

# Effect of the Measured Pulses Count on the Methodical Error of the Air Radio Altimeter

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*Abstract: Radio altimeters are based on the principle of radio location of the earth's surface using a frequency-modulated standing wave. The relatively simple method of measurement consists in the evaluation of the number of pulses generated as resulting from the mixing of the transmitted and received signals. Such a change in the number of modulated pulses within a certain altitude interval, however, is not so simple and is a determinant issue in defining the precision of the radio altimeter. Being knowledgeable of this law in a wider context enables us to enter into discussion on the possibilities of further increasing the precision of measuring low altitudes. The article deals with the law underlying the change in the number of radio altimeter pulses with the changing altitude measured.*

*Keywords: radio altimeter, precision of measuring altitude, frequency modulation, the law of pulse number changes, critical altitude, methodical error*

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## 1 Introduction

The frequency modulated radio altimeter is understood as a radio altimeter that makes uses of a harmonic, frequency-modulated high-frequency signal to measure aircraft altitude over terrain. The altitude measured usually falls between 0 and 1500 m depending on the type of aircraft and the radio altimeter used. Radio altimeters operate on the principle of generating a high-frequency, frequency modulated signal which is then radiated via a transmitter antenna towards the

surface of the earth. The reflected signal subsequently received by a receiver antenna is then fed into the balanced mixer, into which the so-called direct signal from the transmitter has already been sent simultaneously with the transmission. Based on the fact that the reflected signal is received with a time delay, the direct signal frequency is quite different, in every instant, from that of the received one.

On the output of the balanced mixer there arises, following the separation of the undesired components, a differential (beat) signal at a frequency of  $F_r$ . The signal from the balanced mixer is then amplified and the evaluator circuit determines the magnitude of this differential, which corresponds to the aircraft's true altitude. Currently, radio altimeters employ various means of frequency measurement employed as evaluator circuits which are manufactured as spectrum analyzers or analogue and digital devices that ensure direct or indirect measuring of the differential frequency. The principle of these evaluator circuits is usually based on counting the pulses shaped from the differential signal throughout the entire period of modulation. The magnitude of the differential frequency, depending on the time delay of the reflected signal, is expressed as:

$$F_r = \frac{8 \Delta f F_M}{c} H \quad (1)$$

where:  $F_r$  – differential frequency,

$\Delta f$  – frequency swing,

$F_M$  – modulation frequency,

$H$  – altitude.

The equation (1) implies that the measured altitude is directly proportional to the differential frequency  $F_r$ . Actually, it is not that simple, as both the transmitted and the received signal is frequency modulated, and the modulation signal is the periodic function of time. As is given in [1] and [2], the differential signal spectrum is discrete and consists of only those frequency components that are multiples of the  $F_M$  modulation frequency. Consequently, it follows that measuring the differential frequency enables the recording of only those changes in altitude  $\Delta H$  which ensure generation (termination) of the spectral line within the differential signal spectrum. According to [2], the value of  $\Delta H$  is given by expression:

$$\Delta H = \frac{c}{8 \Delta f} \quad (2)$$

This means that the data of the altitude indicator at continuous changes of the altitude are not changing continuously but in jumps.

## 2 Changes in the Pulses Count with the Changing Altitude of Flight

Pulses are generated or terminated at the moment when the envelope of the resulting signal passes through zero. This conclusion can be mathematically illustrated as:

$$U(t_1) = U_v + U_p \cos[\varphi_0 + \varphi_M \cos \Omega_M t_1] = U_v \quad (3)$$

This holds if:

$$\cos(\varphi_0 + \varphi_M \cos \Omega_M t_1) = 0 \quad (4)$$

This condition is met under the condition that:

$$\varphi(t) = \varphi_0 + \varphi_M \cos \Omega_M t_1 = (2k - 1) \frac{\pi}{2} \quad (5)$$

where:  $k$  – arbitrary whole number,

$\varphi(t)$  – phase of a LF differential signal.

From the theoretical description above, one can assume that if the measured altitude changes by a value lower than the  $\Delta H$ , the data on the radio altimeter do not change at all, as the number of steady pulses for the modulation period remains unchanged. Reality, however, is a bit more complex. The conflict exists in that fact that our way of thinking is about the generation and existence of a single steady pulse at changes in the altitude by a value of  $\Delta H$ . At changes in altitude within a range smaller than  $\Delta H$ , one pulse is generated and terminated alternatively.

This alternating generation and termination of a pulse is caused by the unequal phase change of values  $\varphi(t)_{\max} - \varphi(t)_{\min}$ , representing the sum and the difference of the initial phase  $\varphi_0$  and the variable phase  $\varphi_M$ , at changes of the altitude within the interval of  $\Delta H$ . The phase of the  $\varphi(t)$  signal is periodically changing within the interval from the value of:

$$\varphi(t)_{\max} = \varphi_0 + \varphi_M = \frac{4\pi}{\lambda_0} H + \frac{4\pi\xi}{\lambda_0} = \frac{4\pi}{\lambda_0} H(1 + \xi) \quad (6)$$

to the value of:

$$\varphi(t)_{\min} = \varphi_0 - \varphi_M = \frac{4\pi}{\lambda_0} H - \frac{4\pi\xi}{\lambda_0} = \frac{4\pi}{\lambda_0} H(1 - \xi) \quad (7)$$

Around the mean value of:

$$\varphi(t) = \frac{4\pi}{\lambda_0} H \quad (8)$$

With the modulation frequency of  $\Omega_M$ .

As pulses are generated or terminated alternatively, when the modulation envelope is passing through zero, and the condition of the pulse generation

$$\frac{4\pi}{\lambda_0} H_V (1 + \xi) = (2k - 1) \frac{\pi}{2} \quad (9)$$

Similarly, the condition of pulse termination is given by the expression of:

$$\frac{4\pi}{\lambda_0} H_Z (1 - \xi) = (2k - 1) \frac{\pi}{2} \quad (10)$$

From the above one may conclude that pulse generating altitude  $H_V$  is given by the expression:

$$H_V = \frac{\lambda_0}{8} \frac{2k - 1}{1 + \xi} \quad (11)$$

The pulse termination altitude  $H_Z$  is given by the expression:

$$H_Z = \frac{\lambda_0}{8} \frac{2k - 1}{1 - \xi} \quad (12)$$

Analyzing expressions (12) and (13) one can determine the scope of altitude under which the individual signal may exist. Of the given expressions one can determine that the first pulse is generated at an altitude close to the value of  $\lambda/4$  and is terminated on surpassing this altitude, whereas the altitude scope of the existence of this pulse is rather small. With a further increase in altitude, the first pulse is generated and terminated alternatively, but the altitude interval of pulse duration  $L$  is increasing. The interval of duration of individual pulses  $L$  is calculated as the difference in the altitudes at which the pulses are generated and terminated:

$$L = H_Z - H_V = \frac{\lambda_0}{8} \frac{2k - 1}{1 - \xi} - \frac{\lambda_0}{8} \frac{2k - 1}{1 + \xi} = \frac{\lambda_0}{4} (2k - 1) \frac{\xi}{1 + \xi^2} \quad (13)$$

The altitude interval of generating the individual pulses  $V$ , is calculated as the difference between the altitudes, e.g. between the generation of the first and the second pulse:

$$V = H_{V2} - H_{V1} = \frac{\lambda_0}{8} \frac{2(k+1) - 1}{1 + \xi} - \frac{\lambda_0}{8} \frac{2k - 1}{1 + \xi} = \frac{\lambda_0}{4(1 + \xi)} \quad (14)$$

Where:  $H_{V1}$  – altitude of the first pulse generation,

$H_{V2}$  – altitude of the second pulse generation.

The expression (15) makes it apparent that the altitude interval of generating the first and further pulses  $V$  is of a constant value, which is only a bit smaller than the value of the  $\lambda_0/4$ .

The altitude interval of terminating the individual pulses  $Z$  is calculated as the difference in the altitudes, e.g. between the termination of the first and the second pulse.

Expression (16) makes it clear that the altitude interval when the first or the further pulses are terminated  $Z$  is also of constant value, and is only a bit larger than the value of  $\lambda/4$ .

If based on expressions (15) and (16) we develop a graphic function expressing dependence of the number of pulses on altitude, it becomes evident that the values of the pulse generating altitude intervals are lower than those of the altitudes terminating pulses. Consequently, we experience growth throughout the duration of the individual pulses with the altitude  $L$ , until the dependence of the pulses on the minimal altitude  $\Delta H$  start to overlap.

For practical purposes of altitude measurement, it is important that at changes of altitude there exists a steady pulse for the modulation period. Therefore, the change in the altitude must remain within the scope of intervals equalling roughly to  $\lambda_0/4$ , within the pulses generated and terminated alternatively, until the altitude is achieved at which the pulse will exist for good. That altitude corresponds to the critical altitude.

### 3 The Value of the Critical Altitude $\Delta H$

In expressions (14) and (15) it is apparent that the maximum value of the initial phase  $\varphi$  and that of the variable phase  $\varphi_M$  is dependent on the altitude  $H$ . The instant value of the  $\varphi_M$  phase is, however, changing with time in the rhythm of the modulation frequency  $\Omega_M$ , even at constant altitude as expressed in (12). The number of pulses  $N$  generated in this way during a single modulation period, at constant altitude, will be given by the number of the modulation envelope of the  $\varphi_M$  phase passing through zero. This envelope passes through zero at every  $\pi$  radians.

$$N = \frac{\varphi_M}{\pi} = \frac{\frac{8\pi\xi}{\lambda_0} H}{\pi} = \frac{8\pi}{\lambda_0} H \quad (15)$$

Expression (16) leads us to conclude that the number of pulses during a single modulation period is directly proportional to the altitude at which the aircraft is flying.

Further, we analyze by what value is it necessary to change the altitude of flight to increase the number of pulses during a modulation period by a single steady pulse, which, in the end, increases the value on the altitude indicator.

The concrete number of pulses  $N_k$  for a concrete altitude  $H_k$  is given by the expression:

$$N_k = \frac{8\xi}{\lambda_0} H_k \quad (16)$$

Suppose that this number of pulses increases by a single steady pulse if the altitude  $H_k$  increases by a value of  $\Delta H$ .

Then we are able to write it down as:

$$N_{k+1} = \frac{8\xi(H_k + \Delta H)}{\lambda_0} \quad (17)$$

The altitude interval  $\Delta H$  corresponds to the change in the number of pulses by a single pulse, during a single modulation period  $T_M$ . This value of  $\Delta H$  is termed as the critical altitude, representing the minimum change in the altitude recorded by the altimeter. It then follows that if the aircraft changes its value by a magnitude smaller than the value of the  $\Delta H$ , the radio altimeter data remain unchanged, as the number of pulses for a modulation period have not changed either.

This means that the selected principle of the radio altimeter with frequency modulation only allows us to measure altitude on the basis of discrete changes (jumps). Consequently, the altitude indicator reading continuous changes in the measured altitude are actually not changing continuously, but in rather jumps, and the measured altitude can be read only at a precision of  $\pm \Delta H$ .

This critical altitude is to be understood as the methodical error of the radio altimeter operating with frequency modulation. Decreasing the critical altitude, i.e. the methodical error of the altimeter with frequency modulator, is possible only through increasing the value of the frequency swing  $\Delta f$ , as implied by expression (16).

Increasing the frequency swing  $\Delta f$  is only possible by simultaneously increasing the mean value of the carrier frequency  $f_0$ . Thereby, at each value of the carrier frequency, it is possible to realize a limited frequency swing. These boundary frequency limits allows generation of parasite amplitude modulation, negatively affecting the operation of the radio altimeter.

Within the entire scope of the measured altitude  $H$  the pulses are generated and terminated alternatively. As the altitude scope of the duration of pulses  $L$  increases with the altitude, at certain altitudes mutual overlapping occurs. The first overlapping of two pulses takes place at an altitude that immediately surpasses the value of  $\Delta H$ , which is termed as a boundary altitude. The second overlapping of three pulses occurs at an altitude that surpasses the value of  $2\Delta H$ . Each of the subsequent pulse overlappings occurs at altitudes changing by the value of  $\Delta H$ . Overlapping of a certain amount of generated and terminated pulses – unsteady pulses " $N_{\Delta H}$ " – generate steady pulses. The number of steady pulses  $N$

corresponds to the number of overlappings corresponding to the number of overlappings of generated and terminated pulses. It is the number of steady pulses  $N$  that is of importance for altitude measurement.

At altitude scope between 0 as  $\Delta H$ , an  $N_{\Delta H}$  number of unsteady pulses is generated and an  $N_{\Delta H} - 1$  number of pulses are terminated instead. The fact that the number of terminated pulses is lower than that of the generated ones enables us to ensure overlapping pulses between the first and the second altitude intervals known as  $\Delta H$ . The number of generated and terminated pulses  $N_{\Delta H}$  within the altitude scope of  $\Delta H$  can be computed from the equality of the altitude scopes in both cases. The altitude scope of the generating of the individual pulses  $V\varphi$  is given by expression (17). The altitude scope  $\Delta H$  achieved by whole number multiples of the numbers of unstable pulses generated  $\Delta H_V$  is:

$$\Delta H_V = N_{\Delta H} \cdot V \quad (18)$$

The altitude scope of the terminating of the individual pulses  $Z$  is given by (19). The altitude scope of  $\Delta H$  derived by whole number multiples of unstable pulses  $\Delta H_Z$  will be:

$$\Delta H_Z = (N_{\Delta H} - 1) Z \quad (19)$$

Based on the equality of the altitudes, we can write:

$$N_{\Delta H} V = (N_{\Delta H} - 1) Z \quad (20)$$

Introducing expressions (18) and (19) into (20) and making some arrangements, we can determine the number of unstable pulses  $N_{\Delta H}$  within the scope of the boundary altitude  $\Delta H$ .

Substituting the concrete values of the relative frequency swing  $\xi$  of some of the types of RA we can determine the number of unstable pulses  $N_{\Delta H}$  within the altitude scope  $\Delta H$ . At the same time, the last line of the table comprises the serial number of the overlapping pulse  $N$ , where one steady pulse is generated within the next altitude scope  $\Delta H$ .

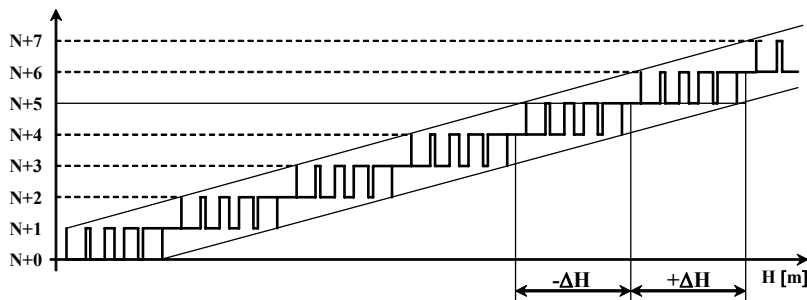


Figure 1

Marking the methodical error of the radio altimeter

## Conclusions

Analysis of the change in the number of pulses at measured aircraft altitude changes has led to the development of a mathematical procedure revealing how this law influences the precision of altitude. The law of the generation and termination of altitude pulses has led to the introduction of the notion of “critical altitude“, which represents the methodical error of the radio altimeter. On the basis of the analysis presented above, it is possible to quantify the number of unsteady pulses within the scope of critical altitude for all types of radio altimeters currently in use. The knowledge of the laws has led to the drawing up of new methods of evaluating the measured altitude of aircraft and then filing several patents to increase the precision of measuring low altitudes.

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