Model Reference Fuzzy Control of Nonlinear Dynamical Systems Using an Optimal Observer

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Abstract: This paper proposes a novel indirect model reference fuzzy control approach for nonlinear systems, expressed in the form of a Takagi Sugeno (TS) fuzzy model based on an optimal observer. In contrast to what is seen in the literature on adaptive observer-based TS fuzzy control systems, the proposed method is capable of tracking a reference signal rather than just regulation. Additionally the proposed algorithm benefits from an adaptation algorithm which estimates the parameters of observer optimally. The stability analysis of the adaptation law and the controller is done using an appropriate Lyapunov function. The proposed method is then simulated on the control of Chua's circuit and it is shown that it is capable of controlling this chaotic system with high performance.

Keywords: fuzzy control; model reference fuzzy control; observer design

1 Introduction

Fuzzy controllers can be viable alternatives to classical controllers when there are experienced human operators who can provide qualitative control rules in terms of vague sentences. Although fuzzy controllers have been successfully used in many industrial applications, in cases when some adaptation is required, there may not be enough expert knowledge to tune the parameters of the controller. This has motivated the design of adaptive fuzzy controllers which can learn from data and one of the early suggestions in this respect is the approach described in [1], named the linguistic self-organizing controller (SOC).Such early fuzzy adaptive systems have suffered from the lack of stability analysis, i.e. the stability of closed-loop system is not guaranteed and the learning process does not lead to well-defined dynamics [2]. In order to overcome this problem, the use of classical controllers have been suggested to complement adaptive fuzzy controllers. The fusion of fuzzy systems with classical control approaches makes it possible to benefit from the general function approximation property of fuzzy systems, as well as its power to use expert knowledge and the well established stability proof of classical control systems. For example in [3], [4] and [5], sliding mode fuzzy controllers are proposed, and in [6], [7] and [8], a H_∞-based fuzzy controller, a fuzzy-identification-based back-stepping controller and a model reference controller with an adaptive parameter estimator based on Takagi Sugeno (TS) fuzzy models are proposed, respectively.

Using a model system to generate the desired response is one of the most important adaptive control schemes [9] studied in such hybrid approaches, and to date, different fuzzy model reference approaches have been proposed. The indirect model reference fuzzy controllers described in [8], [10]-[13] and the direct model reference fuzzy controllers for TS fuzzy models described in [14], [15], and [16] can be cited as some examples. Most of the model reference fuzzy controller schemes existing in the literature assume that the full state measurement of the plant is available [8], [10]-[16]. However, in some practical applications state variables are not accessible for sensing devices or the sensor is expensive, and the state variables are just partially measurable. In such cases it is very essential to design an observer to estimate the states of the system. There has been a tremendous amount of activity on the design of nonlinear observers using fuzzy models, based on approaches like LMI [17]-[19], SPR Lyapunov function [20], [21] and adaptive methods [22].

The TS fuzzy system is one of the most popular fuzzy systems in model-based fuzzy control. A dynamical TS fuzzy system describes a highly nonlinear dynamical system in terms of locally linear TS fuzzy systems. The overall fuzzy system is achieved as a fuzzy blending of these locally linear systems [24]. Using this approach, it is possible to deal with locally linear dynamical systems rather than the original nonlinear dynamical system. When there are difficulties in the measurement of the states, the design of a fuzzy observer using the TS fuzzy model is considered in a number of different papers. In [22] an adaptive approach is proposed to design observer and controller for TS fuzzy system. However the proposed method considers only the regulation problem, tracking control is not addressed.

In this paper, a novel indirect model reference fuzzy controller is described that uses a novel optimal fuzzy observer. The optimality of the fuzzy observer is achieved by finding the optimal solution of an appropriate cost function. The optimal adaptation law used in the design and its stability analysis are quite similar to the optimal adaptation law proposed in [23] and its stability analysis given therein. The superiority of the proposed controller over the one described in [23] is that the current work uses an observer so that full state measurement is not necessary. The stability analysis of the proposed observer and the controller is done using a Lyapunov function. Not only can the proposed indirect model reference fuzzy controller regulate the states of the system under control but also it can make the system track a desired trajectory. This is another benefit of the current work over the observer based model fuzzy controllers available in the literature, such as that described in [22]. To demonstrate the efficacy of the proposed method, it is then used to control a chaotic system. It is shown that by the use of the proposed approach, it is possible to make the chaotic system follow the reference model.

This paper is organized as follows. In Section 2 a brief study of zero-th order TS fuzzy systems is given. The structure of the proposed observer and its optimal adaptation law are introduced in Section 3, continuing by the stability analysis of the observer using a proper Lyapunov function. In Section 4 the proposed optimal observer is used in the design of the model reference fuzzy controller. Simulation results are presented in Section 5. Finally concluding marks are discussed in the next section.

2 Takagi-Sugeno Fuzzy Systems

In 1985 Takagi and Sugeno [25] proposed a new type of fuzzy systemin which the i-th rule of the fuzzy system is as follows:

$$R^i : If\ x_1\ is\ A_{i1}\ and\ x_2\ is\ A_{i2} ... and\ x_n\ is\ A_{in}\ Then\ y = F_i(x_1, x_2, ..., x_n) \tag{1}$$

In this fuzzy system $x_1, x_2, ..., x_n$ are the inputs of the fuzzy system and $F_i(.)$ is a function of inputs. This system can be seen as an extension of singleton fuzzy systems. It is to be noted that only the premise part of a TS fuzzy system is linguistically interpretable and that $F_i(.)$ are not fuzzy sets. The functions $F_i(.)$ can be chosen in different ways. If the functions are chosen as constants (θ_i), a singleton fuzzy system is recovered. This case is generally called a zero-th order TS fuzzy system, since a constant can be seen as a zero-th order Taylor expansion of a function. Another well-known possible selection of $F_i(.)$ is to select the rule consequent as a linear function of the inputs. The resulting fuzzy system is called first order TS fuzzy system.

The output of a TS fuzzy system can be calculated by

$$y = \sum_{i=1}^{m} h_i F_i \tag{2}$$

In which (h_i) is the normalized firing of the i-th rule and (m) is the number of the rules. In this paper we use zero-th order TS fuzzy system. The output of the zero-th order fuzzy system is calculated as:

$$y = \sum_{i=1}^{m} h_i \theta_i \tag{3}$$

3 The Structure of the Optimal Observer and its Stability Analysis

In this section, an optimal indirect adaptive fuzzy observer for a nonlinear system is proposed. The adaptation law for the estimation of the parameters of the nonlinear system is derived. Using an appropriate Lyapunov function, the stability of the proposed observer and the adaptation laws are analyzed.

3.1 The Structure of the Proposed Observer

Let the dynamical equation of the system be in the following form:

$$\begin{aligned} \dot{x} &= Ax + B[u + f(y)] \\ y &= Cx \end{aligned} \tag{4}$$

In which $x \in \mathbb{R}^n$ is the n-dimentional state vector, $A \in \mathbb{R}^{n\times n}$ is the known state matrix, $B \in \mathbb{R}^{n\times 1}$ is the known input matrix, $C \in \mathbb{R}^{1\times n}$ is the known output matrix, $u \in \mathbb{R}$ is the input signal and $f(y)$ is an unknown Lipschitz function of y. The structure of the proposed observer is:

$$\dot{\hat{x}} = A\hat{x} + B[u + \sum_{i=1}^{m} \theta_i h_i(y)] + LCe \tag{5}$$

in which $\hat{x} \in \mathbb{R}^n$ is the estimated value of x, $L \in \mathbb{R}^{n\times 1}$ is the observation gain and $\sum_{i=1}^{m} \theta_i h_i(y)$ is the output of the fuzzy system with m being the number of rules used to estimate $f(y)$. The normalized firing strength of the i-th rule of this fuzzy system is $h_i(y)$, the parameters of the consequent part are θ_i and

$$e = \hat{x} - x \tag{6}$$

is the observation error.

3.2 The Dynamics of the Observation Error

We have:

$$\dot{e} = (A + LC)e + B[\sum_{i=1}^{m} \theta_i h_i(y) - f(y)] \quad (7)$$

It is supposed that the there exist optimal parameters θ_i^* for the fuzzy system such that:

$$f(y) = \sum_{i=1}^{m} \theta_i^* h_i(y) + \varepsilon(y) \quad (8)$$

in which $\varepsilon(y)$ is the approximation error. It can be proved [23] that if $f(y)$ is a Lipschitz function and the control signal u is bounded the time derivative of $f(y)$ is also bounded and we have:

$$\sup_t \left| \frac{df(y)}{dt} \right| < \sigma_f \quad (9)$$

In which σ_f is a positive constant. It is assumed that the time derivative of the approximation error ε is bounded so that:

$$\sup_t |\dot{\varepsilon}(y)| = \sup_t \left| \frac{d\left(\sum_{i=1}^{m} \theta_i^* h_i(y)\right)}{dt} - \frac{df(y)}{dt} \right| < \sigma_\varepsilon \quad (10)$$

where σ_ε is apositive constant. It follows that:

$$\sup_t \left| \frac{dh_i(y)}{dt} \right| \le \eta \quad (11)$$

in which η is a positive constant. The observation error dynamics can be expressed as follows.

$$\dot{e} = (A + LC)e + B\left[\sum_{i=1}^{m} \tilde{\theta}_i h_i(y) - \varepsilon(y)\right] \quad (12)$$

In above $\tilde{\theta}_i = \theta_i - \theta_i^*$. Since the fuzzy systems are proved to be general function approximators, by considering enough number of the rules for the fuzzy system, we have:

$$\sigma_\varepsilon = \sup_t |\varepsilon| \quad (13)$$

It is also assumed that there exists positive definite matrices P and Q such that:

$$\begin{aligned} &(A + LC)^T P + P(A + LC) = -Q \\ &PB = C^T \end{aligned} \quad (14)$$

3.3 The Optimal Adaptation Law for the Observer

In order to design an optimal adaptation law for the observer, the following cost function is defined for the observer.

$$J = \frac{1}{2}\int_0^{t_f} (e-\Delta)^T Q(e-\Delta)dt \tag{15}$$

in which $\Delta = e(t_f)$ and t_f is the final time. In addition, $Q \in \mathbb{R}^{n\times n}$ is a user defined positive definite matrix. This cost function is quite similar to the cost function used in [23]. The difference is that the cost function defined in [23] includes the tracking error while the cost function introduced here includes the observation error. This is an optimal observer design problem and its solution can be obtained by Pontryangin's maximum principle. To solve this optimal problem a Hamiltonian is defined as:

$$H\left(e, \sum_{i=1}^{m} \tilde{\theta}_i h_i\right) = \frac{1}{2}(e-\Delta)^T Q(e-\Delta) + p^T\left((A+LC)e + B\sum_{i=1}^{m}\tilde{\theta}_i h_i + B\sigma_\varepsilon\right) \tag{16}$$

in which p is an adjoint variable, the adjoint equation is given by:

$$\dot{p} = -\nabla H_e^T = -Q(e-\Delta) - (A+LC)^T p \tag{17}$$

The adaptation law for θ_i can be obtained by gradient method as:

$$\dot{\tilde{\theta}}_i = -\gamma_i h_i \nabla H_{\tilde{\theta}_i h_i} = -\gamma_i h_i p^T B \tag{18}$$

in which $\gamma_i > 0$ is the learning rate. The transversality condition requires that [26]:

$$p(t_f) = 0 \tag{19}$$

By letting $p = Pe + S\sum_{i=1}^{m}\theta_i h_i(y)$ and considering (17) we have:

$$\begin{aligned} &\dot{P}e + P\left(\left(A+LC\right)e + B\sum_{i=1}^{m}\tilde{\theta}_i h_i + B\sigma_\varepsilon\right) + \dot{S}\sum_{i=1}^{m}\theta_i h_i \\ &+S\frac{d\left(\sum_{i=1}^{m}\theta_i h_i\right)}{dt} \geq -Q(e-\Delta) - (A+LC)^T\left(Pe + S\sum_{i=1}^{m}\theta_i h_i\right) \end{aligned} \tag{20}$$

This is called sweeping method [26], [27]. In addition, assuming that the adaptation law is stable (the stability analysis will be considered in the next section) we have:

$$\sup_t \left| \frac{d\left(\sum_{i=1}^{m} \theta_i h_i\right)}{dt} \right| = \sup_t \left| \frac{d\left(\sum_{i=1}^{m} \tilde{\theta}_i h_i\right)}{dt} + \dot{\varepsilon}(y) + \frac{df(y)}{dt} \right|$$

$$\leq \sup_t \left| \sum_{i=1}^{m} \dot{\tilde{\theta}}_i h_i + \sum_{i=1}^{m} \tilde{\theta}_i \dot{h}_i \right| + \sigma_\varepsilon + \sigma_f \tag{21}$$

$$\leq \sup \left| -B^T p \sum_{i=1}^{m} \gamma_i h_i^2 \right| + \sup_t \left| \sum_{i=1}^{m} \tilde{\theta}_i \dot{h}_i \right| + \sigma_\varepsilon + \sigma_f$$

The first term of the right hand of (21) is bounded bacause p must be a stable solution to the optimal control problem and h_i is bounded because it is the firing strength of the fuzzy system. The second term must be bounded if the adaptation law of $\tilde{\theta}$ is stable and $\dot{h}_i$ is also bounded. Therefore we have:

$$\sup_t \left| \frac{d\left(\sum_{i=1}^{m} \theta_i h_i\right)}{dt} \right| < \sigma_t \tag{22}$$

From (20) it follows that:

$$\Delta = Q^{-1} \left[PB\left(\varepsilon - \sum_{i=1}^{m} \theta_i^* h_i\right) + S \frac{d\sum_{i=1}^{m} \theta_i h_i}{dt} \right] \tag{23}$$

and also:

$$\begin{aligned} &\dot{P} + P(A+LC) + (A+LC)^T P + Q = 0 \\ &\dot{S} + PB + (A+LC)^T S = 0 \end{aligned} \tag{24}$$

subject to:

$$P(t_f) = 0 \text{ and } S(t_f) = 0 \tag{25}$$

Considering the infinite horizon optimal control $t_f \to \infty$, P and S are in their steady state value ($\dot{P} = 0,\ \dot{S} = 0$) we have:

$$\begin{aligned} &P(A+LC) + (A+LC)^T P = -Q \\ &S = -(A+LC)^{-T} PB \end{aligned} \tag{26}$$

Furthermore:

$$p = Pe - (A+LC)^{-T} PB \sum_{i=1}^{m} \theta_i h_i \tag{27}$$

By substituting (27) in (18) it follows that:

$$\dot{\tilde{\theta}}_i = -\gamma_i h_i \left(e^T P - \vartheta \sum_{i=1}^{m} \theta_i h_i B^T P (A+LC)^{-1} \right) B \tag{28}$$

Considering (14) we obtain:

$$\dot{\tilde{\theta}}_i = -\gamma_i h_i e_y + \gamma_i h_i \vartheta \sum_{i=1}^{m} \theta_i h_i B^T P (A+LC)^{-1} B \tag{29}$$

In which $e_y = C(\hat{x} - x)$ and $\vartheta > 0$ is a design parameter.

3.4 Stability Analysis of the Proposed Observer

In order to analyze the stability of the proposed observer the following Lyapunov function is introduced.

$$V = e^T P e + \sum_{i=1}^{m} \frac{1}{\gamma_i} \theta_i^2 \tag{30}$$

In which P is the solution of (14). The time derivative of the Lyapunov function is obtained as:

$$\dot{V} = \dot{e}^T P e + e^T P \dot{e} + \sum_{i=1}^{m} \frac{2}{\gamma_i} \theta_i \dot{\theta}_i \tag{31}$$

Considering (12) and (29) we have:

$$\begin{aligned} \dot{V} \le\ & e(A+LC)^T P e + e^T P (A+LC) e + 2e^T P B \sum_{i=1}^{m} \tilde{\theta}_i h_i(y) \\ & + 2|e_y|\sigma_\varepsilon + 2\sum_{i=1}^{m} \theta_i h_i \left(e^T P - \vartheta \sum_{i=1}^{m} \theta_i h_i B^T P (A+LC)^{-1} \right) B \end{aligned} \tag{32}$$

$P(A+LC)^{-1}$ can be decomposed into a symmetric part (M) and anti-symmetric part (N) such that [23]:

$$M = \frac{1}{2}\left(P(A+LC)^{-1} + (A+LC)^{-T} P \right) = -\frac{1}{2}(A+LC)^{-T} Q (A+LC)^{-1} < 0 \tag{33}$$

$$N = \frac{1}{2}\left(P(A+LC)^{-1} - (A+LC)^{-T} P \right) \tag{34}$$

Using the property of anti-symmetric matrix (N) we have:

$$B^T N B = 0 \tag{35}$$

So that:

$$\dot{V} \le -e^T Qe + 2e^T PB\left[\sum_{i=1}^{m} \tilde{\theta}_i h_i(y)\right] + 2|e_y|\sigma_\varepsilon$$
$$+2\sum_{i=1}^{m} \theta_i h_i e^T PB - \vartheta \sum_{i=1}^{m} \theta_i h_i B^T (A+LC)^{-T} Q(A+LC)^{-1} B \quad (36)$$

furthermore:

$$\dot{V} \le -e^T Qe + 2e^T PB\sum_{i=1}^{m} \theta_i^* h_i(y) + 2|e_y|\sigma_\varepsilon$$
$$-\vartheta \sum_{i=1}^{m} \theta_i h_i B^T (A+LC)^{-T} Q(A+LC)^{-1} B \quad (37)$$

and

$$\dot{V} \le -\lambda_{\min}(Q)\|e\|^2 + 2\lambda_{\max}(P)\|B\|\|e\|\left[\sum_{i=1}^{m} \|\theta_i^* h_i(y)\| + \|\sigma_\varepsilon\|\right]$$
$$-\vartheta\lambda_{\min}(Q)\left\|(A+LC)^{-1} B\sum_{i=1}^{m} \theta_i h_i\right\|^2 \quad (38)$$

In which $\lambda_{\min}(Q)$ is the smallest eigenvalue of (Q). It follows that in order to have $\dot{V} \le 0$ we should have:

$$-\lambda_{\min}(Q)\|e\|^2 + 2\lambda_{\max}(P)\|B\|\|e\|\left[\sum_{i=1}^{m} \|\theta_i^* h_i(y)\| + \|\sigma_\varepsilon\|\right] \le 0 \quad (39)$$

This equally means that:

$$\frac{2\lambda_{\max}(P)\|B\|\left[\sum_{i=1}^{m} \|\theta_i^* h_i(y)\| + \|\sigma_\varepsilon\|\right]}{\lambda_{\min}(Q)} \le \|e\| \quad (40)$$

Thus V decreases inside a compact set S where:

$$S = \left\{ e \in \mathbb{R}^n \;|\; \frac{2\lambda_{\max}(P)\|B\|\left[\sum_{i=1}^{m} \|\theta_i^* h_i(y)\| + \|\sigma_\varepsilon\|\right]}{\lambda_{\min}(Q)} \le \|e\| \right\} \quad (41)$$

The following theorem summarizes the forthgoing optimal adaptation law and its stability analysis.

Theorem 1. If there exists a positive definite matrix P satisfying (14), the observer given by (5) for the nonlinear dynamical system of (4) with the adaptation law of (29) which is the optimal solution of the cost function (15) ensures that the state estimation error and the estimated values of the fuzzy system θ_i are uniformly bounded. Furthermore, the estimation error can be made to approach an arbitrarily small value by choosing appropriate values for the design constants $\lambda_{\max}(P), \lambda_{\min}(Q)$ and sufficient number of rules for the fuzzy system.

4 The Design of Indirect Model Reference Fuzzy Controller Based on Proposed Observer and its Stability Analysis

In the previous section, the stability of the observer is considered using an appropriate Lyapunov function. In this section the stability of the control system is analyzed and a stable control signal is derived. The goal of the model reference fuzzy controller is to derive the system such that it follows the model reference system in the form of:

$$\dot{x}_m = A_m x_m + B_r r,\ A_m = A + LC + Ba_m^T \tag{42}$$

in which $x_m \in \mathbb{R}^n$ is the n-dimentional state vector of the reference system, $A_m \in \mathbb{R}^{n\times n}$ is the state matrix of the reference system, $B_r \in \mathbb{R}^{n\times 1}$ is the input matrix, $a_m \in \mathbb{R}^{n\times 1}$ is a user defined matrix which determines the dynamics of the reference model. The Lyapunov function is considered as:

$$V_1 = \hat{e}_m^T P_1 \hat{e}_m + e^T Pe + \sum_{i=1}^{m} \frac{1}{\gamma_i}\theta_i^2 \tag{43}$$

In which P_1 is the solution of:

$$\begin{aligned}&(A + LC + Ba_m^T)^T P_1 + P_1(A + LC + Ba_m^T) = -Q_1\\ &P_1 B = C^T\end{aligned} \tag{44}$$

and Q_1 is a positive definite matrix. In addition, $\hat{e}_m$ is the observed tracking error defined as $\hat{e}_m = \hat{x} - x_m$. The Lyapunov function now includes both the observation error and the observed tracking error and it is possible to use it to analyze stability of the tracking error too. Considering (30) we have:

$$V_1 = \hat{e}_m^T P_1 \hat{e}_m + V \tag{45}$$

The time derivative of the Lyapunov function is obtained as:

$$\dot{V}_1 = \dot{\hat{e}}_m^T P_1 \hat{e}_m + \hat{e}_m^T P_1 \dot{\hat{e}}_m + \dot{V} \tag{46}$$

since

$$\dot{\hat{x}} = A\hat{x} + B[u + \sum_{i=1}^{m} \theta_i h_i(y)] + LCe \tag{47}$$

By subtracting (42) from (47) we have:

$$\dot{\hat{e}}_m = A\hat{e}_m + B[u + \sum_{i=1}^{m} \theta_i h_i(y) - a_m^T \hat{x} + a_m^T \hat{x} - a_m^T x_m - b_r r] + LC\hat{e}_m - LCx \tag{48}$$

In which $\hat{e}_m = \hat{x} - x_m$ and:

$$\dot{\hat{e}}_m = A_m \hat{e}_m + B[u + \sum_{i=1}^{m} \theta_i h_i(y) - a_m^T \hat{x} - b_r r] - LCx \tag{49}$$

furthermore:

$$\begin{aligned} \dot{V}_1 &= \hat{e}_m^T \left(P_1 A_m + A_m^T P_1 \right) \hat{e}_m \\ &+ 2\hat{e}_m^T P_1 B[u + \sum_{i=1}^{m} \theta_i h_i(y) - a_m^T \hat{x} - b_r r] - 2\hat{e}_m^T P_1 LCx + \dot{V} \end{aligned} \tag{50}$$

Since for any $\alpha \geq 0$ we have:

$$2\hat{e}_m^T P_1 BLCx \leq \alpha(\hat{e}_m^T P_1 BL)^2 + \frac{1}{\alpha} y^2 \tag{51}$$

We obtain the following.

$$\dot{V}_1 \leq -\hat{e}_m^T Q_1 \hat{e}_m + 2\hat{e}_m^T P_1 B[u + \sum_{i=1}^{m} \theta_i h_i(y) - a_m^T \hat{x} - b_r r] + \alpha(\hat{e}_m^T P_1 L)^2 + \frac{1}{\alpha} y^2 + \dot{V} \tag{52}$$

Considering the indirect model reference fuzzy control signal as:

$$u = a_m^T \hat{x} - \sum_{i=1}^{m} \theta_i h_i(y) - \rho \frac{\hat{e}_{my}}{\hat{e}_{my}^2 + \delta} y^2 + b_r r \tag{53}$$

in which $\rho > 0$ is a design parameter and $B_r = b_r B$. One gets:

$$\dot{V}_1 \leq -\hat{e}_m^T Q_1 \hat{e}_m - 2\rho \hat{e}_m^T P_1 B \frac{\hat{e}_{my}}{\hat{e}_{my}^2 + \delta} y^2 + \alpha(\hat{e}_m^T P_1 L)^2 + \frac{1}{\alpha} y^2 + \dot{V} \tag{54}$$

And further:

$$\dot{V}_1 \leq -\hat{e}_m^T Q_1 \hat{e}_m + \alpha(\hat{e}_m^T P_1 L)^2 + \frac{1 - 2\rho\alpha}{\alpha\left(\hat{e}_{my}^2 + \delta\right)} \hat{e}_{my}^2 y^2 + \frac{\delta}{\alpha\left(\hat{e}_{my}^2 + \delta\right)} y^2 + \dot{V} \tag{55}$$

Taking:

$$0.5\rho^{-1} \leq \alpha \tag{56}$$

and:

$$2\alpha L^T P_1^T P_1 L \leq \lambda_{\min}(Q_1) \tag{57}$$

One obtains:

$$\dot{V}_1 \leq -\frac{1}{2} \lambda_{\min}(Q_1) \left\| \hat{e}_m \right\|^2 + \frac{\delta}{\alpha\left(\hat{e}_{my}^2 + \delta\right)} y^2 + \dot{V} \tag{58}$$

and:

$$\dot{V}_1 \le -\frac{1}{2}\lambda_{\min}(Q_1)\|\hat{e}_m\|^2 + \frac{2\delta}{\alpha\left(\hat{e}_{my}^2+\delta\right)}y_m^2 + \frac{2\delta}{\alpha\left(\hat{e}_{my}^2+\delta\right)}e_y^2 + \frac{2\delta}{\alpha\left(\hat{e}_{my}^2+\delta\right)}\hat{e}_{my}^2 + \dot{V} \quad (59)$$

Considering (38) one obtains:

$$\begin{aligned}\dot{V}_1 \le & -\frac{1}{2}\lambda_{\min}(Q_1)\|\hat{e}_m\|^2 + \frac{2\delta}{\alpha\left(\hat{e}_{my}^2+\delta\right)}y_m^2 + \frac{2\delta}{\alpha\left(\hat{e}_{my}^2+\delta\right)}e_y^2 + \frac{2\delta}{\alpha\left(\hat{e}_{my}^2+\delta\right)}\hat{e}_{my}^2 \\ & -\lambda_{\min}(Q)\|e\|^2 + 2\lambda_{\max}(P)\|B\|\|e\|\left[\sum_{i=1}^m\|\theta_i^* h_i(y)\| + \|\sigma_\varepsilon\|\right] \\ & -\vartheta\lambda_{\min}(Q)\left\|(A+LC)^{-1}B\sum_{i=1}^m\theta_i h_i\right\|^2\end{aligned} \quad (60)$$

Since:

$$\frac{2\delta}{\alpha\left(\hat{e}_{my}^2+\delta\right)} \le \frac{2}{\alpha} \quad (61)$$

We have:

$$\begin{aligned}\dot{V}_1 \le & -\frac{1}{2}\lambda_{\min}(Q_1)\|\hat{e}_m\|^2 + \frac{2}{\alpha}y_m^2 + \frac{2}{\alpha}e_y^2 + \frac{2}{\alpha}\hat{e}_{my}^2 \\ & -\lambda_{\min}(Q)\|e\|^2 + 2\lambda_{\max}(P)\|B\|\|e\|\left[\sum_{i=1}^m\|\theta_i^* h_i(y)\| + \|\sigma_\varepsilon\|\right] \\ & -\vartheta\lambda_{\min}(Q)\left\|(A+LC)^{-1}B\sum_{i=1}^m\theta_i h_i\right\|^2\end{aligned} \quad (62)$$

By taking α as:

$$\max\left\{\frac{8C^TC}{\lambda_{\min}(Q_1)}, \frac{4C^TC}{\lambda_{\min}(Q)}\right\} < \alpha \quad (63)$$

One obtains:

$$\begin{aligned}\dot{V}_1 \le & -\frac{1}{4}\lambda_{\min}(Q_1)\|\hat{e}_m\|^2 + \frac{2}{\alpha}y_m^2 \\ & -\frac{1}{2}\lambda_{\min}(Q)\|e\|^2 + 2\lambda_{\max}(P)\|B\|\|e\|\left[\sum_{i=1}^m\|\theta_i^* h_i(y)\| + \|\sigma_\varepsilon\|\right] \\ & -\vartheta\lambda_{\min}(Q)\left\|(A+LC)^{-1}B\sum_{i=1}^m\theta_i h_i\right\|^2\end{aligned} \quad (64)$$

If

$$\frac{2\lambda_{\max}(P)\|B\|\left[\sum_{i=1}^m\|\theta_i^* h_i(y)\| + \|\sigma_\varepsilon\|\right]}{\lambda_{\min}(Q)} \le \|e\| \quad (65)$$

and

$$y_m^2 \le \frac{\alpha}{8}\lambda_{\min}(Q_1)\|\hat{e}_m\|^2 \quad (66)$$

We have $\dot{V}_1 \le 0$ thus $V_1(t)$ decreases inside a compact set S_1 where:

$$S_1 = \left\{ \hat{e}_m, e \left| y_m^2 \le \frac{\alpha}{8}\lambda_{\min}(Q_1)\|\hat{e}_m\|^2 \text{ and } \frac{2\lambda_{\max}(P)\|B\|\left[\sum_{i=1}^{m}\|\theta_i^* h_i(y)\| + \|\sigma_\varepsilon\|\right]}{\lambda_{\min}(Q)} \le \|e\| \right.\right\} \quad (67)$$

It follows that it is possible to make the observed tracking error ($\hat{e}_m$) and the state estimation error (e) to approach an arbitrarily small value by choosing appropriate values for the design constants $\alpha,\ \lambda_{\max}(P),\ \lambda_{\min}(Q),\ \lambda_{\max}(P_1)\ \ \lambda_{\min}(Q_1)$ and sufficient number of rules for the fuzzy system. But the main concern is to make the tracking error ($e_m = x - x_m$) to approach any small value. Since $e_m = \hat{e}_m - e$ and considering the fact that $\hat{e}_m$ and e can be made arbitrarily small it follows that it is possible to make e_m as small as desired. The following theorem summarizes the forthgoing stability analysis.

Theorem 2. If there exists positive definite matrixes P and P_1 satisfying (14) and (45), the control signal given by (53) together with the observer given by (5) for the nonlinear dynamical system of (4) with the adaptation law of (29) which is the optimal solution of the cost function (15) ensures that the nonlinear dynamical system of (4) follows the reference model of (42) with bounded error. In addition, the state estimation error and the estimated values of the fuzzy system θ_i are uniformly bounded. Furthermore, the tracking error and the state estimation error can be made to approach an arbitrarily small value by choosing appropriate values for the design constants $\alpha,\ \lambda_{\max}(P),\ \lambda_{\min}(Q),\ \lambda_{\max}(P_1)\ \ \lambda_{\min}(Q_1)$ and sufficient number of rules for the fuzzy system.

5 Simulation Results

In this section we use the well-known chaotic system of Chua's circuit to depict the design procedure and verify the effectiveness of the proposed algorithm. The control of the nonlinear chaotic Chua's circuits is an important topic for numerous practical applications since this circuit exhibits a wide variety of nonlinear dynamic phenomena such as bifurcations and chaos [28]. This chaotic circuit possesses the properties of simplicity and universality, and has become a standard prototype for investigation of chaos. In this section we will use the proposed method to control the nonlinear chaotic Chua's circuit.

5.1 Dynamical Equations of Chua's Circuit

The modified Chua's circuit is described by the following dynamical system [28]:

$$\begin{aligned} \dot{x}_1 &= p(x_2 - \frac{1}{7}(2x_1^3 - x_1)) + u_1, \\ \dot{x}_2 &= x_1 - x_2 + x_3 + u_2, \\ \dot{x}_3 &= q.x_2 + u_3, \end{aligned} \tag{68}$$

in which u_1, u_2 and u_3 are the external inputs and x_1, x_2 and x_3 are the states of the system. Considering $q = -\frac{100}{7}$ (as used in many references as [28]) and $u_2 = u_3 = 0$ [28], one obtains the state space equations of the system as:

$$\begin{aligned} \dot{x}_1 &= p(x_2 - \frac{1}{7}(2x_1^3 - x_1)) + u_1, \\ \dot{x}_2 &= x_1 - x_2 + x_3, \\ \dot{x}_3 &= -\frac{100}{7}.x_2, \end{aligned} \tag{69}$$

in which $p = 10$ [28].

5.2 Control of Chua's Circuit

The dynamical equation of Chua's circuit (69) can be viewed as the nonlinear dynamical system in the form of (4) in which

$$A = \begin{bmatrix} 0 & p & 0 \\ 1 & -1 & 1 \\ 0 & -\frac{100}{7} & 0 \end{bmatrix} B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{70}$$

and

$$f(x) = -\frac{1}{7} p(2x_1^3 - x_1), g(x) = 1 \tag{71}$$

In order to model $f(x)$ in the interval of [-2, 2], TS membership functions labeled as *about(-2)*, *about(-1)*, *about(0)*, *about(1)* and *about(2)* are considered. These labels correspond to fuzzy membership functions as: $exp(-(x_1 + 2)^2 / 0.42^2)$, $exp(-(x_1 + 1)^2 / 0.42^2)$, $exp(-x_1^2 / 0.42^2)$, $exp(-(x_1 - 1)^2 / 0.42^2)$ and $exp(-(x_1 - 2)^2 / 0.42^2)$, respectively. The following rules for the TS fuzzy model are considered.

Rule 1: If x_1 is $about(-2)$ then $\dot{x} = Ax + B(a_1^T x + b_1 u)$

Rule 2: If x_1 is $about(-1)$ then $\dot{x} = Ax + B(a_2^T x + b_2 u)$

Rule 3: If x_1 is $about(0)$ then $\dot{x} = Ax + B(a_3^T x + b_3 u)$

Rule 4: If x_1 is $about(1)$ then $\dot{x} = Ax + B(a_4^T x + b_4 u)$

Rule 5: If x_1 is $about(2)$ then $\dot{x} = Ax + B(a_5^T x + b_5 u)$

in which A, B and C are defined as in (70) and:

$$\begin{aligned} &a_1 = a_5 = \begin{bmatrix} -\frac{23}{7}p & 0 & 0 \end{bmatrix}^T, a_2 = a_4 = \begin{bmatrix} -\frac{5}{7}p & 0 & 0 \end{bmatrix}^T, \\ &a_3 = \begin{bmatrix} \frac{1}{7}p & 0 & 0 \end{bmatrix}^T, b_1 = b_2 = b_3 = [1,0,0]^T \end{aligned} \tag{72}$$

are estimated using linearization around the mean point. It should be noted that these values are reported for demonstration and are not used in the design of the controller. The gain of reference model a_m and gain of the observer L are taken as:

$$a_m = [0.01,0,0], L = [-12.5 \quad -0.14 \quad -0.12]^T, \tag{73}$$

respectively. The gain of the reference model a_m correspond to the reference model (A_m) as:

$$A_m = \begin{bmatrix} -12.5 & 10 & 0 \\ -0.14 & -1 & 1 \\ -0.12 & -14.29 & 0 \end{bmatrix} \tag{74}$$

whose eigenvalues are: $\{-12.39, -0.55 \pm 3.77i\}$. The design parameter Q is taken as

$$Q = \begin{bmatrix} 24.98 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \tag{75}$$

in which $I_{3\times3}$ is the identity matrix. The positive definite matrix P which is the solution of (14) is obtained as:

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 76.43 & -5 \\ 0 & -5 & 5.7 \end{bmatrix} \tag{76}$$

Taking Q_1 as:

$$Q_1 = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix} \tag{77}$$

the positive definite matrix P_1 which is the solution of (44) is obtained as:

$$P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 76.43 & -5 \\ 0 & -5 & 5.7 \end{bmatrix} \tag{78}$$

The state matrix of the reference model is set as $A_m = A + Ba_m^T + LC$. Using these design parameters, the regulation performances of the proposed control scheme are tested. Figures 1(a)-1(d) show the results of the regulation of the system as well as the state estimation performance of the observer.

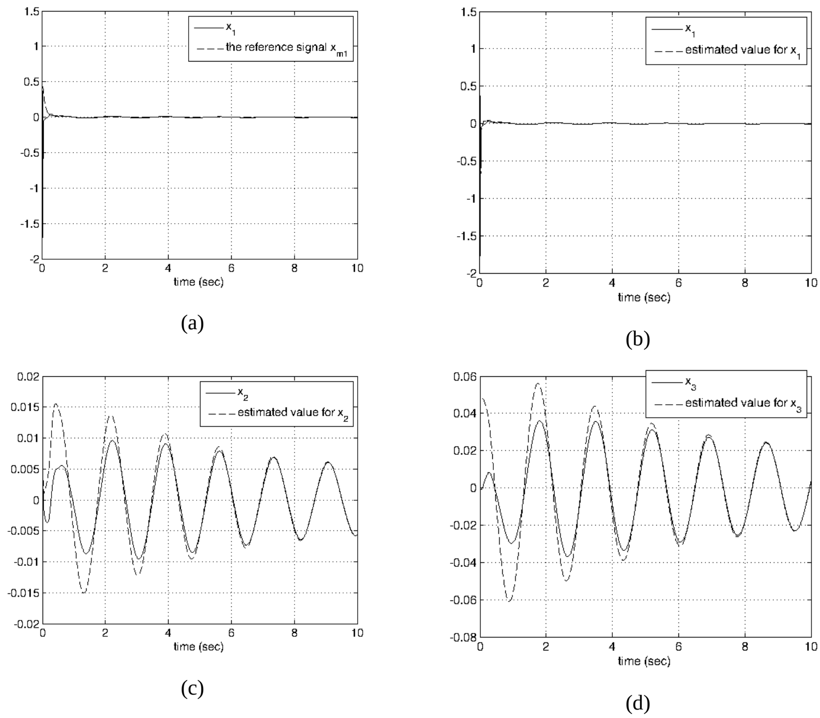
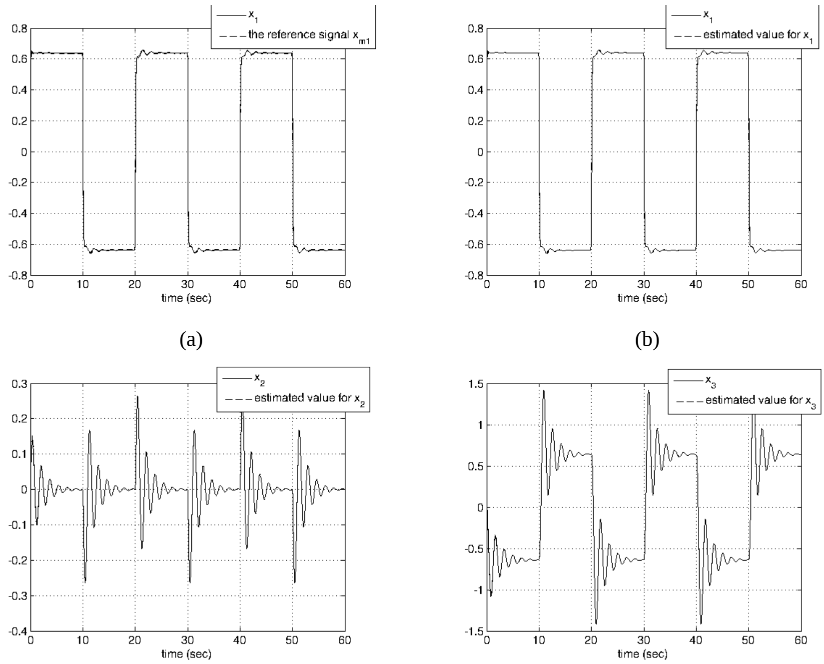


Figure 1

The regulation performance of the proposed observer and the controller when applied to Chua's chaotic system, a) The regulation response of Chua's chaotic system for x_1, b) The performance of the observer for x_1, c) The performance of the observer for x_2, d) The performance of the observer for x_3

The initial values for the states of the system, the observer and the reference model are considered as: $x = [0.5,0,0]^T$, $\hat{x} = [0.4,0,0.05]^T$ and $x_m = [0.55,0,0.1]$ respectively. In addition the tracking performance of the proposed controller is tested. Figures 2(a)-2(d) depict the tracking performance of the controller and the response of the observer. As can be seen from the figures, the tracking performance of the system under control is quite satisfactory.



(a) (b) (c) (d)

Figure 2

The tracking performance of the proposed observer and the controller when applied to Chua's chaotic system, a) The tracking response of Chua's chaotic system for x_1, b) The performance of the observer for x_1, c) The performance of the observer for x_2, d) The performance of the observer for x_3

Conclusions

This paper describes the design of an indirect model reference adaptive fuzzy controller based on an optimal observer for use with nonlinear dynamical systems. The proposed method adopts a TS fuzzy model to represent the dynamics of the system in hand and the adaptive model reference controller. The main contribution of the current work with respect to the previous studies in the field of model reference fuzzy controllers is that the current approach benefits from an optimal observer and therefore full state measurement is no longer needed. Its additional superiority over the observer-based TS adaptive fuzzy controllers seen in the literature is that it is capable of making the system follow a reference model rather than just regulation of the system to zero. Moreover the proposed algorithm calculates the parameters of the TS fuzzy model from data using an optimal adaptation law. The stability of the approach is automatically accomplished with the derivation of the adaptive law by the Lyapunov theory. Lastly, through the application to a Chua's circuit, the applicability of the design to the practical problems of control of chaotic systems is verified. It is shown that the current approach is capable of controlling Chua's circuit with high performance.

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