COLLABORATIONS OF HUNGARIAN MATHEMATICIANS

VERA JOHN-STEINER

University of New Mexico, Albuquerque
USA

This paper focuses on the collaborative partnerships of Hungarian mathematicians in the first half of the twentieth century. They were pioneers in collaboration, an intellectual practice that has become widespread during the last fifty years. Hungarian mathematicians starting with the Bolyais, father and son, worked closely together in co-constructing new knowledge and in supporting each other in developing innovative approaches to mathematics. In the literature Paul Erdős' extraordinary range of jointly authored papers is celebrated by the use of the "Erdős Numbers". This paper discusses different patterns of collaboration and uses as illustration the partnerships of von Neumann and Morgenstern; and Pólya and Szegő. These collaborative partners developed interesting working methods including using different languages for conversation and for writing. Their accounts of their shared activities include key aspects of collaboration including mentorship, complementary skills, domains of expertise, the role of chance versus purpose, and the power of shared vision.

Keywords: mathematical discovery, intellectual partnerships, collaborative patterns, Hungarian mathematicians, mutual support

There is good reason to believe that Hungarian mathematicians were early practitioners of collaborative activities during periods when joint endeavors were less widespread than they are now. I will give a few examples of partnerships in this discussion as it illustrates, in my opinion, one of the many strengths of the achievements of Hungarians both within and outside the country. I am building on work started with Reuben Hersh, and summarized in our article, "A Visit to Hungarian Mathematics". In addition, I am relying on analyses of partnerships drawn from my book *Creative Collaboration.*

If you mention collaboration to a mathematician, you are most likely to be told of the Erdős Numbers. These refer to a network representing joint papers authored by colleagues in the field who have, if they received the number 1, written a paper with Erdős. Anyone receiving the number 2 has written a paper with someone who has been awarded the number 1. The network is large and complex; and it includes some people outside of mathematics as well. For instance, Einstein was given the

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number 2, as he wrote a paper with Ernst Straus, who had written a paper with Erdős. The extraordinary range of close cooperation and jointly authored papers by Pál Erdős is legendary. It reflects his personality and his unceasing passion for his field.

The complexity and frequency of intellectual collaborations have increased rapidly during the last two centuries. This increase is illustrated by the fact that shared Nobel prizes were the norm by the end of the twentieth century, while during the early part of the century solo prizes dominated the field.[1]

One way to define collaboration is that it is the interdependence of thinkers in the co-construction of knowledge, but even when one uses a particular definition, the actual phenomena that such a definition refers to are quite diverse. Thus, it is helpful to distinguish between different patterns of creative partnerships.

One such pattern is *intergenerational* collaboration, which occurs in different contexts including the home, the university and the workplace. In the history of Hungarian mathematics a very interesting example of collaboration across generations is that of Farkas and János Bolyai, father and son. They both addressed issues leading to the development of non-Euclidean geometry, a discovery that had profound mathematical and philosophical implications and which many decades after its original development was used by Einstein in his theory of general relativity.

The story starts in Göttingen, Germany where the elder Bolyai was a student and a close friend of Carl Friedrich Gauss, one of the great mathematicians of the nineteenth century. The two men remained intimate during their entire lives. They shared ideas about the challenge posed by Euclid's parallel axioms. In 1799 Gauss wrote to Farkas Bolyai about his efforts to try to prove this axiom, and bemoaned his difficulty in doing so. Bolyai, the elder, also worked on this problem while he taught mathematics for 47 years at Maros-Vásárhely. We now know that direct attempts to prove this axiom are not demonstrable. Bolyai wrote in his autobiography: "I was obsessed with this problem to the point where it robbed me of my rest, [and] deprived me of my tranquility" (Halsted, xi).

János Bolyai was a gifted child who studied mathematics with his father. His mastery was very rapid, and he pushed his father to present him with increasingly difficult material. At age fifteen he went to study engineering in Vienna. Subsequently, he entered the military, and while posted as a captain at Temesvár, he continued to work on the problems that had preoccupied his father and Gauss. The result was the *Appendix,* a brilliant treatise that stated the basic concepts of non-Euclidean geometry. In presenting this work to his father, who originally warned him against devoting himself to this topic, he expressed his sense of gratitude and closeness to the elder Bolyai, "in certain regards I consider you as a second self" (Halsted, xxviii). His father was eager to have these results published rapidly and he attached them to his own major work, the Tentamen. He also added some of his own reflections and a comparison of Bolyai János' work with that of Lobatchevsky, who also developed what he called "imaginary geometry", and in which his ideas were basically the same as that of Bolyai. Lastly, Gauss responded to Bolyai's work by agreeing to its validity because they corresponded to some of his own, unpublished work.

The foundation of non-Euclidian geometry provides many examples of collaboration, including the sharing of ideas between János Bolyai and his friend Károly Szász. Of greatest interest to this discussion is the way in which father and son demonstrated a deep intellectual and emotional interdependence, starting with János Bolyai learning mathematics from his father. They shared their disappointments and their triumphs with each other, and at a crucial time, the elder Bolyai provided the venue of publication that his son needed.

In the literature of creativity, the development of non-Euclidean geometry is referred to as an example of simultaneous discoveries; that is as an idea that occurs independently to different people at the same time. But the mathematician Morris Kline challenges this interpretation and suggests that there was a kind of thought community between Gauss, Bolyai and the Russian mathematician Lobatchevsky. These three men were linked through the friendship of the elder Bolyai and Gauss, as well as between Lobatchevsky's teacher, Johann Bartels, and Gauss. These mathematicians shared their concerns about Euclid's fifth postulate, which could not be derived from his other postulates. This realization lead Gauss, János Bolyai, and Lobatchevsky to the development of alternatives to the parallel axioms and the foundation of non-Euclidean geometry (Kline, 1972, 878). Gauss was reluctant to publish his own results as he disliked the contentiousness of public debate. In my view, this important episode in mathematical history illustrates the dynamics of discovery, which require both social facilitation and individual originality.

Another form of collaboration is that of *complementarity* which I have defined in the past as consisting of a division of labor based on complementary expertise (including disciplinary knowledge), roles, and temperament. In sustained collaborations such complementarity can result in mutual appropriation, a process in which participants make their own some of their partners' ways of thinking, their specialized expertise and problem solving strategies. John von Neumann, a graduate of the Lutheran gymnasium of Budapest and a winner of the Eötvös prize, was one of the twentieth century's greatest mathematicians. His work ranged from pure mathematics to quantum theory; it included automata theory (critical to computers), axiomatics, and game theory. One of his most interesting collaborations was with the economist Oskar Morgenstern, who described their work together after von Neumann's untimely death (Morgenstern in *Journal of Economic Literature*). Both men published on the topic of games before they started to work together; von Neumann's paper on the theory of games appeared in 1928 and

Morgenstern's first book, which also appeared in 1928, contained some ideas concerning decision making.

The two collaborators met at Princeton University in 1939 and quickly discovered that they shared some interests in common. Morgenstern did not have systematic training in mathematics, but he had studied widely, and was ready to undertake the challenge of mastering some of the concepts in von Neumann's 1928 paper on game theory. The latter was ready to help, and found that discussions with his new Austrian friend were leading him to return to his dormant interests in economics. At first, Morgenstern was going to write a solo paper "showing economists the essence and significance of game theory as it then existed" (Morgenstern, 808). But as he proceeded, von Neumann became increasingly interested in the paper's progress. At one point, he offered to write a joint paper. Morgenstern realized that this was an extraordinary opportunity, and in his description of the beginnings of their collaboration, he refers to this event as "his gift from Heaven". As their work progressed it became a larger and larger treatise. From an article it evolved into a pamphlet and eventually a book. Once they obtained a contract from Princeton University, they no longer worried about the length of the manuscript.

Their practice was to engage in long discussions in German while walking, but once they sat down to write, their text was produced in English. They wrote together. At times, the handwriting changed from one to the other several times on the same page, revealing the extent to which their thinking blended together. They developed the concepts of strategies in two-person and n-person games, a notion first identified in Morgenstern's book, and applied with great success in von Neumann's mathematical Minimax Theorem (Von Neumann, 1944).

In his account of their collaborative work Morgenstern mentions an incident "that shows how chance can influence the direction of scientific work". One day, while working on von Neumann's minimax theorem, Morgenstern took a walk. It was a cold day, and as he was close to the Institute for Advanced Study library, he stepped inside in order to warm up. He picked up a book edited by the French mathematician Borel. He found within that volume a paper by Jean Ville, who presented the minimax theory in mathematically simpler terms than von Neumann. The two used this formulation to advance their own work, and to introduce "the method of convex bodies into economical theory."

I found this incident of interest as the role of chance in scientific discovery is a contemporary topic in the literature of creativity. Keith Simonton has recently published a book in which he places chance as the most important explanatory variable in scientific creativity (2004). He would undoubtedly find Morgenstern's description as support for his theory. I see a somewhat different explanation. In long projects, there are rhythms of clear focus on a topic, and then there are periods when thinkers need an alternative strategy, that of expansion. This incident illustrates the second of these phases. While Morgenstern interprets his picking up Borel's volume as a chance event, this may not be the case entirely. Borel preceded von Neumann in proposing analyses of games of strategy. His treatment was not as formally developed as those of von Neumann in his 1928 paper, but he was in the same general field as the co-authors of the *Theory of Games and Economic Behavior*. His edited volume had to be of interest to Morgenstern. It is in recognizing the value of Ville's approach (which appeared in the Borel volume) as productive and innovative that Morgenstern revealed his readiness to incorporate new material in their treatment. Chance, curiosity, an openness of mind, and judgment all operated together in this simplification and extension of their theory.

The early presentation of their ideas did not resonate immediately with economists. However, the authors were convinced that the role of elegant, formal mathematics in economics was the way of the future, and that their game theory was an important model leading in that direction. When they wrote in their introduction that, "We shall find it necessary to draw upon techniques of mathematics which have not been used heretofore in mathematical economics, and it is quite possible that further study may result in the future in the creation of new mathematical disciplines" (5), they foretold actual developments. Their theory of games and utility were enormously influential and led to new directions in both economics and mathematics.

The way in which these two men worked together represents a mixture of complementary collaboration – the joining of two disciplines – as well as aspects of *integrative collaboration*. "These partnerships require a prolonged period of committed activity. They thrive on dialogue, risk taking, and a shared vision" (John-Steiner, 2000, 203). In the case of von Neumann and Morgenstern, it was their belief in the applicability of formal mathematical models to economics that revealed their shared vision. They also worked so closely together that they switched back and forth in their writing. We have referred to such a pattern as "braided roles." Morgenstern recalled that their collaboration was the most intensive intellectual activity of his life, and he further commented: "We did an enormous amount of work in a very short time, but it was unceasing pleasure and never a time of drudgery" (815). And finally, this partnership has one more feature of integrative collaborations. It is a transformative co-construction: *The Theory of Games and Economic Behavior* has had an enormous impact in the social sciences, and as mentioned above, it gave rise to important new developments in mathematics. In this way it is an illustration of the most intense and consequential form of intellectual collaboration.

In general, intensive collaborations last one decade or less. A notable exception is the partnership of György Pólya and Gábor Szegő. Their lives spanned much of the twentieth century; and their friendship started when Szegő (who was a few years younger than Pólya) was a student of mathematics in Budapest. They met in

1913 when Pólya returned to Hungary while studying abroad. They discussed a conjecture of Pólya's (based on a conjecture about Fourier coefficients) that became the basis of Szegő's dissertation and first publication. Pólya described this "fruitful cooperation" as one in which mathematical theorems emerge through steps. First, there is a productive guess, which is usually followed by lengthy work during which the proof is found. In this case there was a division of labor between these two mathematicians. Pólya provided the conjecture and Szegő worked out the details and found the proof (Alexanderson, 2000).

Subsequently, they co-authored a very influential book on *Problems and Theorems in Analysis* (German edition 1925). The novelty of this book was that the problems were not classified according to topics, but according to the method of solution. As the authors described it in their preface,

> This book is no mere collection of problems. Its most important feature is its systematic arrangement of material, which aims to stimulate the reader to independent work and to suggest to him useful lines of thought. …Above all we aim to promote in the reader a correct attitude, a certain discipline of thought, which would appear to be even of more essential importance in mathematics than in other scientific disciplines (56).

This innovation was the beginning of Pólya's life-long interest in problem-solving heuristics. His book, *How to Solve It,* first appeared in 1945. It was based on this theme and sold over a million copies over the years and was translated into seventeen languages. His contributions influenced the teaching of mathematics (see the 1980 Yearbook of the National Council of Teachers in Mathematics, who rely upon Pólya's approach). Pólya's rules of discovery thinking had a lasting impact both inside and outside of mathematics.

Of their shared work, Pólya wrote,

> It was a wonderful time; we worked with enthusiasm and concentration. We had similar backgrounds. Like all young Hungarian mathematicians of that time, we were both influenced by Lipót Fejér. We were both readers of the same well-directed Hungarian Mathematical Journal for high school students that stressed problem solving. We were interested in the same kinds of questions, in the same topics; but one of us knew more about one topic, and the other more about some other topic. It was a fine collaboration. The book Aufgaben und Lehrsätze aus der Analysis, the result of our cooperation, is my best work and also the best work of Gábor Szegő (Nevai, 1995, 682).

And when asked by an interviewer about this collaboration, Pólya added one more thought: "We completed each other, and that through books and papers, and over many, many years" (Alexanderson, 1985, 250).

This description of their partnership touches on many qualities of creative collaborations. The notion of completion is perhaps the most striking remark from a psychological perspective. It echoes the words of the Russian literary critic, Bakhtin, who wrote; "I cannot do without the other; I cannot become myself without the other; I must find myself in the other, finding the other in me" (as quoted in Wertsch, 1998, 116). One way to think of individual development is that each of us is a subset of human possibilities that are realized in particular cultural and historical circumstances. In a partnership one is stretched intellectually and emotionally as participants appropriate the consequences of their shared experiences. Such a process includes expansion by complementarity, and it creates a safety zone within which both support and constructive criticism are practiced.

In "Our Visit to Hungarian Mathematics" we asked the question: "What is so special about Hungarian mathematics?" Our answer echoed that of Pólya and others who emphasized the role of the high school mathematical journal *A Középiskolai Matematikai Lapok,* the excellent quality of mathematics teaching in some of Hungary's gymnasia and universities, the appeal of a domain of intellectual achievement that requires limited funds in a poor country, and the impact of the Eötvös competition, which many of the mathematicians described in this paper won. To this list I would like to add the facilitating role of collaboration, which resulted in many transformative works, and which among mathematicians carries the imprint of Erdős Pál, whose publications are in excess of 1500 papers, and whose collaborators numbered, according to some estimates, more than 450 individuals.

This paper has focused on mathematicians whose fame was established in the first half of the twentieth century, and whose work has remained influential to this day. I see them as pioneers in an intellectual practice, which has become widespread in the last fifty years. The quality of these partnerships supports the belief that "collaboration thrives on diversity of perspectives and on constructive dialogues between individuals negotiating their differences while creating their shared voice and vision" (John-Steiner, 6).

Notes

1 In physics, with a similar trend in chemistry, the general trend has been from single winners in the early part of the century to two to three physicists sharing the prize in the latter decades. (Source: http://www.slac.stanford.edu/library/nobel/ and http://userpage.chemie.fu-berlin.de/diverse/bib/nobel_chemie_e.html/.)

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