

Stress-Strain Interaction Model of Plasticity

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Abstract: The article firstly investigates a discrete numerical model of finite interaction between successive microstructural bond failures and remaining intact internal bonds in materials. Secondly, it reveals the general linear finite continuous cause and effect interaction concept. The interaction model is examined numerically, experimentally and analytically on an illustrative case of a parallel system of bonds. The general concept is applied to the macroscopic stress-strain interaction model of material plasticity. Examples of metallic materials are elaborated on reported theoretical and experimental strain data.

Keywords: yielding; necking; plasticity; tensile test; metals; interaction

1 Introduction

This research is motivated by the stance that the curve fitting methods based on experimental data about plasticity in engineering of materials often do not have appropriate physical foundation and in some cases are not accurate enough for practical applications. The methods of thermodynamics, continuum mechanics and dislocation physics in materials sciences provide theoretical solutions for complicated problems in engineering plasticity. The article advocates that a comprehensive, more accurate and straightforward definition of non-linear material mechanical properties of plasticity might be of interest in practice, particularly for determination of the ultimate strength of engineering structures. The plasticity model in this article focuses on internal failures of microstructural bonds between discrete material particles rather than on crystallographic defects in shape regularities resulting in dislocation of particles. The article investigates the suitability of an empirical Cause-and-Effect Interaction concept (CEI) for definition of a Stress-and-Strain Interaction (SSI) macrostructural model of plasticity definable by propensity to and intensity of interaction. The applications of the numerical procedure of the analytic model of the SSI concept are illustrated by reported examples of experimental results of plasticity testings.

2 The Linear CEI Concept of Plasticity

The formulations of microstructural processes and applications of thermodynamics in material sciences, e.g. [1], and macroscopic continuum mechanics are in wide use for modelling of plasticity in engineering problems, e.g. [2] [3] [4] [5] [6] [7] [8]. Multilevel theories of irrecoverable deformations, where macro-strains are related to the processes occurring on the micro level of material, provide relatively simple stress-strain and strain-time formulae [9] [10] [11]. The rearrangements of the internal structures within which the particles are being collectively dislocated to new positions of internal equilibrium are frequently explicated in discrete dislocation physics as interactions, e.g. [12] [13] [14] [15] [16]. Simulation methods based on dislocation physics and using finite element analysis, e.g. [17] [18] [19], are important but time-consuming numerical tools. The linear CEI concept [20] [21] [22] in the article holds the internal failures of bonds among discrete particles in materials accountable for the defects on the microstructural level. The starting assumption in this model is that the macrostructural mechanical properties of materials under loading depend on great but finite total number C_R of intact internal elastic bonds intrinsic to the basic material physical microstructure (Fig. 1). The primary effect $E(C)$ induced by successive bond failures C is gradual reduction of strength until yielding of overloaded elastic bonds. The primary effect E (weakening, yielding, plasticity) under loading is linearly related to the cause C (elastic bond failures) as shown:

$$E(C) = p \cdot C \quad (1)$$

Simultaneously the remaining number of intact elastic bonds ($C_R - C$) preserves the residual load-carrying capacity (strength), which is the left-over resistance to deformation after C successive bond failures. From the initial assumption of linearity between the primal cause and effect (1), it follows that the durability $R(C)$ also has to be linearly related to the remaining number of intact bonds:

$$R(C) = r \cdot (C_R - C) \quad (2)$$

The hypothesis of the study is that the weakening $E(C)$ is not just a simple cause-and-effect relation $C \Rightarrow E$ or $E \Leftarrow C$ as in (1 and 2) with respect to the cause C but rather a more complex cause-and-effect interaction $C \Leftrightarrow E$. The weakening $E(C)$ with respect to the durability $R(C)$ is the consequence of the redistribution of internal loads between the numbers of failed C and intact ($C_R - C$) bonds (Table 1). The interaction rate expresses how the weakening $E(C)$ (1) reduces the remaining durability $R(C)$ (2). That in turn interactively accelerates the primary weakening $E(C)$ by the amount of $E(I)$ induced by a secondary cause $I(C)$ due to interaction with the cause C (e.g. Fig. 2). Hence, the interaction rate is simply in proportion i to the ratio of numbers of failed C and intact ($C_R - C$) bonds as shown next:

$$\Delta E[I(C)] = \frac{E(C)}{R(C)} = i \cdot \frac{C}{C_R - C} \quad (3)$$

The secondary weakening $E(I)$ (3) by each successive failure of elastic bonds results in redistribution of load to remaining intact bonds, that is, the weakening of the elastic system of bonds until fracture. However, the overall secondary weakening $E(I)$ by each successive failure of elastic bonds accumulates all the former effects induced by the interaction between the weakening $E(C)$ and the durability of material $R(C)$ that is expressed by the following summation:

$$E[I(C)] = \sum_{C=1}^{C_R} \Delta E[I(C)] \tag{4}$$

The overall plasticity may be viewed as the consequence of the overall weakening $E(C,I)=E(C)+E(I)$ resulting from the primary $E(C)$ (1) and secondary $E(I)$ (4) weakening (e.g. Table 1, Fig. 2).

The two parameters p and $i=p/r$ (1-4) represent the propensity to and the intensity of interaction between the weakening and the durability of material. The work done in weakening $E(I)$ (4) (Fig. 1) is equivalent to the accumulated energy of interaction $U[E(I)]$ attainable by integration of all successive secondary effects of failures of elastic bond commonly available from experiments, as shown below:

$$U[I(C)] = \sum_{C=1}^{C_R} E[I(C)] \tag{5}$$

The exposed CEI concept (1-5) is not in contradiction with the rules in mechanics.

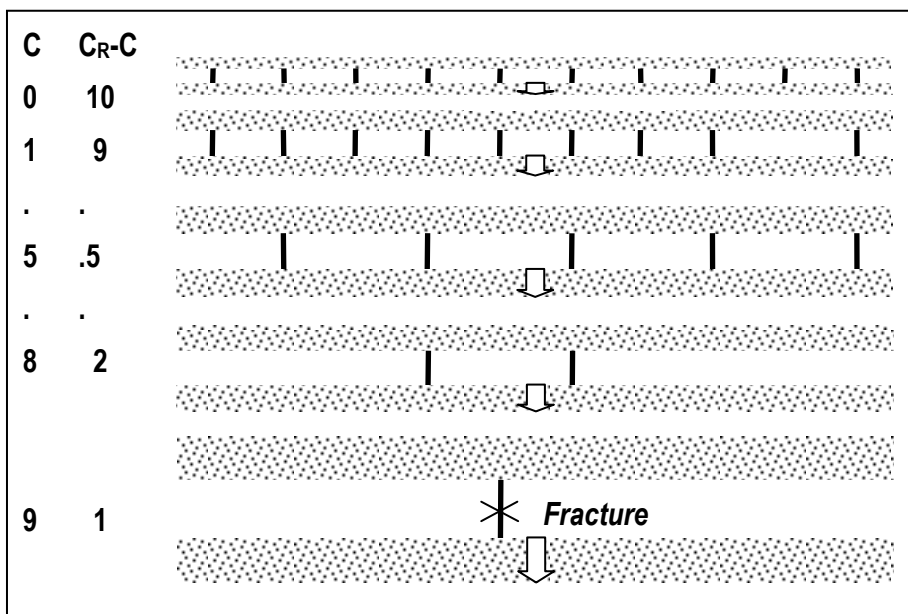


Figure 1

Internal elastic bond failures till fracture in parallel arrangement of bonds ($C_R=10$)

Table 1
CEI calculation for $C_R=10$ initial intact internal bonds in material

C	E(C)	C_R-C	Redistribution	$C/(C_R-C)$	E(I)	E(C,I)	U(I)
0	0	10	10/10=1+0/10	0,00	0,00	0.00	0.00
1	1	9	10/9=1+1/9	0,11	0,11	1.11	0.11
2	2	8	10/8=1+2/8	0,25	0,36	2.36	0.47
3	3	7	10/7=1+3/7	0,43	0,79	3.79	1.26
4	4	6	10/6=1+4/6	0,67	1,46	5.46	2.72
5	5	5	10/5=1+5/5	1,00	2,46	7.46	5.18
6	6	4	10/4=1+6/4	1,50	3,96	9.96	9.14
7	7	3	10/3=1+7/3	2,33	16,2	13.29	15.4
8	8	2	10/2=1+8/2	4,00	10,3	18.29	25.7
9	9	1	10/1=1+9/1	9,00	19,3	28.29	45.0
10	10	0	10/0=1+10/0	∞	∞	∞	∞

3 Mathematical Formulation of the CEI Concept

The direct application of infinitesimal calculus to the massive discrete systems of a great but finite number of micro-structural bonds decomposed into linear (1) and nonlinear (4) parts $E(C,I)=E(C)+E(I)$ provides the following analytical formulation of the general CEI concept (1-5) [20] [21] [22] (Fig. 2) of continuous finite systems on the macroscopic level as shown:

$$\frac{d^2E(C,I)}{d^2C} = i \cdot \frac{1}{(1-c)^2} \quad (6)$$

$$\frac{dE(C,I)}{dC} = p + i \cdot \frac{c}{1-c} \quad (7)$$

$$E(C) = p \cdot \int_0^c dC = C_R \cdot p \cdot c \quad (8)$$

$$E(I) = i \cdot \int_0^c \frac{C}{C_R - C} dC = i \cdot C_R \cdot [-c - \ln(1-c)] \quad (9)$$

$$U(I) = \int_0^c E(I) dC = i \cdot C_R^2 \cdot \left\{ -c^2 / 2 + [c + (1-c) \cdot \ln(1-c)] \right\} \quad (10)$$

The relation $C(E)$ can be obtained from (9) by integration of the inverse derivative of (7), that is, the rate of change of the cause C with respect to the effect E as:

$$\frac{dC}{dE(C,I)} = 1/[dE(C,I)/dC] = \frac{1-c}{p+c(i-p)} \tag{11}$$

The interaction intensity parameter is attainable from the equivalence of the observable work $W(I)=U(I)$ done on interactions and the interaction energy (10):

$$i = W(I)/C_R^2 \cdot \{-c^2/2 + [c + (1-c) \cdot \ln(1-c)]\} \tag{12}$$

In the mathematical formulation of the CEI concept (6-11) $c=C/C_R$ and $e=E/C_R$ are the dimensionless cause C and effect E relative to final value C_R .

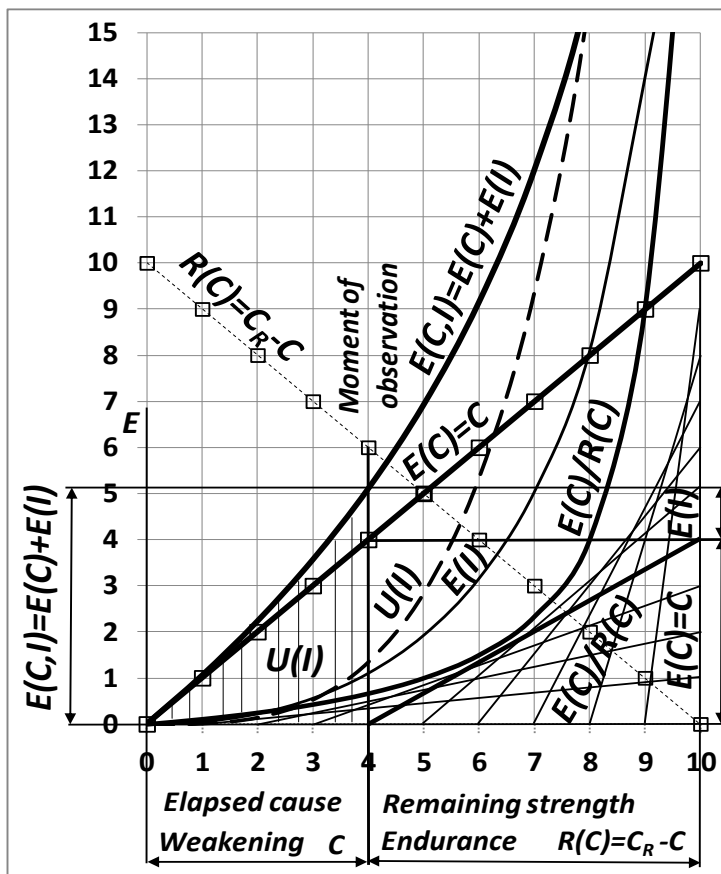


Figure 2

Numeric and analytical example of the CEI concept for $C_R=10$ bonds (Table 1)

In terms of the exposed CEI concept, the strain hardening in polycrystalline materials under excessive mechanical loadings may be viewed as the consequence of overloading of remaining intact bonds after some bonds have failed. The resulting load redistribution increases the internal stresses in microstructural grain boundaries that intensify the massive propagation of dislocations in material followed by observable macroscopic permanent deformations.

4 Experimental Investigation of the CEI Concept

The following experiments physically reproduce the CEI concept for $C_R=10$ bonds simulated by ten elastic rubber band bonds in parallel arrangement under constant load (Fig. 1). Rubber band bonds of lengths $\lambda=4$ cm, $\lambda=6$ cm and $\lambda=12$ cm are investigated in five independent tests each (Fig. 3).

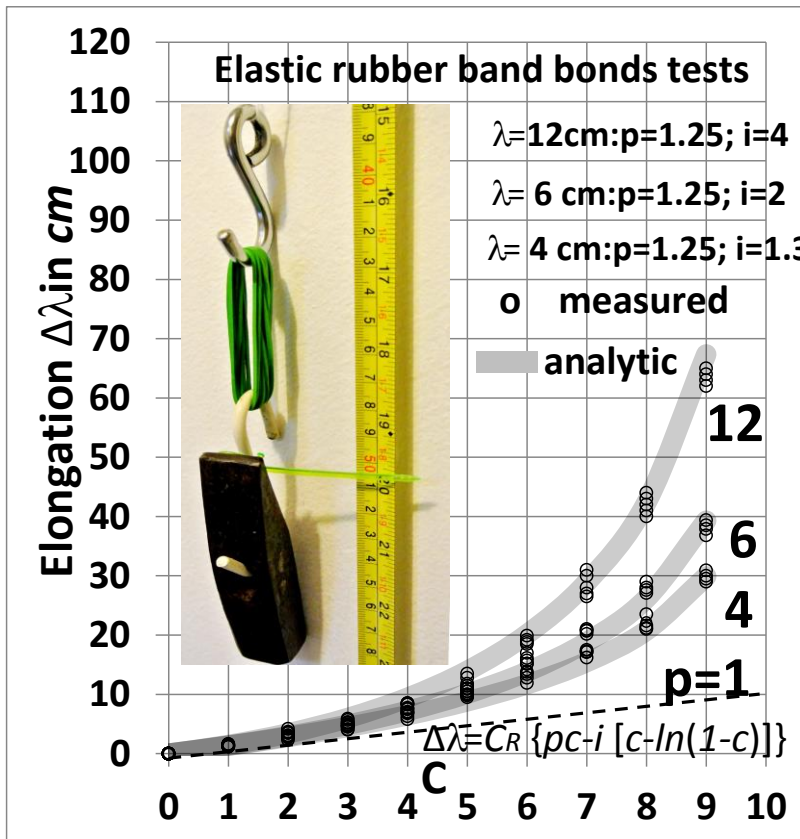


Figure 3

Tension experiments on $C_R=10$ elastic rubber band bonds and CEI analytical results

Each experiment consists of measurements of elongations $\Delta\bar{y}$ after random one-by-one removal of rubber bands. The elongations are used to find the propensity p and intensity i parameters of the CEI model (6-11). The propensity to interaction p is obtained by measurement of the elongation for a single band under the same load. The interaction intensity i is determined from the work done in stretching/extending of the remaining rubber bands obtained by numerical integration of the elongation curves by using the trapezium rule. The experiments confirm the relation between the CEI model and the measured values (Fig. 3).

5 The Appliance of the CEI Concept on the SSI Model

The power rule was proposed earlier for fitting of non-linear $\sigma - \varepsilon$ stress-strain curves: $\sigma = \sigma_o + K \cdot \varepsilon_p^n$ [23], $\varepsilon = \sigma/E + K \cdot (\sigma/E)^n$ [24], $\sigma = K \cdot \varepsilon_p^n$ [25] $\sigma = K \cdot (\varepsilon_o + \varepsilon)^n$ [26] [27]. The exponential rule was suggested as well $\sigma = \sigma_o + \sigma_{sat} \cdot (1 - e^{-m\varepsilon})$ [28]. The study considers the Stress-Strain Interaction (SSI) model of plasticity $\varepsilon \leftrightarrow \sigma$ apparent on the macrostructural level as the manifestation of the interactions between a massive number of failed and intact bonds on the microstructural level patterned after the CEI model (1-5).

The application of the general CEI concept (6-11) to the total plastic strain $\varepsilon_p(\sigma, \sigma_I) = \varepsilon(\sigma) + \varepsilon(\sigma_I)$ composed of the primary linear plastic strain $\varepsilon_p(\sigma)$ induced by stress σ and of the non-linear accumulation of secondary plastic strains $\varepsilon_p(\sigma_I)$ resulting from the interactions of secondary stresses σ_I and plastic strains $\varepsilon_p(\sigma)$ provides the analytical terms for continuous material plasticity as follows:

$$\varepsilon_p'' = \frac{d^2 \varepsilon_p(\sigma, \sigma_I)}{d\sigma^2} = \sigma_R \cdot i \cdot \left(\frac{1}{1-s} \right)^2 \quad (13)$$

$$\varepsilon_p' = \frac{d\varepsilon_p(\sigma, \sigma_I)}{d\sigma} = \sigma_R \cdot \left(p + i \cdot \frac{s}{1-s} \right) \quad (14)$$

$$\varepsilon_p = \varepsilon_p(\sigma, \sigma_I) = \varepsilon_p(\sigma) + \varepsilon_p(\sigma_I) = \sigma_R \cdot \left\{ p \cdot s - i \cdot [s + \ln(1-s)] \right\} \quad (15)$$

$$U(E, I) = U(E) + U(I) = \sigma_R^2 \cdot \left\{ p \cdot s^2 / 2 + i \cdot \left\{ -s^2 / 2 + [s + (1-s) \cdot \ln(1-s)] \right\} \right\} \quad (16)$$

The interaction intensity can be obtained from the work $W(I)$ done in experimentally defined plastic deformations and the theoretical energy in (16) as it is shown below:

$$i = W(I) / \left\{ \sigma_R^2 \cdot \left\{ -s^2 / 2 + [s + (1-s) \cdot \ln(1-s)] \right\} \right\} \quad (17)$$

where $s = \sigma / \sigma_R$ in (13-17) is the stress σ relative to its reference value σ_R .

The parameters $p=1/P$ and $i=1/I$ in (13-17) represent the propensity to and the intensity of plasticity respectively, and can be derived from experimental data. Parameters P and I represent the propensity and intensity module of linear and non-linear plasticity induced by interaction between stresses and strains. The parameters can be obtained directly (17) or numerically using least squares or general nonlinear optimization methods.

The SSI assumption for necking is that the rate of the decrease of stresses induced by changes of the sectional geometry due to interaction between the progressing strains ε and the residual strain capacity $\varepsilon_R - \varepsilon$ can be defined analogously to (14):

$$\sigma_n' = \frac{d\sigma_n(\varepsilon_p, \varepsilon_{pl})}{d\varepsilon} = M + N \cdot \frac{e}{1-e} \quad (18)$$

The application of the CEI concept $\sigma \leftrightarrow \varepsilon$ to the decrease in stresses $\sigma_n(\varepsilon_p, \varepsilon_{pl}) = \sigma_n(\varepsilon_p) + \sigma_n(\varepsilon_{pl})$ due to necking consisting of a primary linear decrease $\sigma_n(\varepsilon_p)$ induced by strain ε_p and of non-linear accumulation of a secondary decrease $\sigma_n(\varepsilon_{pl})$ resulting from the changes in strains ε_{pl} due to interactions with $\sigma_n(\varepsilon_p)$ provides the expression for necking as follows:

$$\sigma_n = \sigma_n(\varepsilon_p, \varepsilon_{pl}) = \sigma_n(\varepsilon_p) + \sigma_n(\varepsilon_{pl}) = \varepsilon_R \cdot \left\{ M \cdot e - N \cdot [e + \ln(1-e)] \right\} \quad (19)$$

where $e = \varepsilon / \varepsilon_R$ in (18, 19) is the strain ε relative to its asymptotic reference value ε_R .

The parameters $m=1/M$ and $n=1/N$ represent the propensity to and the intensity of necking and can be derived from experimental data. Parameters M and N represent the propensity and intensity module of necking, respectively.

6 Examples

The first example demonstrates the appropriateness of the SSI model with respect to tension test results of mild shipbuilding steel (Fig. 4). The propensity to yielding $p=1/P=1/4000=0.00025 \text{ MPa}^{-1}$ is obtained by numerical derivation at the beginning of the yielding. The work done in plasticization is obtained by

numerical integration of the experimental σ - ε curve using the trapezium rule and amounts to $U=24.2$ MPa. The plasticization intensity (17) is obtained from the interaction energy (16) as $i=1/I=1/4145=0.000241$ MPa⁻¹. The SSI expression for plasticity (15) (Fig. 4) in this example is:

$$\varepsilon_p(\sigma, \sigma_1) = 150 \cdot \left\{ 0,00025 \cdot s - 0,000267 \cdot [s + \ln(1-s)] \right\} \quad (19)$$

Ramberg-Osgood power law parameters obtained by the least squares method, $K=11300$ and $n=2,36$, do not match the stress-strain curves over the whole range of the σ - ε curve (Fig. 4). The propensity modulus to necking is $M=0$ MPa. The necking intensity modulus is $N=760$. The SSI expression for necking in this example (Fig. 4) is:

$$\sigma_n(\varepsilon_p, \varepsilon_{pl}) = 474 - 0,15 \cdot 760 \cdot [e + \ln(1-e)] \quad (20)$$

The example of mild shipbuilding steel tested in the Laboratory of Experimental Mechanics of the Faculty of Mechanical Engineering and Naval Architecture shows that the stress-strain curves obtained by the SSI model based on the CEI concept fit the whole range of tests results (Fig. 4).

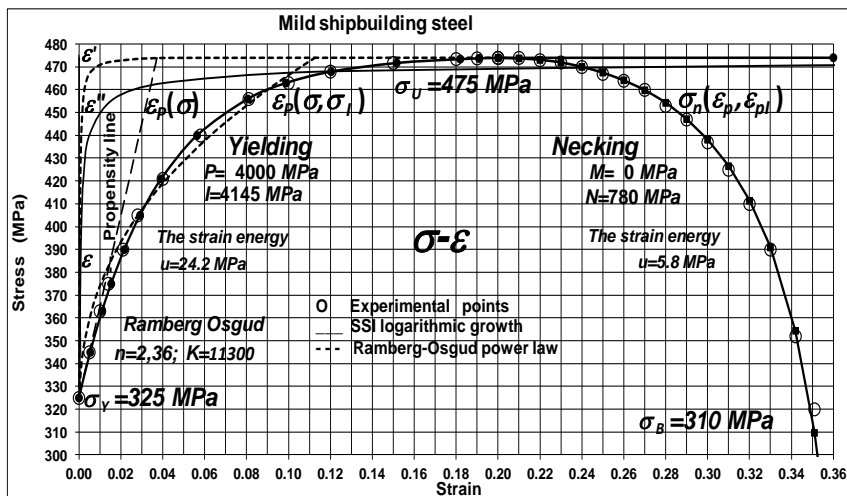


Figure 4

The SSI model of mild shipbuilding steel

The second example applies the SSI model on three types of unclassified cast irons (spheroidal, compacted and flake). The differences between the irons are in the influence of graphite morphology on stress-strain curves that harden with plastic deformation. The example confirms the smooth elastic-plastic transition typical for brittle materials [29] (Fig. 5).

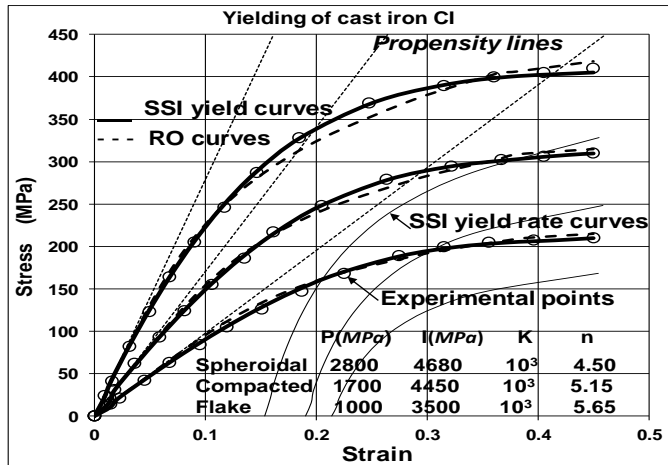


Figure 5
The SSI model of three types of cast irons

The third example compares the SSI model results with the Crystal Plasticity Finite Element (CPFE) [30] numerical study of the polyslip behaviour of single aluminium crystals of different initial crystallographic orientations (111, 112, 123, 100) under tensile loading and with experiments [31] and [32] (Fig. 6).

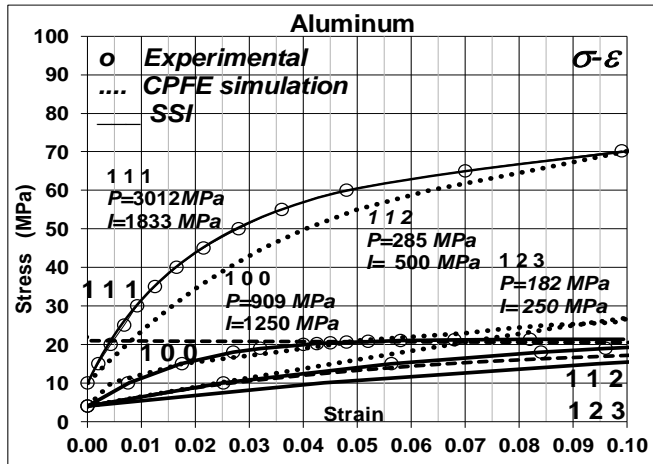


Figure 6
CPFE simulated and experimentally observed orientation-dependence of the stress-strain of single aluminium crystals during tensile loading

The fourth example compares the SSI model results with experimental results [33] and with the CPFE simulation [30] using the assumption of statistically stored dislocations (SSDs) and geometrically necessary dislocation (GND) density addition [34] for different grain diameters (14, 33 and 220 μm) (Fig. 7).

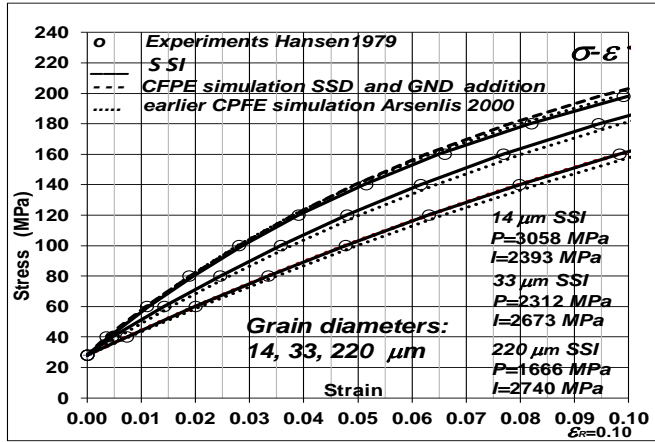


Figure 7
Stress-strain curves for average grain diameters of 14, 33 and 220 μm

The fifth example compares the SSI model results with experimental results obtained by laser extensometer type W-80 from Fiedler Optoelektronik of stress and strain measurements on aluminium specimens in time [35] (Fig. 8).

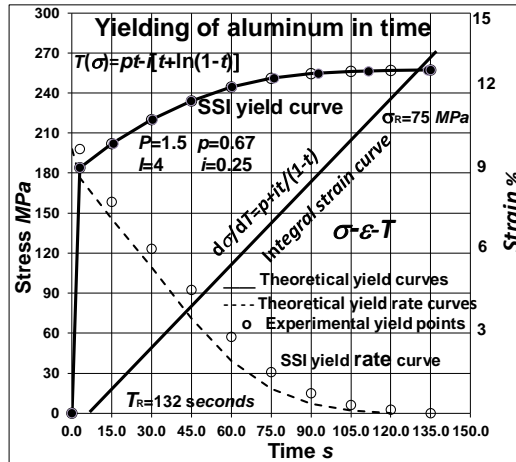


Figure 8
Stress and strain measurements on aluminium specimens in time

Conclusions

The application of the cause-and-effect interaction concept to the time independent stress-and-strain interaction model demonstrates in this paper how the material yielding and plasticity could be viewed as asymptotical growth processes analytically definable as logarithmic function over the whole range of plasticity rather than unlimited power growths in some segments of the stress-strain curves.

The rapid asymptotic growth of the sensitivity to failures may explain sudden and uncertain breaks in continuity of a material's behaviour under increasing loadings due to structural and environmental imperfections and defects in the material. The experience of this study indicates that some time-variant material mechanical properties such as creeping could be investigated in the future as asymptotically propagating processes following the cause-and-effect interaction concept.

The presented model is governed by two unique properties of a material, which are the propensity to and the intensity of interactions, both evident from experiments. The initial propensity represents the starting microstructural constellation of internal bonds between the constituent particles of a material and their consistency normally reflects the initial state of the strength of the material. The interaction intensity parameter stands for the average of massive progressions of internal bond failures relatable to the overall material durability on the macroscopic level. The two interaction parameters are straightforwardly available from standard tensile testing of material mechanical properties. For tensile tests performed in time the interaction parameters can be calibrated in the time scale.

The interaction model of material yielding elaborated in the article is not another curve-fitting method based on experimental data points but rather an implementation of a more general physical concept to investigate the mechanical properties of materials. This physical concept uses the equivalence of theoretical energy of micro-structural interactions between the failed and intact bonds to the experimentally observable stress-strain energy on macro-structural level. The general cause-and-effect interaction concept describes a part of trans-temporal continuum that relates the past and a future time separated by limitation of human's ability of perceptions beyond the instant of observation. The results in this study reveal how the empirical cause-and-effect interaction concept could be a rational approach to an alternative understanding of the non-linear material stress-and-strain interaction model, sufficiently simple and accurate for practical engineering problems of non-linear strains, strain hardening, yielding and plasticity.

Acknowledgement

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