Fuzzy Sets in HR Management

Blanka Zemková

AXIOM SW, s.r.o., 760 01 Zlín, Czech Republic
blanka.zemkova@seznam.cz

Jana Talašová

Faculty of Science, Palacký Univerzity, Olomouc, Czech Republic
talasova@inf.upol.cz

Abstract: The aim of this paper is to demonstrate the different possible applications of fuzzy sets in HR management. This project is currently being carried out by the AXIOM SW company, which specializes in the implementation of the Microsoft Dynamics NAV information system. The evaluation of employees is based on multiple criteria evaluations. The criteria are derived from typical competencies of the employees. A competency model has been created for any given role with different normalized weights assigned to various competencies. The evaluation proceeds in the following manner: Firstly, the appointed evaluators fill in a questionnaire indicating to what extent, in their view, the tested employee meets his/her competencies. These evaluations are expressed using fuzzy scales. Normalized weights assigned to the evaluators of any given employee are set based on the intensity of cooperation between the employee and his/her evaluators. The level of fulfilment of each competency by the given employee is calculated as a weighted average of the fuzzy evaluations, conducted by each of his/her evaluators. Then, the overall fulfilment level of the employee's working role, again as a weighted average of fuzzy numbers, is calculated according to a specified model. This produces an overall evaluation of the employee. The evaluation process is followed by an interview where the employee is informed of his/her evaluation results, the employees gaps are discussed, and possibilities for improvement are proposed.

Keywords: evaluation; fuzzy sets; human resources

1 Introduction

Evaluation has become part of our lives. In some areas, evaluation is easier (based on measurable characteristics), while in others, more complex. There are many different concepts or types of evaluation. The employees evaluation presented in this paper is essentially a multiple criteria evaluation.

In psychometrics, it is common to use discreet scales with sharp real values. The theory of fuzzy sets allows for the use of such linguistic fuzzy scales, where the various scale values are expressed linguistically and modelled by fuzzy numbers. The purpose of using the instruments of linguistic fuzzy modelling is, on the one hand, an exact mathematic data processing that excludes unwanted subjective influence, and, on the other hand, the natural expression of the expertly defined vague evaluations using natural language.

The application of fuzzy sets in psychometrics has been studied by Michael Smithson (see [4]). A similar problem – the employees' evaluation – is also solved e.g. in the article "A multi-granular linguistic model for management decision-making in performance appraisal" (see [1]).

2 Applied Notions and Methods

2.1 Fuzzy Set

A fuzzy set A in a universal set U is uniquely determined by its membership function: $A:U\to[0,1]$. Sets $A_\alpha=\{x\in U, A(x)\geq\alpha\}, \alpha\in[0,1]$ are called α-cuts of the fuzzy set A, a set $Supp\,A=\{x\in U, A(x)>0\}$ is called a support of the fuzzy set A, and a set $Ker\,A=\{x\in U, A(x)=1\}$ is called a kernel of the fuzzy set A. The fuzzy set A is called normal if $Ker\,A\neq\emptyset$.

2.2 Fuzzy Number

A fuzzy number C is a fuzzy set defined in the set of real numbers $\Re$ with the following properties: $Ker\,C\neq\emptyset$, $Supp\,C$ is bounded, α-cuts C_α are for all $\alpha\in(0,1]$ closed intervals. A fuzzy number C is said to be defined in an interval $[A,B]$, if its membership function is equal to 0 outside the interval.

In the presented model, trapezoidal fuzzy numbers defined in an interval $[A,B]$ are used. A trapezoidal fuzzy number C in the interval $[A,B]$ is defined by four points, $(x_1,0),(x_2,1),(x_3,1),(x_4,0)$, where $A\leq x_1\leq x_2\leq x_3\leq x_4\leq B$; the membership function of C depends on the parameters x_1,x_2,x_3,x_4 in the following way: for any $x\in[A,B]$ it holds that $C(x)=0$ for $x<x_1$; $C(x)=(x-x_1)/(x_2-x_1)$ for $x_1\leq x\leq x_2$; $C(x)=1$ for $x_2<x<x_3$; $C(x)=(x_4-x)/(x_4-x_3)$ for $x_3\leq x\leq x_4$; $C(x)=0$ for $x_4<x$.

2.3 Fuzzy Scale (see [5])

We say that fuzzy numbers $T_1,\dots T_S$ defined in the interval $[A,B]$ form a fuzzy partition of the interval $[A,B]$ if $\sum_{i=1}^{S} T_i(x)=1$ for any $x\in[A,B]$.

A fuzzy scale is a set of fuzzy numbers $T_1,\dots T_S$ defined in the interval $[A,B]$ that form a fuzzy partition of this interval and are numbered according to their order.

2.4 Linguistic Variable (see [5])

A linguistic variable is characterized by a quintuple $(V, T(V), X, G, M)$, where V is a name of the variable, $T(V)$ is a set of its linguistic values, $X\subseteq\Re$ is a universal set in which the fuzzy numbers representing the meanings of the linguistic values are defined, G is a syntactic rule for generating linguistic values of V, and M is a semantic rule for setting mathematical meanings, i.e. fuzzy numbers in X, to linguistic values from $T(V)$.

2.5 Linguistic Fuzzy Scale (see [5])

Let us suppose that $(V,T(V),[A,B],G,M)$ is a linguistic variable and that meanings of its linguistic values form a fuzzy scale in $[A,B]$. Then we stipulate that the linguistic variable represents a linguistic scale in $[A,B]$. Linguistic scales meet the properties logically expected from verbally defined scales: meanings of linguistic values that are modelled by fuzzy numbers are linearly ordered and each point of the interval $[A,B]$ either entirely pertains to a certain element of the scale, or its pertaining is divided between two subsequent values on the scale. For modelled linguistic values, the above mentioned trapezoidal fuzzy numbers defined by four points are suitable.

2.6 Weighted Average of Fuzzy Numbers (see [5])

The weighted average of fuzzy numbers $C_1,\dots,C_m$ defined in the interval $[A,B]$, with normalized weights $v_j, v_j\ge 0,\ j=1,\dots,m,\ \sum_{j=1}^{m} v_j=1,$ is a fuzzy number $C=\sum_{j=1}^{m} v_j\cdot C_j$ the membership function of which is for any $c\in[A,B]$ defined by the following relation

$$C(c)=\max\left\{\min\{C_1(c_1),\dots,C_m(c_m)\}\;\middle|\; c=\sum_{j=1}^{m} v_j\cdot c_j,\ c_j\in[A,B], j=1,\dots,m\right\}$$

The weighted average of fuzzy numbers (with real weights) defined in this way is a fuzzification of weighted average of real numbers according to the definition of

the extension principle. In the presented employee evaluation model this definition of weighted average of fuzzy numbers will be used for the aggregation of partial evaluations given by different evaluators.

2.7 Fuzzy Weights (see [3])

Fuzzy numbers $V_j, j = 1,\ldots,m,$ defined in the interval $[0,1]$ are called normalized fuzzy weights, if for each $\alpha \in (0,1]$ and for each $i \in \{1,\ldots,m\}$ the following holds: for each $v_i \in V_{i\alpha}$ (an α–cut of V_i) there exist $v_j \in V_{j\alpha}, j = 1,\ldots,m, i \neq j,$ such that

$$v_i + \sum_{j=1, j\neq i}^{m} v_j = 1 .$$

2.8 Fuzzy Weighted Average (see [3])

Fuzzy weighted average of fuzzy numbers $C_1,\ldots,C_m$ defined in the interval $[A,B]$ with normalized fuzzy weights $V_j, j = 1,\ldots,m,$ is fuzzy number C defined on the interval $[A,B]$, the membership function of which is for each $c \in [A,B]$ defined by the following relation

$$C(c) = \max\left\{ \begin{array}{l} \min\{C_1(c_1),...,C_m(c_m),V_1(v_1),...,V_m(v_m)\} \mid \\ c = \sum_{j=1}^{m} v_j \cdot c_j, \sum_{j=1}^{m} v_j = 1, v_j \geq 0, j = 1,...,m, c_j \in [a,b], j = 1,...,m \end{array} \right\}.$$

In the presented fuzzy model this definition of fuzzy weighted average of fuzzy numbers will be used for the aggregation of partial evaluations against different criteria.

3 Employee Evaluation

First, the procedure of the employee performance evaluation will be demonstrated. The evaluation of employees is based on multiple-criteria evaluations. The criteria are derived using the typical competencies of the employee.

3.1 Competency Model

Competencies are the summary of the knowledge, skills, abilities, attitudes and values necessary for personal development and self-assertion of every member of society. The competency model is always created in terms of the work for a given working role. It reflects the competency composition that is necessary for carrying out a particular type of work.

The competency model was created from multiple sources of information. 13 general competencies were set as the basis (see [2]). A questionnaire was put together for the evaluators to express their opinion on what competencies the employees in their particular roles should have. To these general competencies, specific competencies were added as well for each specified role. Materials from the Microsoft company were used as the basis for the specified competencies in the following roles: project manager, developer, and consultant.

In the Axiom SW company, the following roles were identified: senior executive, project manager, analyst, consultant, developer, dealer, and marketing agent. Since the roles specified in the Microsoft materials do not entirely match with the roles in Axiom, competencies stipulated in these documents were assigned to roles in the Axiom company according to the real classification of working activities.

		Senior Executive	Project manager	Analyst	Consultant	Developer	Dealer	Marketing Agent
Input		0,33	0,33	0,33	0,33	0,33	0,33	0,33
	Eduaction	0,12	0,18	0,18	0,16	0,11	0,1	0,12
	Experience	0,12	0,18	0,16	0,2	0,12	0,12	0,1
	Certification	0,06	0,08	0,08	0,07	0,08	0,06	0,06
	Language Skills	0,08	0,08	0,04	0,04	0,08	0,18	0,16
	Stress Resistance	0,08	0,12	0,06	0,13	0,08	0,12	0
	Creative Thinking	0,06	0	0,14	0,05	0,04	0,06	0,18
	Customer Orientation	0,08	0,1	0	0,12	0,08	0,16	0,12
	Openness toward Changes	0,08	0,08	0	0	0,08	0,06	0,12
	Learning Skills	0,06	0	0,12	0,08	0,11	0	0
	Willingness to learn	0,08	0	0	0,08	0,11	0	0
	Analytical Thinking	0,04	0	0,16	0,07	0,11	0	0
	Vision and Strategy	0,14	0,18	0,06	0	0	0,14	0,14
	TOTAL	1	1	1	1	1	1	1
Output		0,33	0,33	0,33	0,33	0,33	0,33	0,33
	Fullfilment of given goals from previous period	0,2	0,4	0,2	0,06	0,25	0,45	0,45
	Quality of Work	0,4	0,3	0,4	0,5	0,4	0,25	0,3
	Perminf One's Tasks inTime	0,4	0,3	0,4	0,44	0,35	0,3	0,25
	TOTAL	1	1	1	1	1	1	1
Proces		0,34	0,34	0,34	0,34	0,34	0,34	0,34
	Centrality of Work	0,1	0	0,2	0,18	0,2	0,13	0,16
	Interpersonal Sensitivity	0,12	0,28	0,08	0,1	0	0,13	0,08
	Team Work	0,08	0	0,34	0,27	0,4	0	0
	Communication and Influence	0,2	0,2	0,22	0,31	0,3	0,14	0,3
	Integrity	0,2	0,22	0,16	0,14	0,1	0,2	0,26
	Organisational Behaviour, Leadership	0,3	0,3	0	0	0	0,4	0,2
		1	1	1	1	1	1	1

Figure 1

Competency Model

The competencies are divided into three groups: INPUT (knowledge, skills), OUTPUT (results) and PROCESS (behaviour). The weights of competencies can be represented either by real numbers or by fuzzy numbers (for the mathematical structure of normalized fuzzy weights see 2.7).

3.2 Evaluators

The structure of the evaluators is based on the company's organisational structure. A given employee will always be evaluated by those employees he/she most closely cooperates with, i.e. his/her direct supervisor, and collaborators on the same project, and in the case of managers, also by their subordinates.

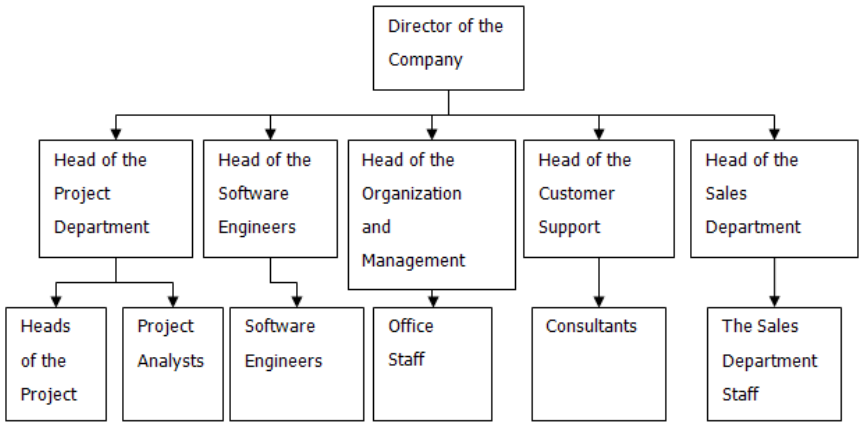

Figure 2
Company's Organizational Structure

The significance attributed to the evaluators will be first divided according to groups, and then to the individual employees in the groups. At both levels the division will be implemented by the Metfessel Allocation method (see [1]). In the end, normalized weights of individual evaluators are set for each employee.

Jan Novák	**Project Analyst**
Evaluation by Supervisors	**40**
Project Manager	100
Evaluation by Collaborators	**60**
Head of the Project	50
Analyst	25
Analyst	25

Figure 3
The Employee's Card – Structure of Evaluators

Since the assignment of employees to particular projects changes over time, it is necessary also to keep the composition of evaluators up-to-date. The division of weights among groups of evaluators must be determined by experts (i.e. it is meaningful to express them with numbers). The division of weights among the collaborators on the project in question must be carried out on the worked hours the employee reports to have spent on tasks in collaboration with the evaluator. (The system of the working-hours record allows for this). In cases where the subordinates evaluate their supervisor, the weights are divided equally amongst them.

3.3 Evaluation Methodology

The process of evaluation proceeds in several steps. First, an employee receives a questionnaire which he/she completes during preparation for the interview. In the questionnaire, the employee articulates his/her self-evaluation. The same questionnaire is handed out to all evaluators who then express their opinion to what extent they see the evaluated employee meeting the stipulated competencies. The questionnaire includes qualitative criteria that is part of the competency model for any given role. The evaluation against the quantitative criteria is taken from the employee's personal record (education, experience, certifications, etc.).

For evaluation against the qualitative criteria, the evaluators have at their disposal a non-uniform six-element linguistic fuzzy scale (see 2.5). Using the scale values, the evaluator expresses to what extent, in his/her view, the evaluated employee has met the required level of a given competence. The individual scale values are modelled with trapezoidal fuzzy numbers that are always given by four points (for the fuzzy scale see 2.3 and Fig. 4).

The scale was intentionally set as a non-uniform one. The reason for this is that people tend rather to avoid extreme values and prefer centre values. Therefore, if anyone chooses an extreme value, it may be assumed that the person is more certain about his/her decision. The extreme values of the used fuzzy scale are therefore less uncertain. The uncertainty rises towards the centre-values.

Table 1
Evaluation Scale

Points	**Descriptors**	**Distinctive points**			
1	Does not meet at all	0	0	0	1
2	In the most part does not meet	0	1	2	3
3	Rather doesn't meet	2	3	4	6
4	Rather meets	4	6	7	8
5	Nearly meets	7	8	9	10
6	Very well meets	9	10	10	10

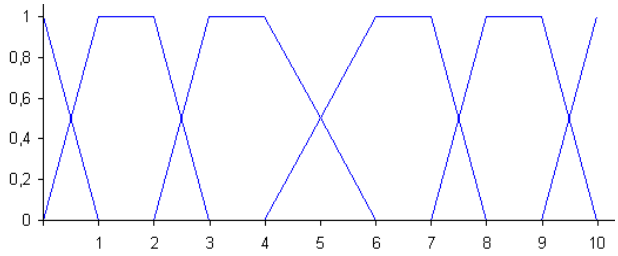

Figure 4
Applied Fuzzy Scale

The evaluation against the quantitative criteria is not part of the questionnaire as values are taken from a personnel record of the employee. Based on meeting the set criteria the employee is given a certain amount of points. For each of the quantitative criterion the maximum amount of points is set. The evaluation itself is then expressed as a level, in which the given employee meets the maximum value. The points are given in the following way (see Table 2):

Table 2

Evaluation against the Quantitative Criteria

Education		**Points**	**Note**
	Secondary school	1	
	Higher Vocational School (DiS.)	2	
	College (Bc.)	3	
	College (Mgr., Ing.)	5	
	College (PhDr., RNDr. etc.)	6	
	College (PhD., CSc.)	8	
Experience			
	Number of months of relevant experience	0-60	
Certification			
	Number of certifications	0-10	1 point for each certification
Language Knowledge			
	Reading comprehension	1	In case of more languages, the points are added, maximum is 6 points
	Ability to make oneself understood	1	
	Writing	1	

For each evaluated employee, there are available evaluations against the qualitative criteria in the form of questionnaires filled in by appointed employees (see Figure 3). The evaluations based on the quantitative criteria are exactly defined. The aggregation of those evaluations is carried out in several steps.

First, the qualitative criteria is assigned an evaluation as a weighted average (see 2.6) of evaluations given by individual employees, where the weights of the co-workers depend on the intensity of collaboration with the employee in question (see Figure 3). The resulting fuzzy number expresses an evaluation of the employee against the given criterion. Other qualitative criteria shall follow the same procedure analogically. It is possible to carry out a linguistic approximation of the resulting evaluation concerning various criteria.

Now we add to the evaluation quantitative criteria based on the specified conditions (education, experience, certification, language skills).

In the next stage, the partial evaluations are aggregated to form an overall evaluation. Firstly, there is the aggregation within groups INPUT, OUTPUT, PROCESS, subsequently followed by the aggregation of these groups. The evaluation tree is defined in accordance with a specified competency model. The evaluation is calculated as a fuzzy weighted average of the partial evaluations against the criteria (see chapter 2.8).

For the purpose of this paper, the evaluation aggregation was carried out using the demo version of the FuzzMe Programme (see http://FuzzMe.wz.cz).

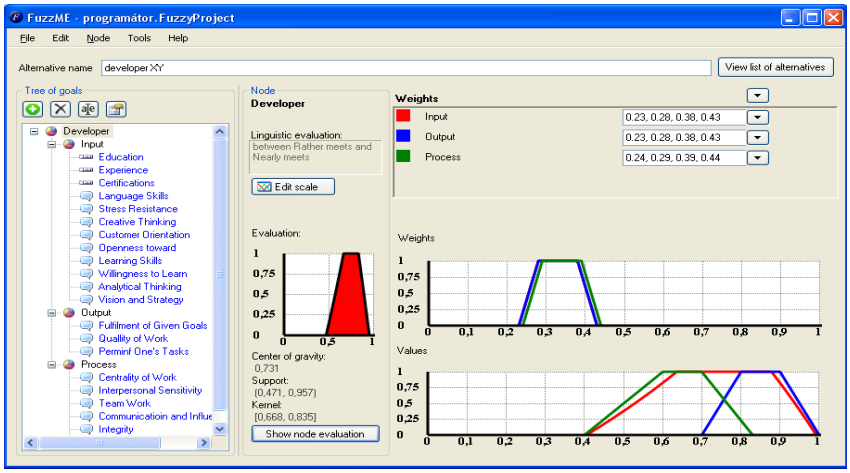

Figure 5

Aggregation of a Partial Evalutions of a Developer

3.4 Outcomes of the Evaluation

The output includes a partial fuzzy evaluation of an employee against the competencies, then the aggregated fuzzy evaluation for the areas INPUT, OUTPUT, and PROCESS, and finally, the overall fuzzy evaluation of the employee (see Table 2, Figure 6).

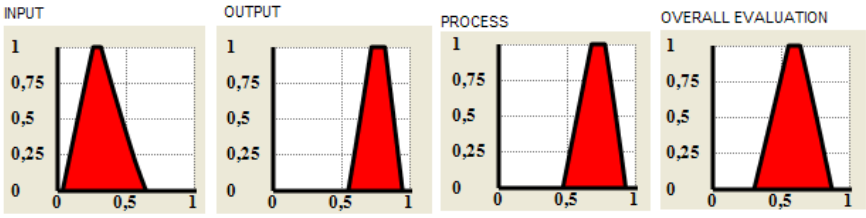

Figure 6

Aggregation against the Groups and Overall Evaluation

Table 2
An Example of Partial Evaluations (Crisp Weights)

JAN NOVÁK		Weights	Evaluation
INPUT		**33**	
	Education	18	5/8
	Experience	16	48/60
	Certifications	8	5/10
	Language Skills	4	4/6
	Stress Resistance	6	Nearly meets
	Creative Thinking	14	Rather meets
	Customer Orientation		Nearly meets
	Openness toward Changes		Rather doesn't meet
	Learning Skills	12	Rather meets
	Willingness to Learn		Nearly meets
	Analytical Thinking	16	Rather meets
	Vision and Strategy	6	Rather meets
	INPUT TOTAL		**0,196;0,26;0,318;0,427**
OUTPUT		**33**	
	Fulfilment of Given Goals	20	Nearly meets
	Quallity of Work	40	Nearly meets
	Perminf One's Tasks in Time	40	Nearly meets
	OUTPUT TOTAL		**0,58;0,72;0,82;0,932**
PROCESS		**34**	
	Centrality of Work	20	Rather meets
	Interpersonal Sensitivity	8	Nearly meets
	Team Work	34	Nearly meets
	Communicatioin and Influence	22	Rather meets
	Integrity	16	Rather meets
	Organisational Behaviour, Leadership		Rather doesn't meet
	PROCESS TOTAL		**0,52;0,68;0,78;0,898**
TOTAL			**0,433;0,555;0,641;0,754**

The evaluations in all mentioned levels are represented by fuzzy numbers or, more exactly, by their linguistic approximations. For further work with the employee the evaluation in groups INPUT, OUTPUT, and PROCESS is particularly important, since interrelations among these evaluations determine the type of the employee, which allows the management to choose the appropriate strategy (Table 3).

Processing the outcomes is followed by a motivation interview with the evaluated employee. First, the employee is asked for his/her self-evaluation. Then the supervisor informs the evaluated employee of the evaluation outcomes. If a striking discrepancy between the self-evaluation and the evaluation by the other employees occurs, an explanation is sought. Then the employee sets his/her objectives for the following time period which he/she must defend before the supervisor. An evaluation of fulfilment of these objectives shall be included into the employee's evaluation for a following time period.

Table 3
The example of Working Types and their Management Strategies

OUTPUT (Performance)	INPUT (Potential)	PROCESS (Behaviour)	WORK TYPE	MOTIVATION STRATEGY
+	+	+	Star	He/She should be given more ambitious tasks, his/her informal authority should be encouraged, should be given as an example, should be delegated to, promoted
+	+	-	Enfant terrible	should be more involved in group tasks or on the contrary trusted with independent tasks, depending on the personality type; requires consistent, unforgiving approach, not-ignoring any assets; needs acceptance by the others, without being preachy;
-	+	+	Promising	more support, stimulate courage, resilience and self-confidence;
-	+	-	Intelligent idler and a badgerer	limits need to be set as well as deadlines for change with clear implications in case of non-compliance; as much feedback as possible;
+	-	+	Agreeable Hard Worker	to instruct and create conditions for self-education;
+	-	-	Free Spirit	shape up;
-	-	+	Nice Lubber	if tutoring by others (of the *infant terrible* type) fails, consider redeployment;
-	-	-	Spoiler	problematic choice; it is necessary to speak expressly about the possibility of departure; the initiative of improvement is to be left solely to the person in question; deadline for visible improvements needs to be pinpointed.

Conclusions

In this paper, the possibilities of the application of fuzzy sets in employee evaluation has been demonstrated. In psychometrics it is common to use discreet scales with sharp integer values. The theory of fuzzy sets allows for the use of linguistic scales where scale values are expressed linguistically and modelled by fuzzy numbers. Similarly, expertly defined weights of criteria can as well be model more adequately by fuzzy numbers, as was shown. The resulting evaluation could also be back-transformed to linguistic expression by means of a procedure

called linguistic approximation. The advantage of the linguistic fuzzy approach is, on the one hand, a mathematical data process excluding subjective bias, and, on the other hand, a natural process of evaluation and natural expression of evaluation resulting in language natural to the evaluator.

References

[1] de Andrés, Rocío, García-Lapresta, José, Martínez, Luis. *A Multi-Granular Linguistic Model for Management Decision-Making in Performance Appraisal.* Soft Computing - A Fusion of Foundations, Methodologies and Applications. Springer Verlag, Heidelberg 2010, pp. 21-34, ISSN 1432-7643

[2] Hroník, František. *Hodnocení pracovníků*. Praha : Grada Publishing, a. s., 2006. ISBN-80-247-0405-6

[3] Pavlačka, Ondřej, Talašová, Jana. The Fuzzy Weighted Average Operation in Decision-Making Models. *Proceedings of the 24th International Conference Mathematical Methods in Economics,* September 13-15, 2006, Plzeň (Ed.L. Lukáš) Typos, Plzeň 2006, pp. 419-426

[4] Smithson, Michael. *Fuzzy Set Analysis for Behavioral and Social Science*. New York : Springer - Verlag, 1987, ISBN 0-387-96431-2

[5] Talašová, Jana. *Fuzzy metody vícekriteriálního hodnocení a rozhodování*. Olomouc : Vydavatelství UP, 2003, ISBN-80-244-0614-4

[6] Talašová, Jana. Fuzzy přístup k hodnocení a rozhodování. *Proceedings of the 5th Conference Aplimat* (Ed.: Kováčová, M.), Bratislava, February 7-10, 2006, Slovak University of Technology in Bratistava, Bratislava, 2006, pp. 221-236

[7] Zemková, Blanka. Application of Fuzzy Sets in Employees' Evaluation. Aplimat – Journal of Applied Mathematics. Bratislava 2008, pp. 579-588, ISSN 1337-6365

[8] Ján Vaščák, Ladislav Madarász: Adaptation of Fuzzy Cognitive Maps – a Comparison Study, in Acta Polytechnica Hungarica, Vol. 7, No. 3, 2010, pp. 109-122

[9] Márta Takács: Multilevel Fuzzy Approach to the Risk and Disaster Management, in Acta Polytechnica Hungarica, Vol. 7, No. 4, 2010, pp. 91-102