

Handling Contradicting Requirements Using Desirability Functions

Zsolt Réthy, Zoltán Koczor, **József Erdélyi**

University of West Hungary, Ady Endre u. 5, 9400 Sopron, E-mail:
rethyzs@wla.hu

Budapest Tech, Doberdó út 6, 1034 Budapest

Abstract: Desirability functions serve to turn multiple response optimization problems into single response problems. A new set of these functions will be introduced within this paper that are based on an analogous probabilistic model for predicting the optimal starting time of a process to meet a given time schedule. We use loss-function-like functions and the probability of not meeting a given target to find the optimal set of input parameters where statistical properties are as we expect.

1 Introduction

Optimizing parameters is a critical issue during the development of any product. For this reason several methodologies have been worked out, most of them multi-criteria decision making techniques. A special set of functions called desirability functions [1][2][3] has been developed for doing this as easily as turning the multiple response problem to a single response problem. We aim to give an alternative from a different point of view, namely for cases where we are able to define loss functions for not meeting the specification limits, and we have information about the distribution of each response variable.

2 Calculating Risk for a Production Time Schedule with Given Loss Functions for Failing to Meet the Specification Limits

Risk can be defined for every activity, as can the processes and decisions we are responsible for. Generally speaking, risk is a function of the probability of an event and the causes of the event: $R(A) = P(A)L(A)$ (1)

where R is the calculated risk, P is the probability of the event A , and L is the calculated total loss [4].

If we define a certain loss or a loss function for exceeding the specification limit for a response variable, and calculate the distribution of the parameter, then we have the probability of meeting the specification limits. So the loss function multiplied by the given probability is the risk function for the parameter.

For the production time schedule we can define two mutually contradicting economic requirements:

- the economic consequences of the delay when we cannot make the time schedule and
- the costs of the early stockpiling and of the additional resources needed for the former.

These requirements can be represented as loss functions.

We can assume that these losses can be calculated simply. The total time, and so the deadline for the production, can be calculated i.e. from process and logistical times. However, there is a certain variation around this time point that we assume to be a normal distribution.

The risk function can be calculated as follows:

$$L(t^*) = L_1(t)(1 - \Phi_{t,s}(T_{kr})) + L_2(t) \int_{-\infty}^{T_{kr}} \varphi_{t,s}(t) dt = L_1(t)(1 - \Phi_{t,s}(T_{kr})) + L_2(t)\Phi_{t,s}(T_{kr}) \quad (2)$$

where

- $\Phi_{t,s}$ is the distribution of the total time with t^* mean and s standard deviation ($t \in N(\mu, \sigma)$)
- $\varphi_{t,s}$ is the density of the total time with t^* mean and s standard deviation
- T_{kr} – the nominal total production time (upper specification limit)
- $L_1(t)$ – the cost of exceeding the specification limit
- $L_2(t)$ – the cost of premature production

We use the fact that the probability of being below T_{kr} is

$$P(t < T_{kr}) = \Phi_{t,s}(T_{kr}) \quad (3)$$

Assuming the same process flow for different start times, the t^* mean can be changed by changing the start time. The risk function can be represented graphically as Figure 1 shows. The optimal start time will be the one where risk is minimal, i.e. minimizing the risk function.

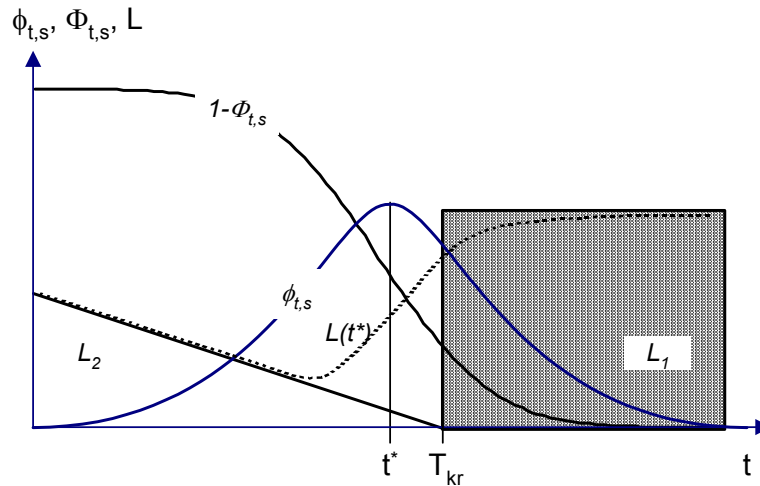


Figure 1

Loss functions and the density function showing the uncertainty of the deadline

3 Defining Desirability Functions Based on Loss Functions

Consider that we have a process with a number of inputs and a number of response variables. The distribution of each response variable is known, and some of them have lower and/or upper specification limits, while others have only a target value defined.

Similarly to the model shown in Figure 1, we can create functions that represent the loss when moving away from the target and the probability of not meeting the target. The latter has the advantage of taking not only the mean but also the standard deviation into account. A couple of papers already handle this in different ways e.g. [5] defines desirability functions based on the mean and deviation to bind the desirability levels to Six Sigma quality levels.

We would like to handle the newly created functions as desirability functions, with the assumption that the response variables are independent and are distributed normally.

First we have to collect information about the process

to have an estimate for the function between the input parameters, denoted \mathbf{x} , and the responses – denoted $\hat{y} = f(\mathbf{x})$ [6];

- to have an estimate for the standard deviation of the responses.
- Further to this, we have to
- define the target value (T) for each response variable;
- define a helper desirability function (δ -function) for each response variable;
- and define a d-function for each response variable based on the δ -function and the probability of not meeting the target;
- both the δ -functions and the modified d-functions should fulfil some requirements such as the image set should be $[0, 1]$.

Similarly to the definition of risk above, we define the desirability as the product of the desirability helper function (based on loss functions but in the opposite manner) and a probability value.

This means that similarly to the previous scheduling model, the loss function has its minimum in the target. However, that means in our interpretation, that the helper desirability function has its maximum at the target.

We define a simple linear helper function (see Figure 2) for each side of the target (this is similar to the desirability function of [2]):

$$\delta_{il}(\mathbf{x}) = \begin{cases} 1 + k_{il}(\hat{y}(\mathbf{x}) - T), & T - \frac{1}{k} \leq y \leq T \\ 0, & \textit{otherwise} \end{cases} \quad (4)$$

for the left side and

$$\delta_{ir}(\mathbf{x}) = \begin{cases} 1 - k_{ir}(\hat{y}(\mathbf{x}) - T), & T \leq y \leq T + \frac{1}{k_{ir}} \\ 0, & \textit{otherwise} \end{cases} \quad (5)$$

for the right side, where k_{il} and k_{ir} are constants. For these constants, we can take into account some multiple of the standard deviation.

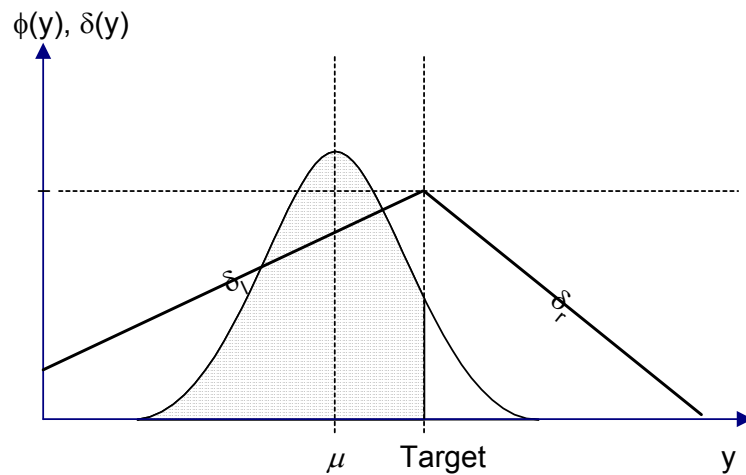


Figure 2

Linear helper functions and the density function of a response variable

Alternatively, we can use helper functions based on Taguchi's quality loss function.

While the traditional quality philosophy binds the loss to the point where a quality characteristic exceeds a specification limit, [7] defines a quadratic function for the loss so that it is only zero if we meet the target value exactly. The quality loss function looks as follows:

$$L(x) = k(x - T)^2 \quad (6)$$

So the helper functions based on this would be (see Figure 3):

$$\delta_{Ti}(\mathbf{x}) = 1 - k_i(\hat{y}(\mathbf{x}) - T)^2; \quad (7)$$

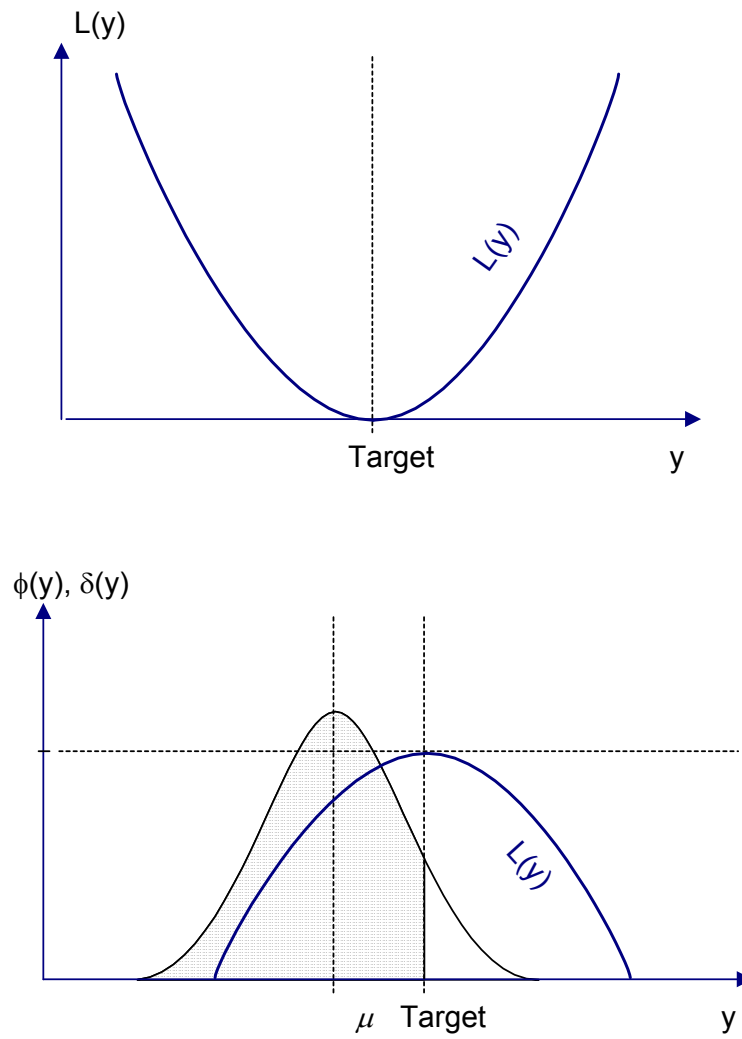


Figure 3:

The Taguchi quality loss function (above) and the helper function created for a response variable (below)

The desirability function will be the sum of each side's desirability:

$$d_i(\mathbf{x}) = \delta_{i_l} \Phi_{\mu, \sigma}(T) + \delta_{i_r} (1 - \Phi_{\mu, \sigma}(T)) \quad (8)$$

where μ is the mean and σ is the standard deviation. In order not to lose generality, the original desirability functions introduced in [2] can be used for the responses where it's more appropriate e.g. for responses with lower/upper specification limits.

Calculation of the composite desirability

In [6] a calculation with individual weights is proposed for each desirability function rather than using $y-d$ value pairs for weighting. This latter is more difficult to use and interpret.

$$D_{Derr} = \sqrt[S]{\prod_{i=1}^q d_i^{w_i}}, \quad (9)$$

where

$$S = \sum_{i=1}^q w_i \quad (10)$$

Based on this calculation method, we also use individual weights and the composite desirability should be maximized.

$$D_s = \sqrt[S]{\prod_{i=1}^q d_i^{w_i}}, \quad (11)$$

Once we have the maximized value, the optimal input parameters can be calculated from it.

Conclusions

Given the generality of the problem a widely usable model can be built that gives the correct answers. As part of our research we created an application that includes expert system features to get an optimal solution. The properties of this application are:

- A standard interface for data input.
- A data exclusion mechanism that “forgets” given extreme values and archives older data - enabling to describe process trends.
- Stores older data sets in the database.
- Practice-oriented outputs.

The usability of the application was tested and proven on a real-life example in practice.

References

- [1] Harrington Jr., E.C.(1965): "The desirability function". *Industrial Quality Control* 21 (10) p. 494-498.
- [2] Derringer, G. and Suich, R. (1980): "Simultaneous Optimization of Several Response Variables". *Journal of Quality Technology* Vol. 12. No. 4. 1980, p. 214-219.
- [3] del Castillo, E. ; Montgomery, D.C. and McCarville, D.R. (1996): „Modified desirability functions for multiple response optimization”. *Journal of Quality Technology* Vol. 28(3) p. 337-344.
- [4] Koczor, Z. ; Marschall, M; Némethné Erdődi, K. and Réthy, Zs. (1996): "Quality assurance methods to determine the optimum of risk based on the product's characteristics and the information on manufacturing". *Anyagvizsgálók Lapja* 1996/4. p. 123-126.
- [5] Ribardo, C. L. (2000): „Desirability functions for comparing arc welding parameter optimization methods and for addressing process variability under Six sigma assumptions”. Dissertation, The Ohio State University, Columbus, OH.
- [6] Derringer, G. C. (1994): "A balancing act: optimizing a product's properties". *Quality Progress*, June 1994, p. 51-58.
- [7] Taguchi, G. (1986): „Introduction to quality engineering”. Asian Productivity Organization, Tokyo.