# Six-Phase Matrix Converter Fed Double Star Induction Motor

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Abstract: two different control strategies applied to a direct AC-AC six-phase matrix converter are investigated in the present paper. The first strategy is derived from the Venturini method, and the second approach is practically an extension of the scalar strategy control. Both strategies were originally applied to the three-phase matrix converter.

The current investigation deals with a comparative performance study of a double star induction motor fed from a six-phase matrix converter using the two above control strategies.

After a theoretical introduction of the six-phase matrix converter, a detailed description of the strategies implementation is presented followed by a discussion of results obtained from simulation results.

Keywords: six-phase; matrix converter; modulation coefficients; double star induction motor

## 1 Introduction

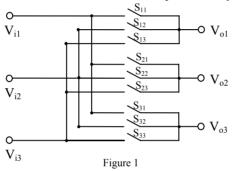
Over the last 20 years induction machines with a double star have been used in many industrial applications due to their performances in high power fields[4]. The double star induction machine requires a double three-phase supply and has many advantages. Not only does give reduced torque oscillations, but it also requires less powerful electronic components as the current flowing in a six-phase machine is less than that flowing in a three-phase machine. However, as the use of an inverter is necessary when feeding a double star induction machine, this may result in supplementary losses, since such an inverter is a harmonic generator[5].

A six-phase matrix converter can be used as a source for these kind machines, but the main problem is in finding the appropriate control method which produces a six-phase system with low electromagnetic torque oscillations.

## 2 Theory of the Matrix Converter

A matrix converter is a direct power converter, generating variable amplitude voltage and frequency from a rigid entry. An intermediate DC-link circuit is not necessary as in the classical inverter case. The principle of such a converter is based on a matrix topology connecting each input phase with each output phase by a two-way power switch, allowing the flow of power in both directions and therefore operating in all four quadrants. By using some well suited laws, the matrix converter can reproduce all the existing electronic power conversions (AC / AC, AC / DC, DC / DC and DC / AC).

The basic diagram of a matrix converter can be represented by Figure 1.



Basic circuit of a matrix converter

The symbol  $S_{ij}$  represents the ideal bidirectional switches, where i represents the index of the output voltage and j represents the index of the input voltage.

Let  $[V_i]$  be the vector of the input voltages given as:

$$[V_i] = V_{im} \begin{bmatrix} \cos(\omega_i t) \\ \cos(\omega_i t - 2\pi/3) \\ \cos(\omega_i t - 4\pi/3) \end{bmatrix}$$
(1)

and  $[V_o]$  the vector of desired output voltages.

$$[V_o] = V_{om} \begin{bmatrix} \cos(\omega_o t) \\ \cos(\omega_o t - 2\pi/3) \\ \cos(\omega_o t - 4\pi/3) \end{bmatrix}$$
(2)

The problem now consists in finding a matrix [M] known as the modulation matrix, such that

$$[V_o] = [M]. [V_i] \tag{3}$$

The development of the equation (3) gives:

$$\begin{bmatrix} V_{o1} \\ V_{o2} \\ V_{o3} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{bmatrix} V_{i1} \\ V_{i2} \\ V_{i3} \end{bmatrix}$$
(4)

where  $m_{ii}$  are the modulation coefficients.

During commutation, the bidirectional switches must function according to the following rules:

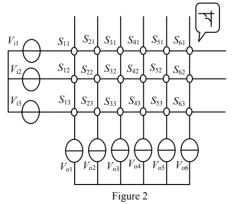
- At every instant t, only one switch  $S_{ij}$  (i = 1,2,3) works in order to avoid short-circuit between the phase.
- At every instant t, at least two switches  $S_{ij}$  (j = 1,2,3) works to ensure a closed loop load current.
- The switching frequency  $f_s = \omega_s/2\pi$  must have a value twenty times higher than the maximum of  $f_i$ ,  $f_a$  ( $f_s >> 20.max(f_i f_a)$ ).
- During the period  $T_s$  known as the sequential period, which is equal to  $1/f_s$ . The sum of the time of conduction being used to synthesize the same output phase must be equal to  $T_s$ .

Now a time  $t_{ii}$ , called the modulation time, can be defined as:

$$t_{ij} = m_{ij} \cdot T_{S} \tag{5}$$

# 3 Theory of a Six-Phase Matrix Converter

The basic scheme of a six-phase matrix converter can be illustrated by Figure 2.



Basic scheme of a six-phase matrix converter

## 3.1 First Method

This method is derived from that proposed by Venturini [1], [2], [4] and has been used to solve the equation (3), from which a six-phase voltage system at the matrix converter output can be similarly obtained and represented by equation (6).

$$\begin{bmatrix} V_{o1} \\ V_{o2} \\ V_{o3} \\ V_{o4} \\ V_{o5} \\ V_{o6} \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \\ m_{41} & m_{42} & m_{44} \\ m_{51} & m_{52} & m_{53} \\ m_{61} & m_{62} & m_{63} \end{bmatrix} \begin{bmatrix} V_{i1} \\ V_{i2} \\ V_{i3} \end{bmatrix}$$

$$(6)$$

The calculation of the  $m_{ij}$  coefficients in equation (6), will be the same as that for the  $m_{ij}$  coefficients for the case of a three phase[1], but expressed as follows:

$$[M] = \begin{bmatrix} m(3,3) & m(4,4) & m(5,5) \\ m(2,4) & m(3,5) & m(4,0) \\ m(1,5) & m(2,0) & m(3,1) \\ m(0,0) & m(1,1) & m(2,2) \\ m(5,1) & m(0,2) & m(1,3) \\ m(4,2) & m(5,3) & m(0,4) \end{bmatrix}$$

$$(7)$$

where the matrix coefficients are given in a general form as:

$$m(x_1, x_2) = \frac{1}{3} \alpha_1 [1 + 2QZ_1^{-1}(x_1)] + \alpha_2 [1 + 2QZ_1^{-1}(x_2)]$$
(8)

where 
$$Q = \frac{V_{om}}{V_{im}}$$
 and  $Z_{\alpha}^{\beta}(y)(t) = \cos[(\alpha\omega_i + \beta\omega_o)t + y\frac{\pi}{3}]$ 

and 
$$\alpha_1 = \frac{1}{2} \left( 1 + \frac{\tan(\theta_i)}{\tan(\theta_0)} \right), \quad \alpha_2 = 1 - \alpha_1$$

with  $\theta_i$  the phase of input voltages and  $\theta_o$  the phase of output voltages.

As an example, the average value of the sixth output phase voltage during the k<sup>th</sup> sequence can be expressed as:

$$V_{o6} = \begin{cases} V_{i1} & 0 \le t - (k - 1)T_s \le m_{61}^k T_S \\ V_{i2} & m_{61}^k T_S \le t - (k - 1)T_s \le (m_{61}^k + m_{62}^k) T_S \\ V_{i3} & (m_{61}^k + m_{62}^k) T_S \le t - (k - 1)T_s \le (m_{61}^k + m_{62}^k + m_{63}^k) T_S \end{cases}$$

$$(9)$$

## 3.2 Second Method

The second method is derived from the scalar strategy control [3], [4]. A straightforward approach to generate the active and zero states of matrix switches in fig.1 consists of using the instantaneous voltage ratio of specific input phase voltages. Let us define the following phase voltages present at the input port as:

$$\begin{cases} V_A(t) = V_{im} \cos(\omega_i t) \\ V_B(t) = V_{im} \cos(\omega_i t - 2\frac{\pi}{3}) \\ V_C(t) = V_{im} \cos(\omega_i t - 4\frac{\pi}{3}) \end{cases}$$

$$(10)$$

At the output port of the converter, the value of any instantaneous output phase voltage may be expressed by equation (11) as follows:

$$V_o(t) = \frac{1}{T_S} (t_K V_K + t_L V_L + t_M V_M)$$
 (11)

where  $t_K + t_L + t_M = T_S$ 

K-L-M are variable subscripts, which may be assigned the variable A, B or C according to the following rules:

<u>Rule 1</u>: At any instant, the input phase voltage which has a polarity different from both others is assigned to "M".

<u>Rule 2</u>: The two input phase voltages which share the same polarity, are assigned to K and L, the smallest one of the two, (in absolute value), being "K". Then  $t_K$  and  $t_L$  are chosen such that:

$$\frac{t_K}{t_L} = \frac{V_K}{V_L} = \rho_{KL} \tag{12}$$

for the interval where

$$0 \le \frac{V_K}{V_I} \le 1 \tag{13}$$

The expressions given by equation (11) and equation (13) are similar to that ones originally proposed by [1]. Equation (12) defines the active time ratio between two out of the three switches, in one commutating leg of the output port (see Fig. 1). This time ratio  $(t_K/t_L)$  is proportional to the instantaneous voltage ratio  $(V_K/V_L)$  of their associated input phases. The ratio must be established with the smaller instantaneous voltage divided by the larger one, as stated in equation (13).

The converter switching pattern depends only on the scalar comparison of input phase voltages and the instantaneous value  $(V_o)$  of the desired output voltage. In

the following, the proper procedure to obtain the respective values of  $t_K$ ,  $t_L$  and  $t_M$  during one period  $T_S$  of the sequence (or the carrier) frequency  $f_S$  is illustrated. For a specific interval where  $0 \le t_K/t_L \le 1$ .

The active times of the three switches associated with the desired output voltage  $V_a$  become:

$$t_K = \frac{T_s(V_o - V_M)}{(\rho_{KL}V_K + V_L - (1 + \rho_{KL})V_M)}$$
(14)

$$t_M = T_s(1 + \rho_{KL}).V_L \tag{15}$$

Using again the current value of  $\rho_{KL}$ , equation (14) can be further developed such as:

$$t_L = \frac{T_s(V_o - V_K)V_L}{(V_K^2 + V_L^2 + V_M^2) + (V_K + V_L + V_M)V_M}$$
(16)

In a balanced three phase system, the summation of the three instantaneous phase voltage is zero. So the following relationships can be obtained:

$$\frac{t_L}{T_s} = \frac{T_s(V_o - V_K)V_L}{(V_K^2 + V_L^2 + V_M^2)} = \frac{T_s(V_o - V_K)V_L}{1.5V_{im}^2}$$
(17)

$$\frac{t_K}{T_s} = \frac{T_s(V_o - V_K)V_K}{1.5V_{im}^2} \tag{18}$$

$$\frac{t_M}{T_s} = 1 - \frac{(t_K - t_L)}{1.5V_{im}^2} \tag{19}$$

The duty cycle of commutators K and L is proportional to the instantaneous value of the corresponding input phase voltage  $V_K$  and  $V_L$  multiplied by the voltage difference between the desired output voltage  $V_o$  and the input phase voltage  $V_M$ . It should be noted at this point that the output voltage  $V_o$ , (i.e  $V_A$ ,  $V_B$ ,  $V_C$ ), can be any kind of waveform, including DC values.

Solving equations (17, 18 and 19) for a given voltage ratio,  $V_{om}/V_{im} = Q \le 0.5$ , will yield a positive value for the times  $t_K$ ,  $t_L$  and  $t_M$  as in the case of the Venturini control algorithm.

The desired output voltages with which the coefficients are calculated are:

$$\begin{cases} V_{o1} = V_{om} \cos(\omega_{o}t) \\ V_{o2} = V_{om} \cos(\omega_{o}t - \frac{\pi}{3}) \\ V_{o3} = V_{om} \cos(\omega_{o}t - 2\frac{\pi}{3}) \\ V_{o4} = V_{om} \cos(\omega_{o}t - 3\frac{\pi}{3}) \\ V_{o5} = V_{om} \cos(\omega_{o}t - 4\frac{\pi}{3}) \\ V_{o6} = V_{om} \cos(\omega_{o}t - 5\frac{\pi}{3}) \end{cases}$$
(20)

The development of equation (3) for the case of a six-phase matrix converter gives:

$$\begin{bmatrix} V_{o1} \\ V_{o2} \\ V_{o3} \\ V_{o4} \\ V_{o5} \\ V_{o6} \end{bmatrix} = \begin{bmatrix} m_{1K} & m_{1L} & m_{1M} \\ m_{2K} & m_{2L} & m_{2M} \\ m_{3K} & m_{3L} & m_{3M} \\ m_{4K} & m_{4L} & m_{4M} \\ m_{5K} & m_{5L} & m_{5M} \\ m_{6K} & m_{6L} & m_{6M} \end{bmatrix} \begin{bmatrix} V_K \\ V_L \\ V_M \end{bmatrix}$$

$$(21)$$

The average value of the voltage of the sixth output phase voltage during the  $K^{th}$  sequence is given as:

$$V_{o6} = \begin{cases} V_K & 0 \le t - (k - 1)T_s \le m_{6K}^k T_S \\ V_L & m_{6K}^k T_S \le t - (k - 1)T_s \le (m_{6K}^k + m_{6L}^k)T_S \\ V_M & (m_{6K}^k + m_{6L}^k)T_S \le t - (k - 1)T_s \le (m_{6K}^k + m_{6L}^k + m_{6M}^k)T_S \end{cases}$$
(22)

## 4 Double Star Induction Machine Modelling

The mathematical model of the machine is written as a set of state equations, both for the electrical and mechanical parts [6], [8], [9]:

$$[V_{abc,s_1}] = [R_{s_1}][I_{abc,s_1}] + \frac{d[\Phi_{abc,s_1}]}{dt}$$
 (23)

$$[V_{abc,s_2}] = [R_{s_2}][I_{abc,s_2}] + \frac{d[\Phi_{abc,s_2}]}{dt}$$
(24)

$$[V_{abc,r}] = [R_r][I_{abc,r}] + \frac{d[\Phi_{abc,r}]}{dt}$$
 (25)

$$J\frac{d\Omega}{dt} = T_{em} - T_r - k_f \Omega \tag{26}$$

where J is the moment of inertia of the revolving parts,  $K_f$  is the coefficient of viscous friction, arising from the bearings and the air flowing over the motor, and  $T_{em}$  is the load torque.

The electrical state variables in the "dq" system are the flux represented by vector  $[\Phi]$ , while the input variable in the "dq" system are expressed by vector [V].

$$\frac{d\Phi}{dt} = [A] \Phi + [B] V$$
 (27)

$$\left[\boldsymbol{\Phi}\right] = \begin{bmatrix} \varphi_{ds_1} \\ \varphi_{ds_2} \\ \varphi_{qs_1} \\ \varphi_{dr} \\ \varphi_{qr} \end{bmatrix} ; \qquad \left[\boldsymbol{V}\right] = \begin{bmatrix} \boldsymbol{V}_{ds_1} \\ \boldsymbol{V}_{ds_2} \\ \boldsymbol{V}_{qs_1} \\ \boldsymbol{V}_{qs_2} \end{bmatrix}$$
 (28)

The equation of the electromagnetic torque is given

$$T_{em} = p \frac{L_m}{(L_m + L_r)(\varphi_{dr}(i_{qs_1} + i_{qs_2}) + \varphi_{qr}(i_{ds_1} + i_{ds_2}))}$$
(29)

The equations of flux are:

$$\Phi_{md} = L_s \left( \frac{\Phi_{ds_1}}{L_{s_1}} + \frac{\Phi_{ds_2}}{L_{s_2}} + \frac{\Phi_{dr}}{L_r} \right)$$
 (30)

$$\Phi_{mq} = L_s \left( \frac{\Phi_{qs_1}}{L_{s_1}} + \frac{\Phi_{qs_2}}{L_{s_2}} + \frac{\Phi_{qr}}{L_r} \right)$$
 (31)

Given that the "dq" axes are fixed in the synchronous rotating coordinates system, we have:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix}$$

$$(32)$$

$$[B] = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$
 (33)

Where:

$$\begin{aligned} a_{11} &= a_{32} = \frac{R_{s1}L_a}{L_{s1}^2} - \frac{R_{s1}}{L_{s1}} \; ; \quad a_{12} = a_{34} = \frac{R_{s1}L_a}{L_{s1}L_{s2}} \; ; \quad a_{56} = -a_{65} = \omega_{gl} \\ a_{11} &= a_{32} = -a_{31} = -a_{42} = \omega_{s} \; ; \quad a_{14} = a_{16} = a_{23} = a_{26} = a_{32} = a_{35} = 0 \; ; \\ a_{41} &= a_{45} = a_{52} = a_{54} = a_{61} = a_{62} = 0 \; ; \quad a_{15} = a_{36} = \frac{R_{s1}L_a}{L_{s1}L_r} \; ; \\ a_{21} &= a_{42} = \frac{R_{s2}L_a}{L_{s1}L_{s2}} \; ; \quad a_{22} = a_{44} = \frac{R_{s2}L_a}{L_{s2}^2} - \frac{R_{s1}}{L_{s1}} \qquad a_{25} = a_{46} = \frac{R_{r2}L_a}{L_{r}L_{s2}} \; ; \\ a_{51} &= a_{62} = \frac{R_{r}L_a}{L_{s1}L_r} \; ; \quad a_{52} = a_{64} = \frac{R_{r}L_a}{L_{s2}L_r} \qquad a_{55} = a_{66} = \frac{R_{r}L_a}{L_r^2} - \frac{R_{r}}{L_r} \; ; \end{aligned}$$

## 5 Simulations and Results Discussion

The simulation was carried out by keeping the supply voltage of the induction motor (i.e the output of the matrix converter) fixed and varying only the frequency  $f_o$  in order to be able to compare the motor performance for both strategies presented before. The six-phase matrix converter presented is being simulated for a desired output frequency of  $f_o$ =50 Hz, with a switching frequency of  $f_s$  = 5 KHz. Both converters are first feeding a passive R-L load (Rs=20  $\Omega$  and Lr=0.04 H) and then a 50 HP, 460 V double star induction motor driving a 100 N.m resistive torque.

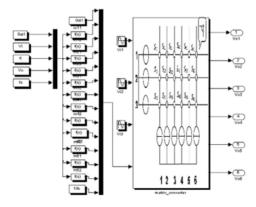


Figure 3
The matrix converter simulink®/Matlab diagram (Venturini method)

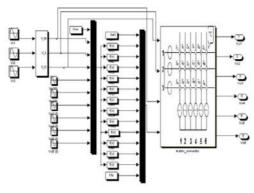


Figure 4

The matrix converter simulink®/Matlab diagram (Scalar strategy control)

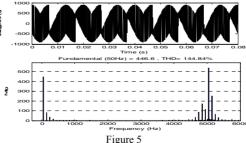
## 5.1 Results Discussion

## **5.1.1** Results of the First Method ( $f_0 = 50 \text{ Hz}$ )

Figure 5 shows the matrix converter output voltage and its spectral analysis. The fundamental harmonic lies at the 50 Hz desired frequency and the higher order harmonics are in the neighbourhood of the 5 KHz switching frequency. The current form in a one phase of the R - L load (Figure 6), shows clearly that it approximates a sine wave shape at the desired frequency (50 Hz). It can be noted that the current shape in the case of a six-phase load illustrated in Figure 7 shows that the system is an unbalanced one.

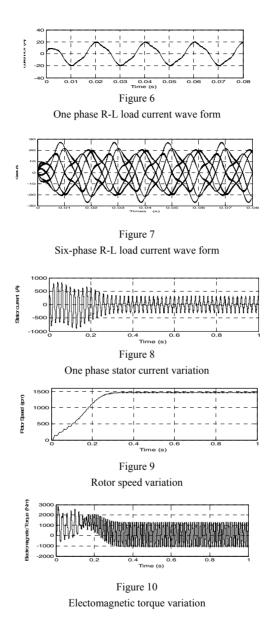
Figure 8 represents a one phase stator current wave form. One can notice the presence of high oscillatory current, resulting from the large electromagnetic torque fluctuations as shows in Figure 10.

The variation of the rotor speed given in Figure 9, shows that its magnitude is not only slightly less than the rotating field speed, but also its shape is an oscillatory one.



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Voltage output and its FFT spectrum



## 5.1.2 Results of the Second Method ( $f_o = 50 \text{ Hz}$ )

Figure 11 represents the matrix converter voltage output and its spectral analysis. It shows a fundamental harmonic lying at the desired frequency (50 Hz) and higher order around the 5 KHz switching frequency.

Figure 12 represents the current form in a phase of the R - L load. This wave form is approaching the sine wave and is at the desired frequency of 50 Hz.

Figure 13 represents the current form in six-phase R-L load. It is visible that it represents a balanced six-phase system.

Figure 14 represents the one phase stator current wave form. In this case, the high oscillations have disappeared leading to a normal operating double star induction machine.

Figure 16 represents the electromagnetic torque wave form. This torque contains very reduced oscillations as that obtained from a balanced voltage system supplying the double star induction machine.

Figure 15 represents the rotor speed variation. This speed is slightly less than the rotating field speed.

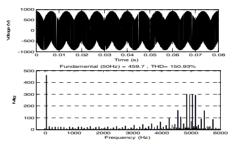


Figure 11 Voltage output and its FFT spectrum

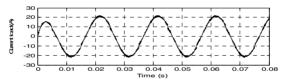


Figure 12
One phase R-L load current wave form

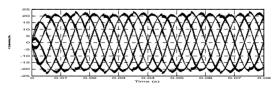
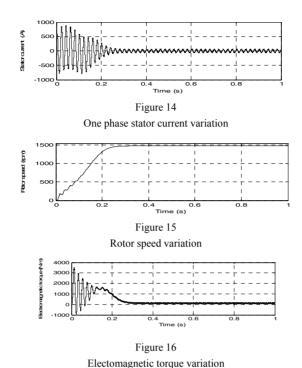


Figure 13
Six-phase R-L load current wave form



#### **Conclusions**

In the present paper, a comparative performance study between two different control strategies applied to a six-phase matrix converter is presented. The first strategy is derived from the Venturini method, and the second one is practically an extension of the scalar strategy control.

This comparative investigation is carried out using first an R-L load and then a double star induction machine. The obtained simulation results have been very useful and helpful in illustrating the merits of each method, though both strategies produce a six-phase voltage system with a load current at the 50 Hz desired frequency.

However, the Venturini method is simple and easy to implement. Unfortunately, the nature of the obtained voltage system is an unbalanced one and needs to be corrected and worked on to produce a balanced one. because an unbalanced voltage system has a direct impact on the motor's performance, giving important electromagnetic torque oscillations and high oscillatory currents in the steady state mode. On the other hand, the scalar strategy is more complex and produces, for the same magnitude, a balanced six-phase voltage system compared to the Venturini method.

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