Stability Analysis Method for Fuzzy Control Systems Dedicated Controlling Nonlinear Processes

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Abstract: This paper presents a new stability analysis method for nonlinear processes with Takagi-Sugeno (T-S) fuzzy logic controllers (FLCs). The design of the FLCs is based on heuristic fuzzy rules. The stability analysis of these fuzzy control systems is performed using LaSalle’s invariant set principle with non-quadratic Lyapunov candidate function. This paper proves that if the derivative of Lyapunov function is negative semi-definite in the active region of each fuzzy rule, then the overall system will be asymptotically stable in the sense of Lyapunov (ISL). The stability theorem suggested in the paper ensures sufficient stability conditions for fuzzy control systems controlling a class of nonlinear processes. The end of the paper contains an illustrative example that describes an application of the stability analysis method.

Keywords: Fuzzy logic controller, Lyapunov stability, nonlinear system, LaSalle’s invariant set principle

1 Introduction

The investigations of the stability of Takagi-Sugeno (T-S) fuzzy control systems begin before 1990 with increased frequency afterwards [1-5]. In principle, for the stability analysis of a fuzzy controller any method can be used witch is suitable for the analysis of nonlinear dynamic systems. Today, there exist preoccupations reported in the literature [6, 7] on the stability analysis and design of T-S fuzzy control systems. The majority of these papers is based on linear matrix inequality (LMI) framework [8] and the stability conditions of fuzzy control systems employs quadratic Lyapunov functions. In this case, there exist two shortcomings:

- first, the linearization can result in uncertainties and inaccuracies of the fuzzy models involved,
- second, using the quadratic Lyapunov functions the stability conditions become usually very restrictive.

This paper presents a new stability analysis method for nonlinear processes with T-S fuzzy logic controllers (FLCs) without process linearization and without using the quadratic Lyapunov functions in the derivation and proof of the stability conditions. The rest of the paper is organized as follows. Section 2 recalls the Takagi-Sugeno fuzzy control systems controlling nonlinear processes. Section 3 gives a stability theorem for nonlinear systems with T-S FLCs and an algorithm for the design of a stable fuzzy control system. An illustrative example presented in Section 4 shows that good control system performance can be obtained by applying the suggested algorithm. Section 5 concludes the paper.

2 One Class of Fuzzy Logic Control Systems

A fuzzy logic control system consists of a process and a fuzzy logic controller as shown in Figure 1. Let $X \subset R^n$ be a universe of discourse. The controlled process is accepted to be characterized by the class of single-input *n*-th order nonlinear system modelled by the state-space equations in (1):

$$\begin{aligned} &\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{b}(\mathbf{x})u, \\ &\mathbf{x}(t_0) = \mathbf{x}_0, \end{aligned} \tag{1}$$

where: $\mathbf{x} \in X$, $\mathbf{x} = [x_1 \quad x_2 \quad \dots \quad x_n]^T$ is the state vector, $n \in IN^*$, $\dot{\mathbf{x}} = [\dot{x}_1 \quad \dot{x}_2 \quad \dots \quad \dot{x}_n]^T$ is the derivative of $\mathbf{x}$ with respect to the time variable *t*, $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}) \quad f_2(\mathbf{x}) \quad \dots \quad f_n(\mathbf{x})]^T$ and $\mathbf{b}(\mathbf{x}) = [b_1(\mathbf{x}) \quad b_2(\mathbf{x}) \quad \dots \quad b_n(\mathbf{x})]^T$ are functions describing the dynamics of the process, *u* is the control signal fed to the process, obtained by the weighted-sum defuzzification method for T-S FLCs.

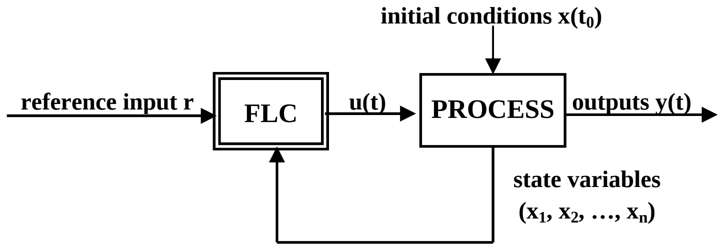

Figure 1
Fuzzy logic control system structure

The FLC consists of r fuzzy rules. The *i*-th IF–THEN rule in the fuzzy rule base of the FLC, referred to as Takagi-Sugeno fuzzy rule, has the following expression:

Rule *i*: IF x_i is $\tilde{X}_{i,1}$ AND … AND x_n is $\tilde{X}_{i,n}$ THEN $u = u_i(x)$, $i = \overline{1,r}$,
$r \in N, r \geq 2$, (2)

where X_{i1}, X_{i2} … X_{in} are fuzzy sets that describe the linguistics terms (LTs) of input variables, $u = u_i(x)$ is the control output of rule *i*, and the function AND is a t-norm. u_i can be a single value or a function of states vector, $x(t)$. Each fuzzy rule generates a firing degree $\alpha_i \in [0,1], i = 1,2,..,r$, according to (3):

$$\alpha_i(\mathbf{x}) = \min(\mu_{\tilde{X}_{i,1}}(x_1), \mu_{\tilde{X}_{i,2}}(x_2), \ldots, \mu_{\tilde{X}_{i,n}}(x_n)). \quad (3)$$

It is assumed that for any $\mathbf{x}$ belonging to the input universe of discourse, X, there exists at least one α_i among all rules that is not equal to zero.

The control signal *u*, which must be applied to the process, is a function of α_i and u_i. By applying the weighted-sum defuzzification method, the output of the FLC is given by:

$$u = \frac{\sum_{i=1}^{r} \alpha_i u_i}{\sum_{i=1}^{r} \alpha_i}. \quad (4)$$

Definition 1: For any input $x_0 \in X$, if the firing degree $\alpha_i(x_0)$ corresponding to fuzzy rule *i* is zero, this fuzzy rule *i* is called an inactive fuzzy rule for the input x_0; otherwise, it is called an active fuzzy rule.

It should be noted that with $x = x_0$, an inactive fuzzy rule will not affect the controller output $u(x_0)$. Hence (4) can be rewritten so as to consider all active fuzzy rules only,

$$u(\mathbf{x}_0) = \frac{\sum_{i=1,\alpha_i \neq 0}^{r} \alpha_i(\mathbf{x}_0) u_i(\mathbf{x}_0)}{\sum_{i=1,\alpha_i \neq 0}^{r} \alpha_i(\mathbf{x}_0)}. \tag{5}$$

Definition 2: An active region of a fuzzy rule *i* is defined as a set $X_i^A = \{\mathbf{x} \in X \,|\, \alpha_i(\mathbf{x}) \neq 0\}$.

3 Stability Analysis of Fuzzy Control Systems Controlling Nonlinear Processes

The stability analysis theorem presented here is based on LaSalle's invariant set principle called also global invariant set theorem and referred in [9]. The premise of the stability criterion in this paper is that, if the control output of each rule to fulfil the same conditions (presented in the next Theorem), the overall system will be stable ISL. The theorem ensures sufficient stability conditions for the fuzzy control systems with the structure described in Section 2. This Section is focused on Theorem 1 that can be expressed also as a useful stability analysis algorithm.

Let $V : R^n \to R$, $V(\mathbf{x}) > 0, \forall \mathbf{x} \neq 0$ be a scalar function with continuous first-order partial derivatives. The time derivative of $V(\mathbf{x})$ along the open-loop trajectory (1) is given by:

$$\begin{aligned} \dot{V}(\mathbf{x}) &= \frac{dV}{dt} = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \frac{dx_i}{dt} = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \left(f_i(\mathbf{x}) + b_i(\mathbf{x}) u \right) = \\ &= \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(\mathbf{x}) + u \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} b_i(\mathbf{x}) = F(\mathbf{x}) + B(\mathbf{x}) u, \end{aligned} \tag{6}$$

where:

$$F(\mathbf{x}) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(\mathbf{x}) \tag{7}$$

and:

$$B(\mathbf{x}) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} b_i(\mathbf{x}). \tag{8}$$

Now, we define:

$$B^0 = \left\{ \mathbf{x} \in X \,\middle|\, B(\mathbf{x}) = 0 \right\}, \tag{9}$$

$$B^+ = \left\{ \mathbf{x} \in X \,\middle|\, B(\mathbf{x}) > 0 \right\}, \tag{10}$$

$$B^- = \left\{ \mathbf{x} \in X \,\middle|\, B(\mathbf{x}) < 0 \right\}. \tag{11}$$

The main result of this paper is given by the following Theorem:

Theorem 1: Let the fuzzy control system consisting of the T-S FLC described in Section 2 and the nonlinear process with the state-space equations (1) with $\mathbf{x} = 0$ an equilibrium point. If there exists a function $V : X \to R$, $V(\mathbf{x}) > 0, \forall \mathbf{x} \neq 0$ with continuous first-order partial derivatives and:

1. $F(\mathbf{x}) \le 0,\ \forall \mathbf{x} \in B^0$,
2. $u_i(\mathbf{x}) \le -\dfrac{F(\mathbf{x})}{B(\mathbf{x})}$ for $\mathbf{x} \in X_i^A \cap B^+$ and $u_i(\mathbf{x}) \ge -\dfrac{F(\mathbf{x})}{B(\mathbf{x})}$ for $\mathbf{x} \in X_i^A \cap B^-$, $i = \overline{1, r}$,
3. the set $S = \left\{ \mathbf{x} \in X \,\middle|\, \dot{V}(\mathbf{x}) = 0 \right\}$ does no contain any state trajectory of the system except the trivial trajectory $\mathbf{x}(t) = 0$ for $t \ge 0$,

then the fuzzy control system is globally asymptotically stable ISL at the origin.

Proof

Further on, we will prove that the derivative of V with respect to time, $\dot{V}$, is negative semi-definite in terms of employing (1) in order to obtain the closed-loop system structure.

Consider an arbitrary input $\mathbf{x_0} \in X$. Then three cases will be considered as follows.

Case 1: If $\mathbf{x_0} \in X_i^A \cap B^+ \neq 0$ then $B(\mathbf{x}_0)$ is strictly positive. From the condition two of Theorem 1 it results that:

$$u_i(\mathbf{x_0}) \le -\frac{F(\mathbf{x}_0)}{B(\mathbf{x_0})} \Rightarrow$$

$$\Rightarrow u(\mathbf{x_0}) = \frac{\sum_{i=1, \alpha_i \neq 0}^{r} \alpha_i(\mathbf{x_0}) u_i(\mathbf{x_0})}{\sum_{i=1, \alpha_i \neq 0}^{r} \alpha_i(\mathbf{x_0})} \le \frac{-\frac{F(\mathbf{x_0})}{B(\mathbf{x_0})} \sum_{i=1, \alpha_i \neq 0}^{r} \alpha_i(\mathbf{x_0})}{\sum_{i=1, \alpha_i \neq 0}^{r} \alpha_i(\mathbf{x_0})} = -\frac{F(\mathbf{x_0})}{B(\mathbf{x_0})} \Rightarrow$$

$$\Rightarrow \dot{V}(\mathbf{x_0}) = F(\mathbf{x_0}) + B(\mathbf{x_0})u(\mathbf{x_0}) \le F(\mathbf{x_0}) + B(\mathbf{x_0})\left(-\frac{F(\mathbf{x_0})}{B(\mathbf{x_0})}\right) = 0. \tag{12}$$

Therefore, $u_i(\mathbf{x_0}) \le -\frac{F(\mathbf{x_0})}{B(\mathbf{x_0})} \Rightarrow \dot{V}(\mathbf{x_0}) \le 0, \forall \mathbf{x_0} \in X_i^A \cap B^+ \ne 0.$ (13)

Case 2: If $\mathbf{x_0} \in X_i^A \cap B^- \ne 0$ then $B(\mathbf{x_0})$ is strictly negative. From the condition two of Theorem 1 it results that:

$$u_i(\mathbf{x_0}) \ge -\frac{F(\mathbf{x_0})}{B(\mathbf{x_0})} \Rightarrow$$

$$\Rightarrow u(\mathbf{x_0}) = \frac{\sum_{i=1,\alpha_i \ne 0}^{r} \alpha_i(\mathbf{x_0}) u_i(\mathbf{x_0})}{\sum_{i=1,\alpha_i \ne 0}^{r} \alpha_i(\mathbf{x_0})} \ge \frac{-\frac{F(\mathbf{x_0})}{B(\mathbf{x_0})} \sum_{i=1,\alpha_i \ne 0}^{r} \alpha_i(\mathbf{x_0})}{\sum_{i=1,\alpha_i \ne 0}^{r} \alpha_i(\mathbf{x_0})} = -\frac{F(\mathbf{x_0})}{B(\mathbf{x_0})} \Rightarrow$$

$$\Rightarrow \dot{V}(\mathbf{x_0}) = F(\mathbf{x_0}) + B(\mathbf{x_0})u(\mathbf{x_0}) \le F(\mathbf{x_0}) + B(\mathbf{x_0})\left(-\frac{F(\mathbf{x_0})}{B(\mathbf{x_0})}\right) = 0. \tag{14}$$

Therefore, $u_i(\mathbf{x_0}) \ge -\frac{F(\mathbf{x_0})}{B(\mathbf{x_0})} \Rightarrow \dot{V}(\mathbf{x_0}) \le 0, \forall \mathbf{x}_0 \in X_i^A \cap B^- \ne 0$. (15)

Case 3: If $\mathbf{x}_0 \in B^0$. In this case we have, from condition 3 of Theorem 1, that $F(\mathbf{x_0}) \le 0$. Therefore:

$$\dot{V}(\mathbf{x_0}) = F(\mathbf{x_0}) + B(\mathbf{x_0})u(\mathbf{x_0}) = F(\mathbf{x_0}) \le 0, \forall \mathbf{x_0} \in B^0 . \tag{16}$$

From above three cases one may conclude that $\dot{V}(\mathbf{x}) \le 0, \forall \mathbf{x} \in X$.

Summarizing, $\dot{V}$ is negative semi-definite.

Condition 3 of theorem ensures the fulfilment of LaSalle's invariant set principle. Both the condition of regarding the sign of $\dot{V}$ and the condition 3 satisfy the conditions from LaSalle's global invariant set theorem. Therefore, the equilibrium point at the origin is globally asymptotically stable. The proof is now complete ■

The above stability theorem ensures sufficient stability conditions regarding the accepted class of fuzzy control systems described briefly in Section 2.

Theorem 1 proves that if the Lyapunov function is negative semi-definite in the active region of each fuzzy rule then the overall system will be asymptotically ISL.

3.1 The Stability Analysis Algorithm

The stability analysis algorithm ensuring the stability of the class of fuzzy logic control systems considered in Section 2 is based on Theorem 1. It consists of the following steps:

1 *Determine the state-space equations of the nonlinear process,*

2 *Determine the membership function of the LTs in the T-S FLC structure,*

3 *Determine the premise of each fuzzy rule,*

4 *Set the* V *function, calculate its derivative and the expression of the functions* $F(x)$ *and* $B(x)$ *and the sets* B^0 *,* B^+ *and* B^0 *as well,*

5 *If* $F(x) \le 0, \forall x \in B^0$ *then go to step 6. Else go to step 4.*

6 *For each fuzzy control rule i determine* u_i *such that* $u_i(\mathbf{x}) \le -\dfrac{F(\mathbf{x})}{B(\mathbf{x})}$ *for* $\mathbf{x} \in X_i^A \cap B^+$ *and* $u_i(\mathbf{x}) \ge -\dfrac{F(\mathbf{x})}{B(\mathbf{x})}$ *for* $\mathbf{x} \in X_i^A \cap B^-$, $i = \overline{1,r}$,

7 *Check that the set* $S = \{\mathbf{x} \in X \,|\dot{V}(\mathbf{x}) = 0\}$ *does not contain any state trajectory of the system except the trivial one,* $\mathbf{x}(t) = 0$ *for* $t \ge 0$.

4 Design Example

This Section presents an example that deals with one chaotic Lorenz system to be controlled by a Takagi-Sugeno FLC. Modern discussions of chaos are mainly based on the works about the Lorenz attractor. The Lorenz equation is commonly defined as three coupled ordinary differential equations expressed in (17) to model the convective motion of fluid cell, which is warmed from below and cooled from above:

$$\frac{dx}{dt} = \sigma(y - x), \quad \frac{dy}{dt} = x(\rho - z) - y, \quad \frac{dz}{dt} = xy - \beta z , \tag{17}$$

where the three parameters $\sigma, \rho, \beta > 0$ are called the Prandtl number, the Rayleigh number, and a physical proportion, respectively. These constants determine the behaviour of the system and these three equations exhibit chaotic behaviour i.e. they are extremely sensitive to initial conditions. A small change in initial conditions leads quickly to large differences in corresponding solutions.

The classic values used to demonstrate chaos are $\sigma = 10$ and $\beta = \frac{8}{3}$. Let $X = [-40, 40] \times [-40, 40] \times [-40, 40]$.

4.1 Design of Stable Fuzzy Logic Control System

The algorithm presented in Section 3 will be applied in the sequel in order to find the values of u_i for which the system (17) can be stabilized by the above described T-S FLC. A similar fuzzy logic control system has been designed in [10] but involving Barbashin-Krasovskii's theorem.

Step 1: The design of the fuzzy logic control system with TS FLC starts with rewriting the ordinary differential equation (17) in the following form representing the state-space equations of the controlled process with $x_1 = x$, $x_2 = y$ and $x_3 = z$:

$$\dot{\mathbf{x}} = \begin{pmatrix} \sigma(x_2 - x_1) \\ x_1(\rho - x_3) - x_2 \\ x_1 x_2 - \beta x_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u, \quad \mathbf{x_0}(t) = \mathbf{x_0}. \tag{18}$$

Step 2: The first two equations are considered in the T-S FLC design. The fuzzification module of T-S FLC is set according to Figure 2 showing the membership functions that describe the LTs of the linguistic variables of x_1 and x_2. The LTs representing Positive, Zero and Negative values are noted by P, Z and N, respectively. The inference engine employs the fuzzy logic operator AND modelled by the *min* t-norm.

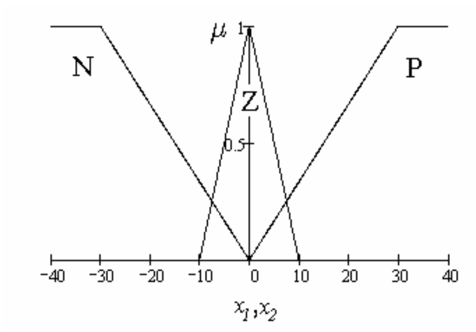

Figure 2
Membership functions of x_1 and x_2

Step 3: The inference engine is assisted by the complete set of fuzzy control rules illustrated in Table 1.

Table 1
Fuzzy Control Rule Base

Rule	Antecedent		Consequent
	x_1	x_2	u
1	P	P	u_1
2	N	N	u_2
3	P	N	u_3
4	N	P	u_4
5	P	Z	u_5
6	N	Z	u_6
7	Z	P	u_7
8	Z	N	u_8
9	Z	Z	u_9

Step 4: Let $V(\mathbf{x}) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right)$ be a Lyapunov function candidate, which is a continuously differentiable positive function on the domain *X*. The time derivative of *V* along the trajectories of the system (18) is given by:

$$\dot{V}(\mathbf{x}) = -\sigma x_1^2 - x_2^2 - \beta x_3^2 + x_1 x_2(\sigma + \rho) + x_1 u . \tag{19}$$

Then (19) results in $F(\mathbf{x}) = -\sigma x_1^2 - x_2^2 - \beta x_3^2 + x_1 x_2(\sigma + \rho)$, $B(\mathbf{x}) = x_1$ and

$B^0 = \left\{\begin{pmatrix}0 & x_2 & x_3\end{pmatrix} \in R^3\right\}$, $B^+ = \left\{\begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix} \in R^3 \middle| x_1 > 0\right\}$,

$B^- = \left\{\begin{pmatrix}x_1 & x_2 & x_3\end{pmatrix} \in R^3 \middle| x_1 < 0\right\}$.

Step 5: Since $F(\mathbf{x}) \le 0, \forall \mathbf{x} \in B^0$, the step 6 continues is applied.

Step 6: Further on, we will analyze each fuzzy control rule.

For rule 1: x_1 is P, x_2 is P and $X_1^A \times [-40, 40] \cap B^+ = (0,40] \times (0,40] \times [-40,40]$, $X_1^A \cap B^- = \varnothing$. In this case we must have that $u_1(\mathbf{x}) \le -\frac{F(\mathbf{x})}{B(\mathbf{x})} = \sigma x_1 + \frac{x_2^2 + \beta x_3^2}{x_1} - x_2(\sigma + \rho)$. From this inequality we set $u_1(\mathbf{x}) = -x_2(\sigma + \rho)$.

For rule 2: x_1 is N, x_2 is N and $X_2^A \times [-40,40] \cap B^- = [-40,0) \times [-40,0) \times [-40,40]$, $X_2^A \cap B^+ = \varnothing$. In this case we must have that $u_2(\mathbf{x}) \ge -\frac{F(\mathbf{x})}{B(\mathbf{x})} = \sigma x_1 + \frac{x_2^2 + \beta x_3^2}{x_1} - x_2(\sigma + \rho)$. From this inequality we set $u_2(\mathbf{x}) = -x_2(\sigma + \rho)$.

For rule 3: x_1 is P, x_2 is N and $X_3^A \times [-40,40] \cap B^+ = (0,40] \times [-40,0) \times [-40,40]$, $X_3^A \cap B^- = \varnothing$. In this case we must have that $u_3(\mathbf{x}) \le -\dfrac{F(\mathbf{x})}{B(\mathbf{x})} = \sigma x_1 + \dfrac{x_2^2 + \beta x_3^2}{x_1} - x_2(\sigma + \rho)$. From this inequality we set $u_3(\mathbf{x}) = -1$.

For rule 4: x_1 is N, x_2 is P and $X_4^A \times [-40,40] \cap B^- = [-40,0) \times (0,40] \times [-40,40]$, $X_4^A \cap B^+ = \varnothing$. In this case we must have that $u_4(\mathbf{x}) \ge -\dfrac{F(\mathbf{x})}{B(\mathbf{x})} = \sigma x_1 + \dfrac{x_2^2 + \beta x_3^2}{x_1} - x_2(\sigma + \rho)$. From this inequality we set $u_4(\mathbf{x}) = 1$.

For rule 5: x_1 is P, x_2 is Z and $X_5^A \times [-40,40] \cap B^+ = (0,40] \times (-10,10) \times [-40,40]$, $X_5^A \cap B^- = \varnothing$. In this case we must have that $u_5(\mathbf{x}) \le -\dfrac{F(\mathbf{x})}{B(\mathbf{x})} = \sigma x_1 + \dfrac{x_2^2 + \beta x_3^2}{x_1} - x_2(\sigma + \rho)$. From this inequality we set $u_5(\mathbf{x}) = \sigma x_1 + \dfrac{x_2^2 + \beta x_3^2}{x_1} - 10(\sigma + \rho)$.

For rule 6: x_1 is N, x_2 is Z and $X_6^A \times [-40,40] \cap B^- = [-40,0) \times (-10,10) \times [-40,40]$, $X_6^A \cap B^+ = \varnothing$. In this case we must have that $u_6(\mathbf{x}) \ge -\dfrac{F(\mathbf{x})}{B(\mathbf{x})} = \sigma x_1 + \dfrac{x_2^2 + \beta x_3^2}{x_1} - x_2(\sigma + \rho)$. From this inequality we set $u_6(\mathbf{x}) = \sigma x_1 + \dfrac{x_2^2 + \beta x_3^2}{x_1} + 10(\sigma + \rho)$.

For rule 7: x_1 is Z, x_2 is P. Two cases should be considered:

a. $X_7^A \times [-40,40] \cap B^- = [-10,0) \times (0,40] \times [-40,40]$. In this case we must have that $u_7(\mathbf{x}) \ge -\dfrac{F(\mathbf{x})}{B(\mathbf{x})} = \sigma x_1 + \dfrac{x_2^2 + \beta x_3^2}{x_1} - x_2(\sigma + \rho)$.

b. $X_7^A \times [-40,40] \cap B^+ = (0,10] \times (0,40] \times [-40,40]$. In this case we must have that $u_7(\mathbf{x}) \le -\dfrac{F(\mathbf{x})}{B(\mathbf{x})} = \sigma x_1 + \dfrac{x_2^2 + \beta x_3^2}{x_1} - x_2(\sigma + \rho)$.

From both cases, set $u_7(\mathbf{x}) = -x_2(\sigma + \rho)$.

For **rules 8 and 9** a similar reasoning to that of rule 7 swill be applied with the result $u_8(\mathbf{x}) = u_9(\mathbf{x}) = u_7(\mathbf{x})$.

Step 8: We note that $\dot{V}_i(\mathbf{x}) = F(\mathbf{x}) + B(\mathbf{x})u_i$ and $S_i = \{\mathbf{x} \in X \,|\, \dot{V}_i(\mathbf{x}) = 0\}$. Use (4) result that $\dot{V}(\mathbf{x}) = \frac{\sum_{i=1}^{n} \alpha_i(\mathbf{x}) \dot{V}_i(\mathbf{x})}{\sum_{i=1}^{n} \alpha_i(\mathbf{x})}$. We prove now that $S \subseteq \bigcup_{i=1}^{n} S_i$. We suppose that there exists $\mathbf{x}_0 \in S$. Then:

$$\dot{V}(\mathbf{x_0}) = 0 \Rightarrow \frac{\sum_{i=1}^{n} \alpha_i(\mathbf{x}) \dot{V}_i(\mathbf{x})}{\sum_{i=1}^{n} \alpha_i(\mathbf{x})} = 0 \Rightarrow \sum_{i=1}^{n} \alpha_i(\mathbf{x}) \dot{V}_i(\mathbf{x}) = 0 \,. \tag{20}$$

It is important to highlight that the interpretation of (20) is that there exists at least one rule index i such that $\dot{V}_i(\mathbf{x}) = 0$. Therefore $S \subseteq \bigcup_{i=1}^{n} S_i$. Since $S_i = \varnothing$ for $i = \overline{1,8}$ and $S_9 = \{0\}$, the result is $S = \{0\}$. Thus, $S = \{\mathbf{x} \in X \,|\, \dot{V}(\mathbf{x}) = 0\}$ does not contain any state trajectory of the system except the trivial one, $\mathbf{x}(t) = 0$ for $t \geq 0$. Concluding, due to Theorem 1 it results that the system composed by this T-S FLC and the Lorenz process described by (18) is globally asymptotically stable ISL at the origin.

4.2 Simulation Results

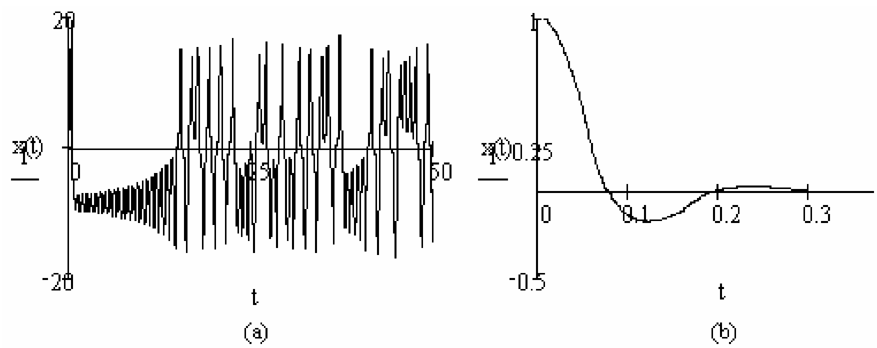

Figure 3

State variable x_1 versus time of Lorenz chaotic system without FLC (a) and with FLC (b)

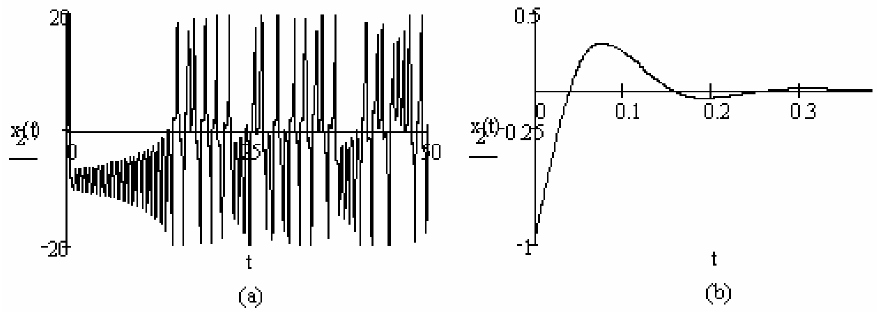

Figure 4

State variable x_2 versus time of Lorenz chaotic system without FLC (a) and with FLC (b)

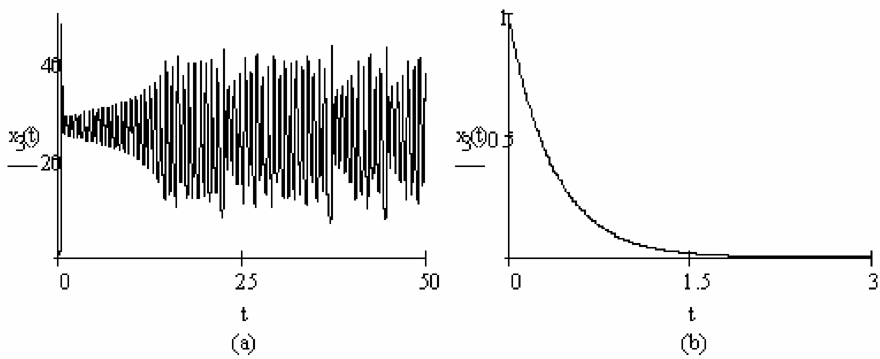

Figure 5

State variable x_3 versus time of Lorenz chaotic system without FLC (a) and with FLC (b)

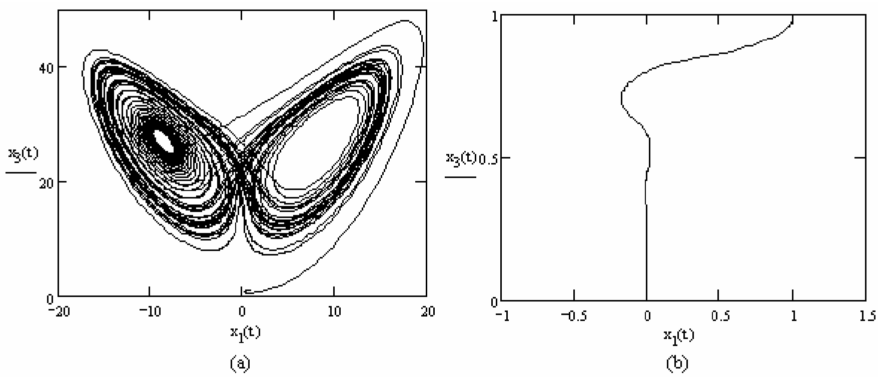

Figure 6

2D phase portraits of Lorenz system without control (a) and with FLC (b)

Considering the values of process parameters $\sigma = 10$, $\rho = 28$, $\beta = \frac{8}{3}$, the initial state $x_1(0) = 1$, $x_2(0) = -1$ and $x_3(0) = 1$, the responses of x_1, x_2 and x_3 versus time in the closed-loop system are illustrated in Figures 3 to 7.

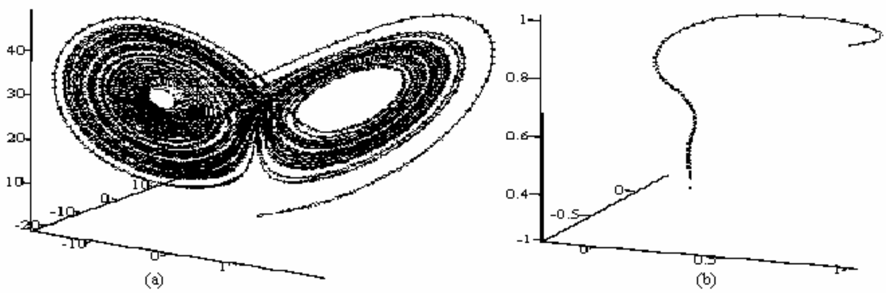

Figure 7
3D phase portrait Lorenz system without control (a) and with FLC (b)

Conclusions

A new approach to the globally asymptotically stability analysis of fuzzy control systems employing T-S FLCs dedicated to a class of nonlinear processes has been introduced. The new stability analysis approach is different to Lyapunov's theorem in several important aspects and allows more applications. In particular, it is well-suited to controlling processes where the derivative of the Lyapunov function candidate is not negative definite, therefore applying the LaSalle's invariant set principle to nonlinear processes controlled by T-S FLCs can be applied to a wide area of nonlinear dynamic systems. Using the proposed stability analysis approach makes the inserting of a new fuzzy rule (with the index r+1) become very easy because this needs only the fulfilment of the condition $\dot{V}_{r+1}(x) \leq 0$.

The stability analysis algorithm proposed in this paper, based on Theorem 1, guarantees sufficient stability conditions for the fuzzy control systems. This algorithm can result in a design method, which is advantageous because the stability analysis decomposed to the analysis of each fuzzy rule. Therefore, the complexity of system analysis is reduced drastically.

This paper has shown, by the Lorenz system, how the stability analysis algorithm can be applied to the design of a stable fuzzy control system for a nonlinear process. Our stability approach can be applied also in situations when the system has an equilibrium point different to the origin and / or the reference input of the fuzzy control system is non-zero by appropriately defined state transforms.

Future research will be focused on increasing the area of applications [11-16]. But this must be accompanied by the derivation of transparent design methods for low-cost fuzzy logic controllers.

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