaft and bushing), which will allow the calculation in a more realistic way, the costs of some changes in the design variables.

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List of Symbols

Name of symbol	Unit	Short description
$h(x,z)$, $h(\varphi)$	[mn]	Gap function of the bearing (can be function of coordinates x, z or of angle φ .
$p(x,z)$, $p(\varphi)$	[MPa]	Pressure function in the lubricant film.
x, y, z		Axis of the global coordinate system.
η	[Pas]	Absolute viscosity of the lubricant.
U	[mm/s]	Velocity of the relative motion.
t	[s]	Time.
F, F_n	[N]	Load of the bearing, normal load.
h_o	[μm]	Minimum gap distance.
e	[mm]	Eccentricity between the shaft and the bushing.

n	[rpm]	Rotation speed of the shaft.
r	[mm]	Radius of the shaft.
R	[mm]	Radius of the bushing.
b	[mm]	Width of the bearing.
O_b		Center point of the bushing.
O_s		Center point of the shaft.
$h_{i,j}$	[mm]	Nodal values of the gap function.
p_{ij}	[MPa]	Nodal values of the pressure function.
K	[1/mm]	Coefficient matrix containing nodal values of gap function.
p	[MPa]	Vector of nodal pressure values.
g	[N/mm^3]	Vector of constants.
$A_{i,j}$, $B_{i,j}$, $C_{i,j}$		Auxiliary parameters.
$D_{i,j}$, $E_{i,j}$, $G_{i,j}$		
F_1	[N]	Load component in direction $\varphi = 0$.
F_2	[N]	Load component perpendicular to F1.
Name of symbol	Unit	Short description
F_f	[N]	Friction force.
ω	[rad/s]	Angular velocity.
φ	[^{o}]	Angle describing the position where the gap is measured.
β	[^{o}]	Lubrication angle.
P	[kW]	Input power.
d = 2r	[mm]	Shaft diameter.
T_{max}	[^{o}C]	Maximum permissible operational temperature.
R_{eH}	[MPa]	Yield stress of the shaft material.
$\bar{p}_{max}$	[MPa]	Maximum permissible value of the pressure in the lubricant film.
$\bar{p}$	[MPa]	Average pressure in the lubricant film.

R_{a1}, R_{a2}	[μm]	Average surface roughness of the shaft and bushing.
P_j		The j^th population in the RVA algorithm.
$\{x_i\}_j$		Variables of the j^th member of the population
y_k		k^th variable of the "new" member.
x_k		k^th variable of the "old" member.
R_k		Random number having a value between 0 and 1.
q^*		Spreading parameter
up_i		Upper limit for the explicit constraint of the i^th design variable.
lw_i		Lower limit of the explicit constraint of the i^th design variable.
μ		Friction factor