Multicriteria Decision Making and Rankings Based on Aggregation Operators

(Application on Assessment of Public Universities and Their Faculties)

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Abstract: In order to solve decision making problem we have to compare and rank a finite set of alternatives. In this paper we want to show some approaches to creating the preference structure (ranking) of alternatives. These approaches lead us to use chosen multicriteria decision methods and aggregation operators. ARRA (Academic ranking and rating agency) uses one fixed way to create a ranking of public universities and their faculties. In this paper many more approaches to creating such rankings (or preference structures) of alternatives are discussed and the results are compared.

Keywords: Alternative; preference structure; multicriteria method; aggregation operator

1 Introduction

ARRA (Academic ranking and rating agency) publishes every year an assessment report concerning public universities and their faculties. The higher education institution assessment procedure consists of the following steps:

- the selection of indicators (criteria) for the quality of education and research in individual universities and the assignment of a certain number of points to each faculty for the performance in the particular indicator (indicators are arranged into groups and each group of indicators gains a certain number of points),
- the partition of faculties into six groups according to the so-called Frascati Manual in order to compare only faculties that have the same orientation and similar working conditions,

- assigning a point score to faculties (the ranking is based on the average points score in the individual groups of indicators),
- calculating the point scores for the higher education institutions in individual Frascati groups (the ranking of the institution in the given group is given by the average assessment of all its faculties included in that group). For more details, see [8].

Assessment criteria created by ARRA [8] are shown in the table below.

Table 1
Criteria created by ARRA

Area	Code	Description
Science and research	VV1	Number of publications in WoK for the years 1996 – 2005 per creative worker
	VV2	Number of citations in WoK for the years 1996 – 2005 per creative worker
	VV2a	Number of citations in WoK per publication in WoK for the years 1996 – 2005
	VV3	Number of publications in WoK having at least 5 citations in WoK for the years 1996 – 2005 per creative worker
	VV3a	Number of publications in WoK having at least 25 citations in WoK for the years 1996 – 2005 per creative worker
	VV4	Number of full-time PhD students per professor or associate professor in 2005
	VV5	Average annual number of PhD graduates in 2003 – 2005 in proportion to the number of professors and associate professors
	VV6	The number of full-time PhD students divided by the number of bachelor's and master's degree full-time students
	VV7	Grant funding from the KEGA and VEGA agencies per creative worker in 2005
	VV8	Grant funding from the APVV agency per creative worker in 2005
	VV9	Funding from foreign grants and state programmes per creative worker
	VV10	Total grant funding from agencies per creative worker
Study and education	SV1	Proportion of the number of full-time and part-time students per teacher in 2005
	SV2	Proportion of the number of full-time and part-time students per professor or associate professor in 2005
	SV3	Proportion of professors, associate professors and other teachers with PhD to the total number of teachers
	SV4	Proportion of professors and associate professors to all teachers
	SV5	Average age of active professors
	SV6	Ratio of the actual number of applications received to the planned number in 2005
	SV7	Ratio of registered and admitted students in 2005
	SV8	Proportion of foreign students
	SV9*	Proportion of graduates unemployed for longer than 3 months of institution's graduates in 2005
	SV10*	Number of students taking part in study abroad (SAIA administered scholarship programmes and the Socrates EC programme) per 100 students

For simplicity in the examples we have chosen seven faculties (alternatives) a_i, $i=1,\dots,7$ (from a total number of 24 technical specialized faculties evaluated by ARRA) and five evaluation criteria C_j, $j=1,\dots,5$, one from every set: SV1-SV4 ⇨ C_1, SV6-SV8 ⇨ C_2, VV1-VV3a ⇨ C_3, VV4-VV6 ⇨ C_4, VV7-VV10 ⇨ C_5.

In the following examples the differences between the ARRA rankings (preference structures) and other ranking methods are shown.

Example 1

Take a set of alternatives **A** = ($a_1,\ldots,a_7$). Every alternative a_i is described by five criteria $C_1,\ldots, C_5$ see table below.

Table 2
Input data

	C_1	C_2	C_3	C_4	C_5
a_1	2,78	0,75	0,03	0,32	10,6
a_2	2,73	0,69	2,19	0,31	146,3
a_3	4,05	0,5	0,36	0,33	124,6
a_4	3,4	0,69	0,71	0,31	54,7
a_5	3,06	0,67	7,61	0,22	93,3
a_6	2,81	0,61	0,1	0,3	36,4
a_7	3,8	0,55	0,2	0,28	78

Our aim is to create a final ranking on these alternatives.

2 The Basics of Preference Relations Notations

When the decision maker chooses between two alternatives (which are not incomparable) described by score vectors **x** and **y** within his choice set, he is able to say that he prefers **x** to **y** (or vice-versa) or he has the possibility to say that two alternatives are indifferent (equivalent). We will not distinguish alternative **a** and its corresponding score vector **x**.

- Strict preference (*P*) - a couple of alternatives **x**, **y** belongs to the relation *P*, if and only if the decision maker strictly prefers **x** to **y** (**x** *P* **y**): $\mathbf{x} \succ \mathbf{y}$.
- Indifference (*I*) - a couple of alternatives **x, y** belongs to the relation *I*, if and only if the decision maker is indifferent between alternatives **x** and **y** (**x** *I* **y**): $\mathbf{x} \approx \mathbf{y}$.
- Incomparability (*J*) - a couple of alternatives **x**, **y** belongs to the relation *J*, if and only if the decision maker is unable to compare **x** and **y** (**x** *J* **y**). (But this is not our case.)

3 Multicriterial Methods Used in the Assessment of Public Universities

In this section we recall three of the often used social choice procedures and their application on the assessment. Note that these methods are usually used in voting systems to find a winner, and they can be applied to create rankings, as well.

3.1 Borda Count Method

For the Borda Count Method, each candidate (alternative) gets 1 point for each last place vote received, 2 points for each next-to-last point vote, etc., all the way up to *m* points for each first place vote (where *m* is the number of candidates/alternatives). The candidate with the largest point total wins the election.

$$B_i=\sum_{j=1}^{m} B_{ij},\quad i=1,2,\dots,n \tag{1}$$

B_{ij} is a number assigned by j-th expert (criterion) to i-th candidate (alternative).

The ranking is done using B_i as the utility function value for alternative (candidate) i.

3.2 Plurality Voting

The idea of plurality voting is simply to declare as the social choices the alternatives with the largest number of first-place rankings in the individual preference list. The successive application of the plurality voting method, omitting its actual winners, can serve for ranking purposes.

3.3 The Hare System

This system is based on the idea of arriving at a social choice by successive deletions of less desirable alternatives. If any alternative occurs at the top of at least half of the preference lists, then it is declared to be a social choice [7]. Ranking can be achieved by using a similar strategy as plurality voting.

Example 2

The preference structures by the criteria C_j (see Table 1) are as follows:

C_1: $a_3 \succ a_7 \succ a_4 \succ a_5 \succ a_6 \succ a_1 \succ a_2$,

C_2: $a_1 \succ a_2 \approx a_4 \succ a_5 \succ a_6 \succ a_7 \succ a_3$,

C_3: $a_5 \succ a_2 \succ a_4 \succ a_3 \succ a_7 \succ a_6 \succ a_1$,

C_4: $a_3 \succ a_1 \succ a_2 \approx a_4 \succ a_6 \succ a_7 \succ a_5$,

C_5: $a_2 \succ a_3 \succ a_5 \succ a_7 \succ a_4 \succ a_6 \succ a_1$.

The final preference structures via the usage of the mentioned methods are:

Borda count method (B.C.): $a_3 \succ a_2 \succ a_4 \succ a_5 \succ a_1 \approx a_7 \succ a_6$.

Plurality voting (P.V.): $a_3 \succ a_1 \succ a_2 \succ a_4 \succ a_5 \succ a_7 \succ a_6$.

Hare system (H.S.): $a_3 \succ a_1 \succ a_2 \succ a_4 \succ a_5 \succ a_7 \succ a_6$.

These three methods satisfy monotonicity and the Pareto condition.

4 Linear Normalization of Input Data

Data normalization consists of rescaling the attribute values of the data into a single specified range, such as from 0 to 1 or from 0 to 100.

In the following table are shown 3 basic arithmetic processes of linear normalization.

Table 3

Linear normalization

	Procedure 1	Procedure 2	Procedure 3
Definition	$v_i = \frac{x_i}{\max x_i}$	$v_i = \frac{x_i - \min x_i}{\max x_i - \min x_i}$	$v_i = \frac{x_i}{\sum x_i}$
Normalized vector*	$0 \le v_i \le 1$	$0 \le v_i \le 1$	$0 \le v_i \le 1$
Constraints	$\max v_i = 1$	$\min v_i = 0, \max v_i = 1$	$\sum v_i = 1$
Proportionality conserved	Yes	No	Yes
Interpretation	% of maximum x_i	% of range $(\max x_i - \min x_i)$	% of total $\sum x_i$

Conditions under which the methods can be applied: $x_i \ge 0$,

$x_1, \ldots, x_n$ are the input values and the values $v_1, \ldots, v_n$ are normalized outputs.

* except for some pathological values of x_i (if $x_1 = \ldots = x_n$, then the corresponding criterion C_j gives no information and it can be omitted).

Example 3

We continue in Example 1, i.e., we consider Table 2. In the following tables we show the normalized inputs according to three different Procedures 1, 2, 3.

Table 4
Procedure 1 of linear normalization

	C_1	C_2	C_3	C_4	C_5
a_1	68,64	100,00	0,39	96,97	7,25
a_2	67,41	92,00	28,78	93,94	100,00
a_3	100,00	66,67	4,73	100,00	85,17
a_4	83,95	92,00	9,33	93,94	37,39
a_5	75,56	89,33	100,00	66,67	63,77
a_6	69,38	81,33	1,31	90,91	24,88
a_7	93,83	73,33	2,63	84,85	53,32

Table 5
Procedure 2 of linear normalization

	C_1	C_2	C_3	C_4	C_5
a_1	3,79	100,00	0,00	90,91	0,00
a_2	0,00	76,00	28,50	81,82	100,00
a_3	100,00	0,00	4,35	100,00	84,01
a_4	50,76	76,00	8,97	81,82	32,50
a_5	25,00	68,00	100,00	0,00	60,94
a_6	6,06	44,00	0,92	72,73	19,01
a_7	81,06	20,00	2,24	54,55	49,67

Table 6
Procedure 3 of linear normalization

	C_1	C_2	C_3	C_4	C_5
a_1	12,28	16,82	0,27	15,46	1,95
a_2	12,06	15,47	19,55	14,98	26,90
a_3	17,90	11,21	3,21	15,94	22,91
a_4	15,02	15,47	6,34	14,98	10,06
a_5	13,52	15,02	67,95	10,63	17,15
a_6	12,42	13,68	0,89	14,49	6,69
a_7	16,79	12,33	1,79	13,53	14,34

Remark 1

The linear normalization of input data saves the preferences between alternatives in individual criteria, but for different aggregation methods the final preference structures may be different.

5 Chosen Aggregation Operators

In many decision making problems, a number of independent attributes or criteria are often used to individually rate an alternative from the decision maker's perspective and then these individual ratings are combined to produce an overall assessment.

In decision making, values to be aggregated are typically preference or satisfaction degrees. A preference degree tells to what extent an alternative **x** is preferred to an alternative **y**, and thus is a relative evaluation. A satisfaction degree expresses to what extent a given alternative is satisfactory with respect to a given criterion.

For more information about aggregation operators and their properties see [1], [4].

5.1 Basic Aggregation Operators

- Arithmetic mean

The simplest and most common way to aggregate data is to use a simple arithmetic mean A_M (average).

$$A_M(x_1,\dots,x_n) = \frac{1}{n}\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} \frac{1}{n}.x_i \tag{2}$$

This operator is interesting because it gives an aggregated value that is between *max* $(x_1,\dots,x_n)$ and *min* $(x_1,\dots,x_n)$. The result of aggregation is "a middle value". The average is often used since it is simple and satisfies the properties of monotonicity, continuity, symmetry, idempotence and stability for linear transformations.

- Weighted arithmetic mean

Unlike the arithmetic mean, the weighted arithmetic mean reflects the possibly different importance of single criteria in multi-criteria decision making.

For n-ary operators, the weights form an n-dimensional weighting vector **w**

w = $(w_1,\dots,w_n) \in [0,1]^n$, $\sum_{i=1}^{n} w_i = 1$. If a weighted arithmetic mean W : $\bigcup_{n\in N}[0,1]^n \to [0,1]$ is an operator for any input tuples, it is necessary to know the relevant weights for all possible input cardinalities n and, therefore, it is necessary to have a weighting triangle $\Delta = (w_{in} \mid n \in N, i \in \{1,\dots, n\})$ such that all $w_{in} \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$ for all $n \in N$, see [1].

- Ordered weighted arithmetic mean

Definition 4: Let Δ be a given weighting triangle. An aggregation operator $A_{W\Delta}$: $\bigcup_{n\in N}[0,1]^n \to [0,1]$ defined by

$$A_{W\Delta}(x_1,\dots,x_n) = \sum_{i=1}^{n} w_{in}.x_i \qquad (3)$$

is called a weighted arithmetic mean associated with Δ.

Notice that the arithmetic mean AM is the only symmetric weighted mean associated with weighted triangle $\Delta = (w_{in}) = \left(\frac{1}{n}\right)$.

Table 7

Chosen aggregation operators

	C_1	C_2	C_3	C_4	C_5	R average	R Aw
PROCEDURE 1 OF NORMALIZATION							
a_1	68,64	100,00	0,39	96,97	7,25	6	5
a_2	67,41	92,00	28,78	93,94	100,00	2	3
a_3	100,00	66,67	4,73	100,00	85,17	3	2
a_4	83,95	92,00	9,33	93,94	37,39	4	4
a_5	75,56	89,33	100,00	66,67	63,77	1	1
a_6	69,38	81,33	1,31	90,91	24,88	7	7
a_7	93,83	73,33	2,63	84,85	53,32	5	6
PROCEDURE 2 OF NORMALIZATION							
	C_1	C_2	C_3	C_4	C_5		
a_1	3,79	100,00	0,00	90,91	0,00	6	3
a_2	0,00	76,00	28,50	81,82	100,00	2	4
a_3	100,00	0,00	4,35	100,00	84,01	1	1
a_4	50,76	76,00	8,97	81,82	32,50	4	2
a_5	25,00	68,00	100,00	0,00	60,94	3	6
a_6	6,06	44,00	0,92	72,73	19,01	7	7
a_7	81,06	20,00	2,24	54,55	49,67	5	5
PROCEDURE 3 OF NORMALIZATION							
	C_1	C_2	C_3	C_4	C_5		
a_1	12,28	16,82	0,27	15,46	1,95	7	6
a_2	12,06	15,47	19,55	14,98	26,90	2	2
a_3	17,90	11,21	3,21	15,94	22,91	3	4
a_4	15,02	15,47	6,34	14,98	10,06	4	3
a_5	13,52	15,02	67,95	10,63	17,15	1	1
a_6	12,42	13,68	0,89	14,49	6,69	6	7
a_7	16,79	12,33	1,79	13,53	14,34	5	5

There are some methods of generating weighting triangles [2], [3]. The method proposed in [3] is based on monotone real functions called quantifiers q: $[0,1] \to [0,1]$, such that $\{0,1\} \subseteq \mathrm{Ran}\, q$. The weighting triangle

$$\Delta_q = (w_{in}), \quad w_{in} = q\left(\frac{i}{n}\right) - q\left(\frac{i-1}{n}\right) \tag{4}$$

is defined for non-decreasing quantifiers q.

Example 4

In this example we show a comparison of arithmetic and weighted arithmetic mean application. The weighting vector used by weighted arithmetic mean (A_w) is in this example $\mathbf{w} = \left(\frac{1}{4}, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}\right)$.

The Ordered Weighted Averaging Operators (OWA operators) were originally introduced by Yager [6] to provide the means for aggregating scores associated with the satisfaction of multiple criteria, which unifies in one operator the conjunctive and disjunctive behaviour.

Definition 5: Let A_W : $\bigcup_{n \in N} [0,1]^n \to [0,1]$ be a weighted arithmetic mean associated with the weighted triangle $\Delta = (w_{in})$.

The operator $A_{W'}$: $\bigcup_{n \in N} [0,1]^n \to [0,1]$ given by

$$A_{W'}(x_1,\dots,x_n) = \sum_{i=1}^{n} w_{in} \cdot x_i', \tag{5}$$

where $(x_1',\dots,x_n')$ is a non-decreasing permutation of the input n-tuple $(x_1,\dots,x_n)$ is called an OWA operator associated with Δ.

A fundamental aspect of this operator is the re-ordering step, in particular an aggregate x_i is not associated with a particular weight w_i but rather a weight is associated with a particular ordered position of an aggregate.

Example 5

Let $A_{W'\Delta q}(\mathbf{x}) = \sum_{i=1}^{5} \left(q\left(\frac{i}{5}\right) - q\left(\frac{i-1}{5}\right)\right) x'_i$, be an OWA operator with non-decreasing permutation of the input 5-tuple $(x_1,\dots, x_n)$. For the inputs see Table 3.

(i) Let a quantifier q_1 : $[0,1] \to [0,1]$ by given by $q_1(x) = x^p$, $p > 1$. Then OWA operator $A_{W'\Delta q1}$ prefers the "high score" of the inputs. Take $p = 2$.

(ii) Let a quantifier q_2: $[0,1] \to [0,1]$ by given by $q_2(x) = x^p$, $p \in]0,1[$. Then

OWA operator $A_{W'\Delta q2}$ prefers the "low score" of the inputs. Take $p = 0{,}1$.

(iii) Let a quantifier q_3: $[0,1] \rightarrow [0,1]$ by given by $q_3(x) = [1 + (2x\text{-}1)^p]/2$, $p = 1/(2k+1)$ and $k \in N$. Then OWA operator $A_{W'\Delta q3}$ prefers the "average score" of the inputs. Take $k = 2$.

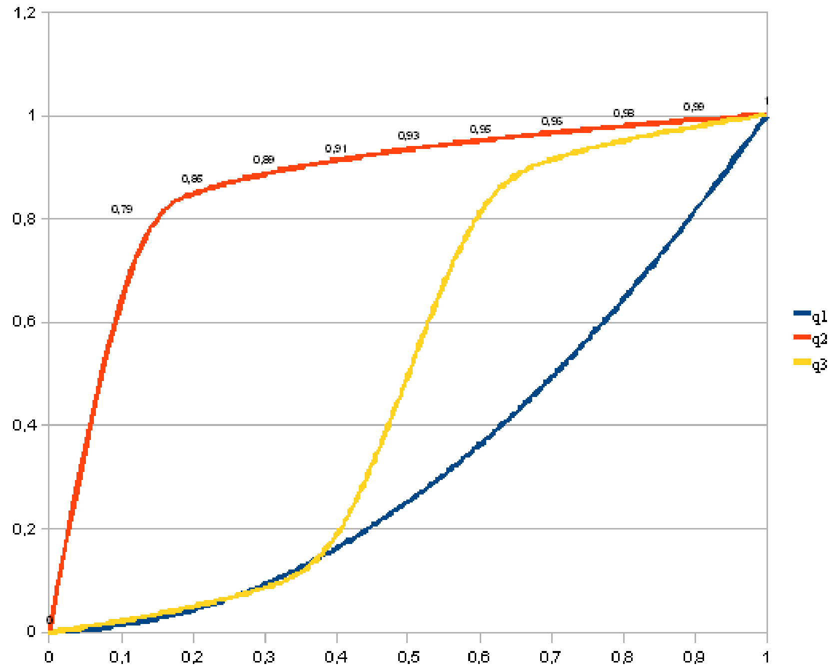

Figure 1
Quantifiers

Conclusion

We have introduced some other decision making methods to show another view of the ranking of the public universities and their faculties. We have used all three procedures of linear input data normalization and applied five chosen aggregation operators and three multicriteria methods to it, in order to compare them with the ARRA ranking. ARRA creates its ranking based on the normalization Procedure 1 and on the simple arithmetic mean (average) as an aggregation method. It is one of the possible choices for composing them.

As we have seen in the previous examples, the results (rankings) by using the same aggregation methods for different normalization procedures are not the same. We have shown that they depend firstly on the choice of the normalization procedure and secondly on aggregation method. The chosen aggregation operators show us that Procedure 3 of linear input data normalization seems to be the best way, because by using different aggregation methods, rankings are very similar.

Recall that all of these methods and many others are only a support for a decision maker.

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Table 8

Summary of the rankings based on chosen aggregation operators and multicriteria methods

	PROCEDURE 1 OF NORMALIZATION					*R*	*R*		*R*		*R*		*R*		*R*	
	C_1	C_2	C_3	C_4	C_5	average	Aw	OWA_1		OWA_2		OWA_3		B.C.	P.V.	H.S.
a_1	68,64	100,00	0,39	96,97	7,25	6	5	44,58	7	67,41	6	21,51	6	5	2	2
a_2	67,41	92,00	28,78	93,94	100,00	2	3	81,79	1	68,91	5	45,54	2	2	3	3
a_3	100,00	66,67	4,73	100,00	85,17	3	2	71,61	3	94,04	1	27,26	4	1	1	1
a_4	83,95	92,00	9,33	93,94	37,39	4	4	56,03	5	80,87	3	29,21	3	3	4	4
a_5	75,56	89,33	100,00	66,67	63,77	1	1	75,37	2	76,81	4	93,14	1	4	5	5
a_6	69,38	81,33	1,31	90,91	24,88	7	7	47,21	6	67,16	7	20,87	7	7	7	7
a_7	93,83	73,33	2,63	84,85	53,32	5	6	56,03	4	87,99	2	23,14	5	5	6	6
	PROCEDURE 2 OF NORMALIZATION															
	C_1	C_2	C_3	C_4	C_5			OWA_1		OWA_2		OWA_3		B.C.	P.V.	H.S.
a_1	3,79	100,00	0,00	90,91	0,00	6	3	37,61	6	11,86	5	17,18	5	5	2	2
a_2	0,00	76,00	28,50	81,82	100,00	2	4	73,73	1	10,20	7	39,56	2	2	3	3
a_3	100,00	0,00	4,35	100,00	84,01	1	1	63,11	2	89,93	1	21,00	4	1	1	1
a_4	50,76	76,00	8,97	81,82	32,50	4	2	47,55	4	51,18	3	24,60	3	3	4	4
a_5	25,00	68,00	100,00	0,00	60,94	3	6	51,10	3	30,56	4	82,71	1	4	5	5
a_6	6,06	44,00	0,92	72,73	19,01	7	7	32,92	7	10,32	6	12,28	7	7	7	7
a_7	81,06	20,00	2,24	54,55	49,67	5	5	39,24	5	72,93	2	14,61	6	5	6	6
	PROCEDURE 3 OF NORMALIZATION															
	C_1	C_2	C_3	C_4	C_5			OWA_1		OWA_2		OWA_3		B.C.	P.V.	H.S.
a_1	12,28	16,82	0,27	15,46	1,95	7	6	7,59	7	11,97	7	3,76	7	5	2	2
a_2	12,06	15,47	19,55	14,98	26,90	2	2	20,13	2	12,96	2	18,78	2	2	3	3
a_3	17,90	11,21	3,21	15,94	22,91	3	4	15,41	3	16,99	3	6,73	4	1	1	1
a_4	15,02	15,47	6,34	14,98	10,06	4	3	11,54	4	14,61	4	8,52	3	3	4	4
a_5	13,52	15,02	67,95	10,63	17,15	1	1	25,08	1	15,67	1	53,02	1	4	5	5
a_6	12,42	13,68	0,89	14,49	6,69	6	7	8,78	6	11,99	6	4,08	6	7	7	7
a_7	16,79	12,33	1,79	13,53	14,34	5	5	11,46	5	15,81	5	5,11	5	5	6	6

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