

# A New Method for Determining the Reliability Testing Period Using Weibull Distribution

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*Abstract: In this paper, we present a calculation methodology of the testing duration of the products' reliability, using the Weibull distribution, which allows the estimation of the mean duration of a censored and/or complete test, as well as of the confidence intervals for this duration. By using these values we can improve the adequate planning and allocation of material and human resources for the specific testing activities. The proposed methodology and the results' accuracy were verified using the Monte Carlo data simulation method.*

*Keywords: reliability; test plan; Weibull distribution; Monte Carlo simulation*

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## 1 Introduction

The reliability theory is a technological discipline closely related to the probability theory and mathematical statistics [1, 2, 3]. The data regarding the reliability of the products are obtained mainly through the following three methods: following the behavior of the products in real operation; during the laboratory tests; by using the data simulation through the Monte Carlo method. During the laboratory tests, we tried to emulate, as much as possible, the conditions in real operation, by reproducing the range of internal stresses, as well as the environmental stresses. The most important laboratory tests are the reliability tests [4, 5].

### 1.1 Background on Reliability Test

The reliability tests have a great importance, aiming either to determine, either to check the reliability characteristic of a product, if this is established in a predictive way. The reliability tests are extremely necessary and they have a decisive role in

improving the technical solutions and in increasing the performances. The essential problem of reliability tests is the testing duration, which is generally comparable with the product's useful life time [6, 7, 8, 9].

The most used reliability tests are the following [7, 10, 11, 12, 13]:

- Complete tests (type  $n$  out of  $n$ ) - in these tests  $n$  products of the same kind, the experiment being considered finished when all of the  $n$  products have failed.
- Censored tests (type  $r$  out of  $n$ ) - are commonly used and they consist of subjection to testing of  $n$  products of the same type, the experiment being considered finished after the failure of  $r < n$  tested products; obviously, the  $r$  number is previously determined, usually by technical, economical and statistical considerations [2, 14, 15].
- Truncated tests (with a fixed testing time) - a  $n$  number of products are subjected to testing, but the experiment doesn't stop according to the number of failed elements, but according to a  $t_r$  time, previously set, a period during which the testing takes place.

The testing methodology about to be used has a direct economical impact, because in every test the following terms intervene: the cost of the tested product; the total cost of the experiment; the time consumed for testing and for the statistical processing of the data resulted from testing. Therefore, the selection of a specific type of test is a managerial decision that has to be taken by a responsible authority. Also, in the follow-up of the results of the statistical processing, we will propose certain corrective technical and economical actions, aimed directly at the quality and reliability of the product in question [3, 6, 16, 17, 18, 19, 20].

In order to realize reliability tests we must take into account the following aspects:

- a. A previously determined  $n$  number of products that will be subjected to testing.
- b. A testing plan that includes the following aspects: the selection of stress parameters, which will determine the failure mechanisms specific to the product.
- c. Instructions regarding the adequate type of test and the methods of calculation, in order to estimate the reliability parameters.
- d. A test chart where the experimental data are recorded and the statistical calculations, as well as the chronological recording of observations and interventions, are made.
- e. Testing stands, testing equipment, auxiliary materials and qualified staff in order to realize the test.

## 1.2 Nomenclature

$W(t, \beta, \eta)$  - the Weibull distribution, having the function of distribution:

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta};$$

$\beta$  - the shape parameter of Weibull distribution;

$\eta$  - the scale parameter of Weibull distribution;

$B(x, n, p)$  - the binomial distribution, having the function of probability:

$$\Pr(X = x) = C_n^x \cdot p^x \cdot (1 - p)^{n-x};$$

$N$  - the number of simulations;

$n$  - the sample volume;

$r$  - the level of censorship;

$F_n(t_i)$  - the empirical function of distribution correspondent to the operation time until failure;

$T_{n/n}$  - the duration of a complete test realized on a volume sample  $n$ ;

$T_{r/n}$  - the duration of a censored test at level  $r$ , realized on volume  $n$  sample;

$1 - \alpha$  - confidence level;

$\alpha$  - significance level;

$Q_{p, v_1, v_2}$  - the  $p$  quantile of the Fisher - Snedecor, with  $v_1$  and  $v_2$  degrees of freedom;

$U = 1 - \alpha/2$  - the index used to note the superior confidence limit; represents the value of the probability corresponding to the estimation of the superior limit of the testing duration, [%];

$L = \alpha/2$  - the index used to note the inferior confidence limits; represents the value of the probability corresponding to the estimation of the inferior limit of the testing duration, [%].

## 1.3 Review on the Calculus of the Testing Period

When using the Weibull distribution in the modeling of the products' reliability, these is estimated, in the majority of situations, on the basis of experimental results obtained following their testing on stands, using censored tests or complete tests. The organization and the process of reliability tests of the products represent

complex activities from an organizational standpoint and also big resources consumers. The calculation relations of the duration of a complete or a censored reliability test, found in the specialty literature, are based on a series of equations which allow the estimation of the empirical distribution function [9]:

$$F_n(t_r) = \frac{r}{n+1}. \quad (1)$$

The equation (1) gives mean values of the empirical distribution function. The mean values of the empirical distribution function can be obtained using:

$$F_n(t_r) = \frac{r-0.3}{n+0.4}, \quad (2)$$

or:

$$F_n(t_r) = \frac{i - 0.30685 - 0.3863 \cdot \left(\frac{r-1}{n-1}\right)}{n}, \quad (3)$$

for  $n > 20$  and:

$$F_n(t_r) = 1 - 2^{\frac{1}{n} \left(\frac{r-1}{n-1}\right)} \cdot \left[ 2^{\left(1-\frac{1}{n}\right)} - 1 \right], \quad (4)$$

for  $n \leq 20$ .

The value of the duration of a reliability test censored at level  $r$  is obtained using the inverse function of distribution of the considered statistical model [4, 21, 22]:

$$T_{r/n} = \eta \cdot \left\{ \ln \left[ \frac{1}{1 - F_n(t_r)} \right] \right\}^{\frac{1}{\beta}}, \quad (5)$$

in this case being the Weibull distribution. The equation (5) results from the logarithmation of the Weibull distribution function, written as follows:

$$e^{-\left(\frac{t}{\eta}\right)^\beta} = 1 - F(t). \quad (6)$$

we obtain:

$$\left(\frac{t}{\eta}\right)^\beta = \ln \left[ \frac{1}{1 - F(t)} \right]. \quad (7)$$

We notice that after a series of algebraic calculations, the equation (7) can be written in the form of equation (5). Also, in the equation (5), instead of  $F(t)$  we used the value determined by using one of the (1) ÷ (4) relations. Thus we obtained mean or median values of the duration of testing. For complete test case in relations (1) ÷ (4), parameter  $r$  is replaced by the value of the sample volume used  $n$ . Consequently, the objective of this paper is to present an estimation modality of the mean duration for censored and/or complete reliability tests, as well as of the confidence intervals for this duration. Knowing these values allows for the careful planning of the testing activities [4, 23, 24].

## 2 Statistical Calculation Model

The value obtained for a reliability test doesn't offer important data regarding the real duration of a test, because the time of operation until failure of a tested product represents a random variable.

For this situation, a favorable solution consists is the determination of the confidence intervals of the duration of the test. These intervals contain the real value of the test, with a  $1 - \alpha$  probability [25, 26]:

$$\Pr(T_L \leq T_{r/n} \leq T_U) = 1 - \alpha. \quad (8)$$

The calculation of the confidence intervals is realized in the conditions of a Bernoulli extraction (the scheme of the urn with returned balls). Thus, the median value of probability at which from  $n$  products subjected to testing, a number of  $r$  products fail, results as a solution to the equation [4, 27, 28]:

$$\sum_{i=r}^n C_n^i \cdot F_{Me}^i \cdot (1 - F_{Me})^{n-i} = 0.5. \quad (9)$$

The difficulties in calculation which can occur solving the equation (9), depending on  $F_{Me}$ , can be eliminated by using an approximate value:

$$F_{Me} = \frac{1}{1 + \frac{n-r+1}{r} \cdot Q_{0.5, 2(n-r+1), 2r}}. \quad (10)$$

The equation (10) represents the connecting relation that can be established between the binomial distribution and the Fisher-Snedecor distribution [4, 11, 26].

Using the  $F_{Me}$  solution, obtained by solving one of the (9) or (10) equations, along with to equation (5), leads to the obtaining of the duration of the reliability test. In fact, the equations (1) ÷ (4) are nothing more than regression relations of the solutions of equation (9), for different combinations of the parameters  $n$  and  $r$ .

Because, by definition, the distribution function is an ascending function, the confidence level of the duration of testing period results by using the solutions of the equations:

$$\sum_{i=r}^n C_n^i \cdot F_L^i \cdot (1 - F_L)^{n-i} = \frac{\alpha}{2}, \quad (11)$$

and

$$\sum_{i=r}^n C_n^i \cdot F_U^i \cdot (1 - F_U)^{n-i} = 1 - \frac{\alpha}{2}, \quad (12)$$

together with (5), namely:

$$T_L = \eta \cdot \left\{ \ln \left[ \frac{1}{1 - F_L} \right] \right\}^{\frac{1}{\beta}}, \quad (13)$$

and

$$T_U = \eta \cdot \left\{ \ln \left[ \frac{1}{1 - F_U} \right] \right\}^{\frac{1}{\beta}}, \quad (14)$$

A similar value of the  $F_L$  and  $F_U$  probabilities can be obtained by approximating the binomial distribution through the Fisher-Snedecor distribution:

$$F_L = \frac{1}{1 + \frac{n-r+1}{r} \cdot Q_{1-\frac{\alpha}{2}, 2(n-r+1), 2r}}. \quad (15)$$

and

$$F_U = \frac{1}{1 + \frac{n-r+1}{r} \cdot Q_{\frac{\alpha}{2}, 2(n-r+1), 2r}}. \quad (16)$$

For the case of the complete tests, in the calculation relations (10) and (15), (16) the parameter  $r$  is replaced with the value of the used sample volume  $n$ .

### 3 Simulation Study

#### 3.1 Program Description

To verify the precision of reasoning and of the mathematic model proposed at point 2, we used the Monte Carlo simulation method. The method implies, in this case, generating a very high number of samples ( $N \gg 1000$ ), that belong to a completely specified Weibull population,  $W(t, \beta, \eta)$ . This database is then subjected to a statistical analysis that is aimed at the duration of a reliability test using a censored plan with the  $n$  and  $r$  parameters. For the development of this study we created a Mathcad calculus programme. The logical chart of this programme is presented in Figure 1. The running of the programme implies the determination of the following entry data:  $N$ ,  $n$ ,  $r$ ,  $\beta$ ,  $\eta$  and  $\alpha$ . The program generates a matrix with  $n$  lines and  $N$  columns, using the generator of random, uniform and continuous numbers within the  $[0,1]$  interval. The values of simulated failure times are obtained by using the inverse function of distribution of the Weibull statistical:

$$t = \eta \cdot \ln \left( \frac{1}{1 - rnd(1)} \right)^{\frac{1}{\beta}}. \quad (17)$$

Thus, we obtain a matrix with  $n \times N$  dimensions, in which every column represents a reliability test. In order to determine the duration of censored tests at level  $r$ , the calculation program sets in ascending order the columns of the previously generated matrix.

Also, the  $r$ -th line of this matrix is extracted at the end. The  $N$  values contained in this line represent the simulated durations of the reliability tests ( $t_{r,i}$ ). The calculation of the median and mean durations of the testing duration is made by determining the median and the mean of these values:

$$t_{Me} = \begin{cases} t_{\left(\frac{n+1}{2}\right)}, & \text{if } N \text{ is even} \\ \frac{t_{\left(\frac{n}{2}\right)} + t_{\left(\frac{n}{2}+1\right)}}{2}, & \text{if } N \text{ is odd} \end{cases} \quad (18)$$

and

$$m = \frac{\sum_{i=1}^N t_{r,i}}{N} \quad (19)$$

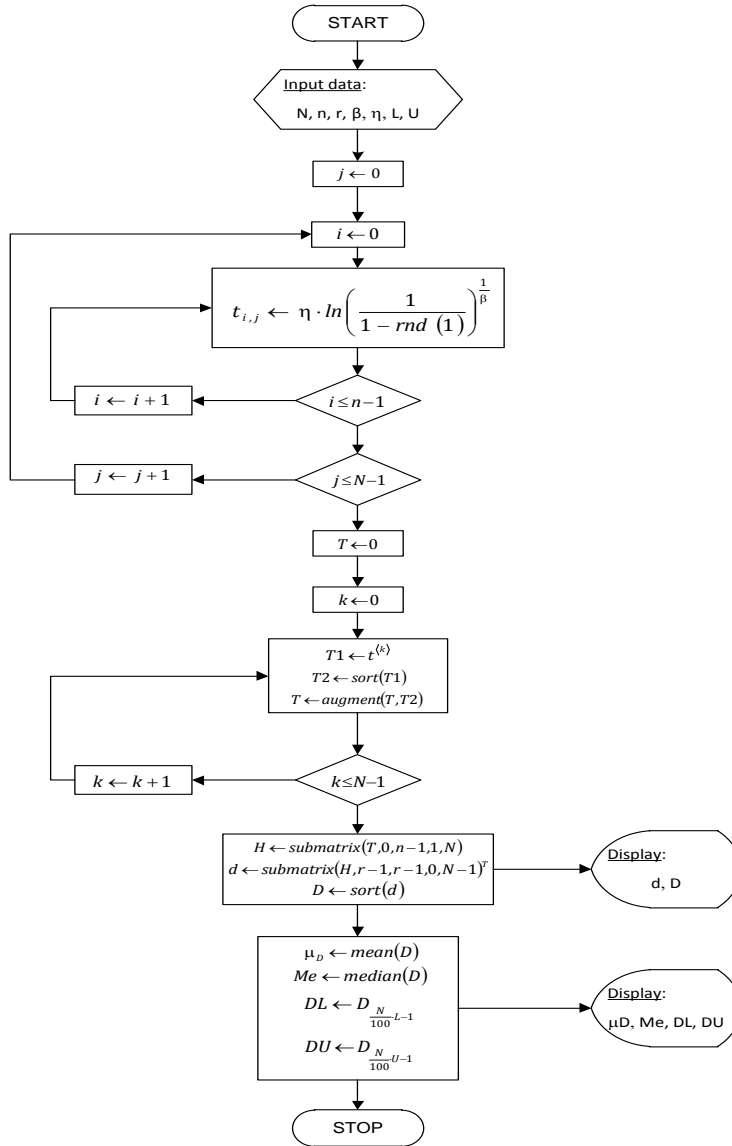


Figure 1

The logical scheme of the Monte Carlo numerical simulation program

In the previous equations, we noted  $t_{(p)}$  as the  $p$  quantile of the  $t$  variable. The determination of confidence limits for the duration of the tests is realized by determining the  $t_{L/100}$  and  $t_{U/100}$  quantiles of the truncation durations for the  $N$  simulated tests. The calculation method used for the determination of  $p$  quantiles applies the equation:



$$t_p = t_{[p \cdot (N+1)]}. \quad (20)$$

If the  $p \cdot (n+1)$  expression doesn't generate an integer value, then for the determination of the  $p$  quantile we recommend the use of linear interpolation. We assume that, after the evaluation of the  $p \cdot (n+1)$  expression, we find that the value of the  $t_p$  quantile is included in the  $[t_{(k)}, t_{(k+1)}]$ . To determine the value of the  $t_p$  quantile, we use the relation:

$$t_p = t_{(k)} + [p \cdot (n+1) - k] \cdot [t_{(k+1)} - t_{(k)}] \quad (21)$$

For high volumes of sample (such is the case with the realized application), instead of the previous relation we can use equation (22).

$$t_p = t_{[p \cdot (N+1)]}. \quad (22)$$

In equation (22), by  $t_{[p]}$  we noted the integer part of the value of the expression between brackets.

## 3.2 Monte Carlo Simulation Data

To demonstrate the way of using the calculation methodology presented in the third point of this paper, we present further several case studies, determined for different values of the Weibull distribution parameters  $\beta$  and  $\eta$ , as well as for different testing schemes  $n$  and  $r$ .

The solving of the equations (9), (11) and (12) was made using the specialized functions existent in Mathcad 14. The solving accuracy of these equations was established at  $10^{-15}$ . In parallel, we presented the values obtained by using the approximate relations (10), (15) and (16).

The obtained  $F_{Me}$ ,  $F_L$  și  $F_U$  probabilities are then used to determine the median duration of the reliability test, eq. (5) and the limits of the confidence interval  $(1-\alpha)$  for this duration ( $T_L$  și  $T_U$ ).

The values for these limits are obtained by using the equations (13) and (14). The significance level was established at the value of  $\alpha = 10\%$ . In Table 1 values for different combinations of the Weibull distribution's parameters and different censored testing plans,  $n/r$  are presented.

The accuracy of the obtained results, by using the proposed calculation methodology, was verified using the Monte Carlo simulation.

For this purpose we used the MathCAD 14 software, which is described at point 3.1. The calculus programme was run for the same combinations of values of the Weibull distribution's parameters, as in the previous case. Also, the number of simulations was established at the value  $N=10000$  and the confidence interval  $1-\alpha$  at 90%.

Table 1

The determination of the testing durations with the reliability tests using the calculation relations

Test plan		The calculated duration of the reliability test					
		$T_{r/n}$		$T_L$		$T_U$	
$n$	$r$	eq. (8)	eq. (9)	eq. (10)	eq. (14)	eq. (11)	eq. (15)
$\beta=1.5, \eta=50$							
10	5	35.605	35.605	19.926	19.926	56.218	56.218
10	10	97.035	97.035	61.118	61.118	151.520	151.520
20	5	20.417	20.417	11.472	11.472	32.023	32.023
20	10	37.332	37.332	25.278	25.278	51.936	51.936
20	15	58.860	58.860	42.591	42.591	78.556	78.556
20	20	112.601	112.601	78.641	78.641	164.494	164.494
$\beta=1.5, \eta=100$							
10	5	71.211	71.211	39.854	39.854	112.438	112.438
10	10	194.069	194.069	122.236	122.236	303.040	303.040
20	5	40.834	40.834	22.945	22.945	64.045	64.045
20	10	74.664	74.664	50.556	50.556	103.871	103.871
20	15	117.720	117.720	85.182	85.182	157.112	157.112
20	20	225.201	225.201	157.282	157.282	328.989	328.989
$\beta=2, \eta=50$							
10	5	38.760	38.760	25.080	25.080	54.595	54.595
10	10	82.213	82.213	58.126	58.126	114.841	114.841
20	5	25.541	25.541	16.576	16.576	35.796	35.796
20	10	40.161	40.161	29.978	29.978	51.445	51.445
20	15	56.508	56.508	44.333	44.333	70.166	70.166
20	20	91.917	91.917	70.223	70.223	122.139	122.139
$\beta=2, \eta=100$							
10	5	91.917	91.917	70.223	70.223	122.139	122.139
10	10	164.425	164.425	116.252	116.252	229.681	229.681
20	5	51.082	51.082	33.152	33.152	71.592	71.592
20	10	80.322	80.322	59.956	59.956	102.890	102.890
20	15	113.015	113.015	88.667	88.667	140.332	140.332
20	20	183.835	183.835	140.446	140.446	244.279	244.279

Under these conditions, we determined the median values, the mean values and the confidence intervals for the testing duration. The results obtained by Monte Carlo numerical simulation are presented in Table 2.

Table 2

The determination of the testing durations related to the reliability tests using the Monte Carlo method

Test plan		Values obtained by Monte Carlo simulation			
		$T_{r/n}$	$T_L$	$T_U$	$\tilde{T}_{r/n}$
$n$	$r$				
$\beta=1.5, \eta=50$					
10	5	35.744	19.998	55.945	36.562
10	10	96.820	61.412	152.861	100.609
20	5	20.385	11.680	31.748	20.885
20	10	37.301	25.362	52.071	37.843
20	15	58.992	42.416	78.349	59.594
20	20	112.252	78.947	165.037	115.808
$\beta=1.5, \eta=100$					
10	5	71.140	39.118	111.946	72.881
10	10	193.087	121.367	301.877	200.325
20	5	40.782	22.834	63.778	41.770
20	10	74.810	50.408	103.875	75.681
20	15	118.342	85.409	158.009	119.295
20	20	225.390	157.378	328.730	231.811
$\beta=2, \eta=50$					
10	5	38.614	25.034	54.654	39.101
10	10	82.338	58.557	115.534	83.971
20	5	25.502	16.507	35.797	25.793
20	10	40.211	29.926	51.526	40.393
20	15	56.463	44.296	70.287	56.759
20	20	91.957	70.221	121.616	93.419
$\beta=2, \eta=100$					
10	5	91.957	70.221	121.616	93.419
10	10	164.752	116.708	228.715	167.494
20	5	51.307	33.260	71.201	51.696
20	10	80.452	59.513	102.790	80.616
20	15	113.174	88.908	139.999	113.716
20	20	183.619	140.521	244.166	186.904

The using mode of this method for the estimation of the durations of the complete and/or at  $r$  level censored reliability tests is presented in Figure 2.

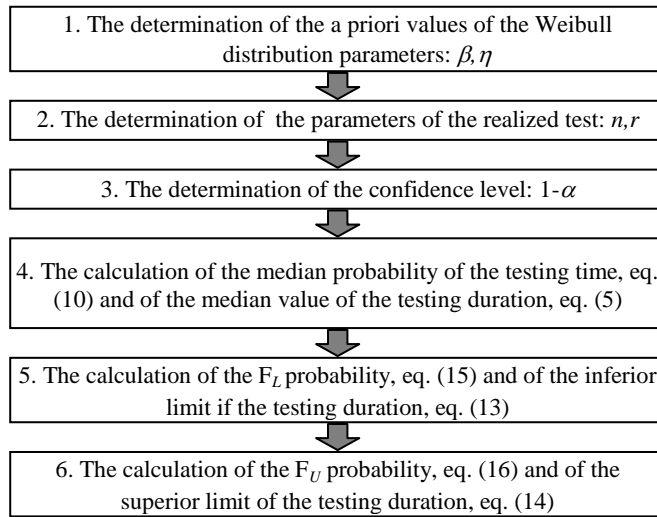


Figure 2

The calculation algorithm for the duration of the reliability tests

## Conclusions

Based on the results presented in Tables 1 and 2, we will realize several comparative studies to show the correctness of the proposed calculus method. In Figure 3 the results of the median duration of a censored test at level  $r$  are presented, realized on a sample of  $n$  volume, on a graphical form for the case study  $\beta=1.5$ ,  $\eta=50$ . The inferior and superior limits of the testing duration for the case study in question ( $\beta=1.5$ ,  $\eta=50$ ) are presented in Figures 4 and 5.

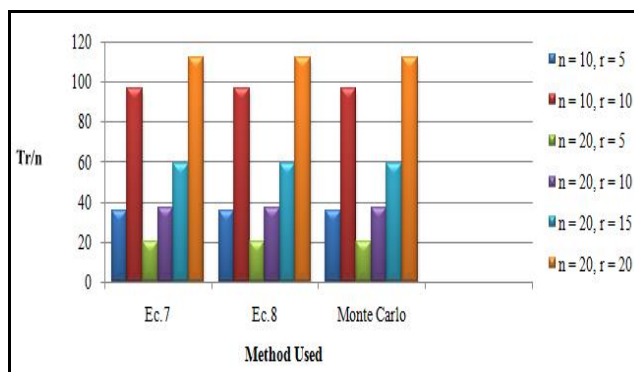


Figure 3

The duration of a censored test level  $r$ , realized on sample size  $n$  ( $T_{nr}$ )

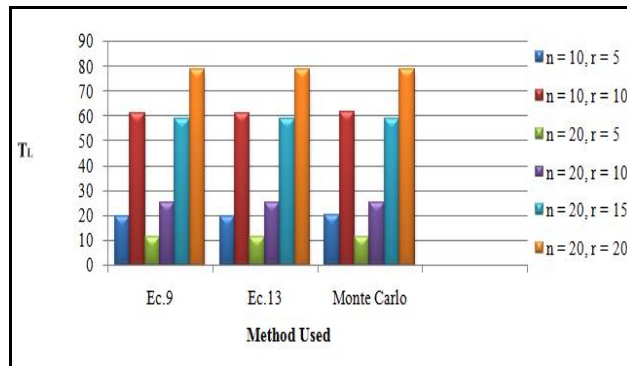


Figure 4

Limit inferior duration of a censored test level  $r$ , realized on sample size  $n$  ( $T_L$ )

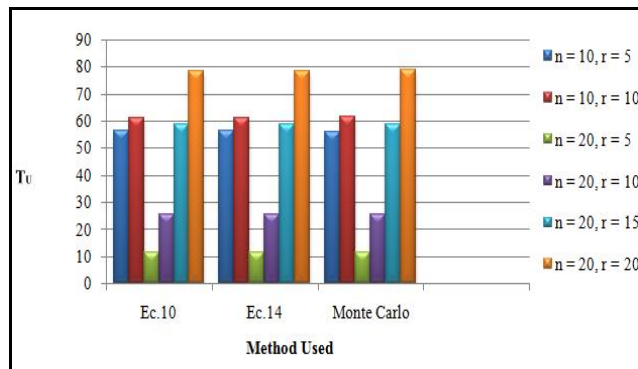


Figure 5

Limit superior duration of a censored test level  $r$ , realized on sample size  $n$  ( $T_U$ )

The presented calculation model allows the obtaining of accurate estimated values, because the differences towards the simulated values are very small. If the number of simulations would grow, the resulting differences would be insignificant. The approximate relations (10), (15) and (16) lead to the obtaining of some values, which, at the results' display accuracy of  $10^{-3}$ , don't differ from the real values obtained through the equations (9), (11), (12). Based on the presented results we found the significant reductions in time that can be made by using the censored testing plans.

Given the powerful competition on the industrial market, we can no longer imagine the realization of a product without a rigorous quality and reliability control of the product, based on different types of tests, in all the stages of the products' existence, from the raw materials being used, up to their use. In these types of tests, we put special emphasis on the reliability tests. The testing

laboratories, that possess modern testing equipment and highly-trained personnel, have acquired an ever increasing development. Today, almost every company has a reliability test laboratory, adequately equipped to the type of products it realizes. The application of censored testing plans, using the Monte Carlo method, determines the testing duration of the products in a shorter time and in conditions of economic efficiency.

The proposed method of calculation has applicability in laboratories that are specialized in testing the materials or the products from different fields of use. These fields are given by the particular versatility of the Weibull distribution's model:

- ❖ the tear resistance, the corrosion resistance, the wear resistance, the fatigue resistance and the contact fatigue resistance of textile and metallic materials;
- ❖ the modeling of the materials properties: steels, titanium, semi-conductor materials, tungsten, ceramic, glass, plastic materials, porcelain, graphite, paper, textile fibers, composite materials;
- ❖ the modeling of the durability of mechanical components: bearings, engines, motor vehicles' structures, tools;
- ❖ the modeling of the functioning times of relays, passive electronic components (resistors and capacitors) and active electronic components (transistors, integrated circuits);
- ❖ the modeling of the life times of subsystems, made of identical component elements, in series connected and whose behaviour is described using the gamma distribution.

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