MATHEMATICS IN LANGUAGE

© Zsolt FÜLÖP
(University of Szeged, Szeged, Hungary)

fulop.zs32@freemail.hu

Received: 24.07.2013; Accepted: 19.12.2013; Published online: 16.08.2014

Within this paper, we are focusing on the relationships between Language and Mathematics. Both Language and Mathematics operate with sentences. The problem of formulating sentences is wider and richer in Language, this kind of problem in Mathematics is narrow but more precise. Considering a statement we set up the problem of its precise negation. In general we get various variants of answers, but Mathematics accepts only one precise answer. The main objective of this study is to analyse three statements and their negations with the tools of mathematics, such as conjunction, disjunction, implication, etc. More precisely, we attempt to highlight the students' way of thinking related to the problem of negation. A sample of 78 primary school students (8-th grade) and 65 high school students (11-th grade) participated in the study. The test-paper contained the statements and the students had to choose the perfect negation of the statement from six versions of answer. We have to mention that only one is considered the perfect negation, if we argue with the tools of Mathematics. The aim of the research was to find out how the students can handle this kind of problem by the use of their Language and Grammar knowledge, because of the fact that the tools of Mathematics necessary to solve the problems are contained in the 12-th grade curriculum. The results show that a small part of the students gave the right answer, namely the perfect negation of the statement. Our conclusion is that the Language and Grammar knowledge is not enough to find out the perfect negation of the statement, it is necessary the students be acquainted with the tools of Mathematics, especially with the tools of Mathematical logic. Our suggestion is that it is necessary to improve the students' logical thinking and their inclination to manipulate the rigorous rules of the Mathematics.

Keywords: Mathematical methods used in Language, perfect negation, elements and operations of the Mathematical logic, conjunction, disjunction, implication

Mathematics is a tool in which students learn how to deal with problems, and how to apply their knowledge into real life problems, they improve their ability about logical thinking and reasoning. The elements of the Mathematical logic and the Grammar rules are interrelated. Many interesting Grammar problems are answerable by the use of the tools of Mathematical logic. An interesting problem is the problem of the negation, namely we consider a statement and we set up the problem of its perfect negation. The problem of the negation in the common language is more complicated than in the field of Mathematical logic. We can say that the Language is wider and richer, the Mathematics is narrow but more precise. If we set up the problem of the negation of a statement we can get more variants of answer in the common language, there are several versions to negate a sentence in Grammar (Kiss et al., 1998), but Mathematics accepts only one answer (and its mathematically equivalent formulations). We call the perfect negation the variant of answer what we can obtain with the rigorous rules of the Mathematical logic.

Theoretical framework

A mathematical statement is a sentence that states a fact or contains a complete idea and it can be judged to be true or false.

A truth table is a mathematical table used in Mathematical logic to compute the functional values of logical expressions on each of their functional arguments, that is, on each combination of values taken by their logical variables. In particular, truth tables can be used to tell whether a propositional expression is true or false for all legitimate input values. Practically, a truth table is composed of one column for each input variable, and one final column for all of the possible results of the logical operation that the table is meant to represent. Each row of the truth table therefore contains one possible configuration of the input variables (for instance, p=true q=false), and the result of the operation for those values (see Truth table – Wikipedia, the free encyclopedia).

In order to find the perfect negation of a statement we have to use the following logical operations. In the next we give a short introduction in the field of mathematical operations and we also give a mathematical form of sentences and connectives.

- Logical negation is an operation on one logical value, typically the value of a statement, that produces a value of *true* if its operand is false and a value of *false* if its operand is true.
  The logical negation of the statement p is denoted by $\neg p$ (that means *not p*) and its truth table is the following:

Table 1.

p	$\neg p$
true	false
false	true

- Logical conjunction is an operation on two logical values, typically the values of two propositions, that produces a value of *true* if both of its operands are *true*, otherwise it produces a value of *false*.
  The logical conjunction is denoted by $p \wedge q$ (that means *p and q*) and its truth table is the following:

Table 2.

p	q	$p \wedge q$
true	true	true
true	false	false
false	true	false
false	false	false

- Logical disjunction is an operation on two logical values, typically the values of two propositions, that produces a value of *true* if at least one of its operands is *true*, otherwise it produces a value of *false*.
  The logical conjunction is denoted by $p \vee q$ (that means *p or q*) and its truth table is the following:

Table 3.

p	q	$p \vee q$
true	true	true
true	false	true
false	true	true
false	false	false

- Logical implication is associated with an operation on two logical values, typically the values of two propositions, that produces a value of *false* just in the singular case the first operand is *true* and the second operand is *false*.
  The logical conjunction is denoted by $\mathrm{p} \Rightarrow \mathrm{q}$ (that means *if p then q*) and its truth table is the following:

Table 4.

p	q	$p \Rightarrow q$
true	true	true
true	false	false
false	true	true
false	false	true

In the Language "the conditional sentences are sentences expressing factual implications, or hypothetical situations and their consequences. They are so called because the validity of the main clause of the sentence is *conditional* on the existence of certain circumstances, which may be expressed in a dependent clause or may be understood from the context. A full conditional sentence therefore contains two clauses: the dependent clause expressing the condition, called the *protasis*, and the main clause expressing the consequence, called the *apodosis*. An example of such a sentence (in English) is "If it rains, the picnic will be cancelled." Here the condition is expressed by the clause *if it rains*, this being the protasis, while the consequence is expressed by *the picnic will be cancelled*, this being the apodosis. In terms of logic, the protasis corresponds to the antecedent, and the apodosis to the consequent. Language uses a variety of grammatical forms and constructions in conditional sentences. The forms of verbs used in the protasis and apodosis are often subject to particular rules as regards their tense and mood. "*There are various ways of classifying conditional sentences. One distinction is between those that state an implication between facts, and those that set up and refer to a hypothetical situation. There is also the distinction between conditional that are considered factual or predictive, and those that are considered counterfactual or speculative* *(referring to a situation that did not or does not really exist).*" (See Conditional sentence – Wikipedia, the free encyclopedia: http://en.wikipedia.org/wiki/Conditional_sentence). In general Mathematics deal with conditional sentences expressing an implication, what essentially states that if one fact holds, then so does another, and the facts are usually stated in whatever grammatical tense is appropriate to them. Such sentences may be used to express a certainty, a universal statement, a law of science, etc. (in this cases *if* may often be replaced by *when*). An example of such a sentence is "If it snows (then) I do not go to cinema." Mathematics also deals with sentences used for logical deductions about particular circumstances (which can be in various mixtures of past, present and future), such as the sentences "If it's raining now, then your laundry is getting wet." , „If it's raining now, there will be mushrooms to be picked next week." and „If it's raining here now, then it was raining on the West Coast this morning." An important mathematical problem related to conditional sentences is the problem of the plausible inference, treated by *György Pólya* (1988).

The main problem in this work is how to find out the perfect negation of a statement if we operate with the tools of the Mathematical logic? For example, we consider the sentence "It is snowing and Winnie the Pooh is cold." and we set up the problem of its perfect negation. In general we have two possible variants of answer:

- It is not snowing *and* Winnie the Pooh is not cold. (*Variant 1.*)
- It is not snowing *or* Winnie the Pooh is not cold. (*Variant 2.*)

We use the methods of Mathematical logic to choose the perfect negation. We have to deal with two statements:

Statement "It is snowing" is denoted by "p"

Statement "Winnie the Pooh is cold" is denoted by "q"

The statement "It is snowing *and* Winnie the Pooh is cold." is denoted by "$p \wedge q$ "and its negation is denoted by "$\neg(p \wedge q)$". First of all we create the truth table of the negation:

Table 5.

p	q	$p \wedge q$	$\neg(p \wedge q)$
true	true	true	false
true	false	false	true
false	true	false	true
false	false	false	true

The 4-th column of the *Table 5.* contains the output values of the perfect negation. The correct variant of answer (from the *Variant 1.* or *Variant 2.* mentioned above) must have the same output values.

We consider the *Variant 1.* "It is not snowing *and* Winnie the Pooh is not cold.", this is denoted by "$\neg p \wedge \neg q$". We have to create the truth table of the *Variant 1.*

Table 6.

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
true	true	false	false	false
true	false	false	true	false
false	true	true	false	false
false	false	true	true	true

The 5-th column of the *Table 6.* contains the output values of the *Variant 1.*, and we can see that this output values differs from the output values of the perfect negation (the 4-th column of the *Table 5.*), namely there are errors in the 2-nd and 3-rd rows. Therefore, the *Variant 1.* is not the perfect negation of the statement.

Let us deal with the *Variant 2.* "It is not snowing *or* Winnie the Pooh is not cold." , this is denoted by "$\neg p \vee \neg q$". The truth table of the *Variant 2.* is the following:

Table 7.

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$
true	true	false	false	false
true	false	false	true	true
false	true	true	false	true
false	false	true	true	true

We can see that the output values of the *Variant 2.* (the 5-th column of the *Table 7.*) are identical with the output values of the perfect negation. We draw the conclusion that "It is not snowing *or* Winnie the Pooh is not cold." is the perfect negation of the statement "It is snowing and Winnie the Pooh is cold."

In the following we consider other possible variants of answer:

It is not snowing and Winnie the Pooh is cold. (denoted by $\neg p \wedge q$)

It is not snowing or Winnie the Pooh is cold. (denoted by $\neg p \vee q$)

It is snowing and Winnie the Pooh is not cold. (denoted by $p \wedge \neg q$)

It is snowing or Winnie the Pooh is not cold. (denoted by $p \vee \neg q$)

We can see that none of the variants mentioned above is the perfect negation. In order to prove this we have to create the truth table of the statements:

Table 8.

p	q	$\neg p$	$\neg q$	$\neg p \wedge q$	$\neg p \vee q$	$p \wedge \neg q$	$p \vee \neg q$
true	true	false	false	false	true	false	true
true	false	false	true	false	false	true	true
false	true	true	false	true	true	false	false
false	false	true	true	false	true	false	true

We can observe that all of the statements have a different output as compared to the output of the perfect negation (4-th column of the *Table 5.*), therefore none of these statements is the perfect negation.

Let us consider another example, the statement "If it snows then I do not go to cinema.". In this case we have to cope with a problem of implication, namely the statement is denoted by $p \Rightarrow q$, where the statement "It snows" is denoted by *p* and the statement "I do not go to cinema" is denoted by *q*. It is interesting to mention, that the negation of the statement "I do not go to cinema" is "I go to cinema", and it is denoted by $\neg q$.

The perfect negation of the statement "If it snows then I do not go to cinema." is denoted by $\neg(p \Rightarrow q)$, and we can see its output values in the *Table 9.*

Table 9.

p	q	$p \Rightarrow q$	$\neg(p \Rightarrow q)$
true	true	true	false
true	false	false	true
false	true	true	false
false	false	true	false

In order to find the perfect negation we have to consider the following six possible variants, and to create their truth table:

- It does not snow and I go to cinema. (denoted by $\neg p \wedge \neg q$)
- If it does not snow then I do not go to cinema. (denoted by $\neg p \Rightarrow q$)
- It does not snow and I do not go to cinema. (denoted by $\neg p \wedge q$)
- If it snows then I go to cinema. (denoted by $p \Rightarrow \neg q$)
- It snows and I go to cinema. (denoted by $p \wedge \neg q$)
- If it does not snow then I go to cinema. (denoted by $\neg p \Rightarrow \neg q$)

Table 10.

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$\neg p \Rightarrow q$	$\neg p \wedge q$	$p \Rightarrow \neg q$	$p \wedge \neg q$	$\neg p \Rightarrow \neg q$
true	true	false	false	false	true	false	false	false	true
true	false	false	true	false	true	false	true	true	true
false	true	true	false	false	true	true	true	false	false
false	false	true	true	true	false	false	true	false	true

We can see that the operations $p \wedge \neg q$ and $\neg(p \Rightarrow q)$ have the same output values. Therefore the perfect negation of the statement "If it snows I do not go to cinema." is the statement "It snows and I go to cinema."

In the same way *Szalay* proved, that the perfect negation of the proverb „*Early bird catches the worm*" is the statement „*The bird is early and it does not catch any worm*" (Szalay, 2010).

The student survey. Results and discussion

This study was conducted with the purpose of analyzing the students' approach to the problem of the negation. The aim of the research was to survey whether the Grammar and Language knowledge of the students is enough to find out the mathematically perfect negation or the knowledge related to Mathematical logic is necessary to solve this kind of problem.

The survey was carried out with a group of 78 primary school students (8-th grade) and 65 high school students (11-th grade) in Budapest. The worksheet contained three statements, all of them with six variants of negation. The students had to choose the negation of the statement from these six variants of answer. The test-paper is presented in the Appendix. The elements of the Mathematical logic, necessary to find out the mathematically perfect negation, are contained in the 12-th grade curriculum

(Kosztolányi et al., 2006). Therefore the students (from 8-th grade and 11-th grade) did not know the rules of the Mathematical logic and they could use only their knowledge related to Grammar and Language, in order to choose the negation of the statement. Our aim was to find out how many students choose the mathematically perfect negation, in order to draw the conclusion that the Grammar knowledge is enough to find out the perfect negation or it is necessary the knowledge related to Mathematical logic.

The first statement on the test-paper is “Early bird catches the worm”. Szalay proved that the mathematically perfect negation is “*The bird is early and it does not catch any worm*” (Szalay, 2010), namely the variant C on the test-paper. The students’ result is presented in the *Table 11.*

Table 11.

Variants of negation	Primary school	High-school
A. The bird is late and it does not catch any worm.	18	0
B. The bird is late and it catches the worm.	3	0
C. The bird is early and it does not catch any worm.	6	5
D. If the bird is late then it does not catch any worm.	36	51
E. If the bird is late then it catches the worm.	0	0
F. If the bird is early then it does not catch any worm.	15	9
Total	78	65

We can observe that a little part of the students chose the variant C (the mathematically perfect negation).

It is interesting to analyse why the majority of students chose the variant D? Let us begin to find out what is the meaning of the proverb “Early bird catches the worm.”? Everyone can choose between two versions:

- The bird is early and it catches the worm.
- If the bird is early then it catches the worm.

Szalay (2010) chose the second version to prove that the variant C is the perfect negation. Our assumption is the majority of students thought in the same way, namely they thought the meaning of the proverb is “If the bird is early then it catches the worm”. We try to pursue the students’ way of thinking. They had to deal with two statements (“the bird is early” and “it catches the worm”) and the connective *if…then* (it represents an implication). They thought that it is necessary to negate both of the statements and the construction of the sentence with *if…then* remains (they did not know that the entire structure of the sentence may be changed in the process of the negation). 24 students chose the variant F. This variant is closer to the perfect negation, since only the statement “it catches the worm” is negated. But the structure of the sentence (*if…then*) remains and this is the source of the error. 18 students chose the variant A. In this case the connective *and* is the correct one, but both of the statements are negated and this is the error. We have to mention that only 11 students gave the right answer.

The second statement is “It is snowing and Winnie the Pooh is cold.” In the theoretical framework we proved that the mathematically perfect negation is “It is not snowing *or* Winnie the Pooh is not cold.” (the variant C on the test-paper). The *Table 12.* contains the distribution of the students’ answers. We can observe that the majority of students chose the variant A

("It is not snowing *and* Winnie the Pooh is not cold."), namely they thought that they have to negate both of the sentences and the connective *and* remains. The situation is different, according to the rules of the Mathematical logic, namely the negation transforms the connective *and* in the connective *or* (as we proved in the theoretical framework). We have to mention that only a small number of students chose one of the variants, which contains the connective *or*, and only 9 students gave the mathematically right answer.

Table 12.

Variants of negation	Primary school	High-school
A. It is not snowing and Winnie the Pooh is not cold.	48	52
B. It is not snowing and Winnie the Pooh is cold.	11	6
C. It is not snowing or Winnie the Pooh is not cold.	5	4
D. It is snowing and Winnie the Pooh is not cold.	9	3
E. It is not snowing or Winnie the Pooh is cold.	0	0
F. It is snowing or Winnie the Pooh is not cold.	5	0
Total	78	65

The third statement is "If it snows then I do not go to cinema." and its perfect negation is "It snows and I go to cinema" (variant E on the test-paper). This is the most difficult one, because the initial statement contains the word *not* ("If it snows I do *not* go to cinema."), and we have to find out the *negation* of the statement (the word *not* means a negation in Language and its presence in the initial statement may create difficulties). The negation of the statement "I do not go to cinema" is "I go to cinema". This fact is quite trivial if we operate with the rules of the Mathematical logic, but in the Language it is strange to say. *Table 13.* contains the distribution of the students' answers. The majority of the students chose the variant F ("If it does not snow then I go to cinema."). In this variant both of the clauses ("it snows" and "I do not go to cinema") are negated and the structure *if…then* remains. Only a small part of the students gave the right answer (variant E).

Table 13.

Variants of negation	Primary school	High-school
A. It does not snow and I go to cinema.	17	0
B. If it does not snow then I do not go to cinema.	6	0
C. It does not snow and I do not go to cinema.	5	4
D. If it snows then I go to cinema.	6	4
E. It snows and I go to cinema.	6	3
F. If it does not snow then I go to cinema.	38	54
Total	78	65

We can observe that, in all of the cases, the initial statement was a complex sentence, which contains two clauses and a connective. The negation of a sentence is an operation, which changes the connective of the statement. Majority of the students thought that the connective remains intact in the process of the negation, and it was the main source of the errors. Many students committed the mistake that they negated both of the clauses. This is another source of error, because it is not necessary, in all of the cases, to negate both of the clauses in order to find the perfect negation.

We have to underline the small number of the right answers. This is a very good proof that the students' Grammar and Language knowledge is not enough to manipulate the rigorous rules of Mathematical logic related to statements and sentences. In order to find out the perfect negation it is necessary to know the elements, rules and operations of Mathematical logic.

Conclusion

The Mathematics and the Language are interrelated. The tools of Mathematical logic are very useful in other areas, such as the Grammar and Language, as we can use the rigorous rules of Mathematics to solve some problems related to Language. The problem of the negation is wider and richer in Language, but more precise in the field of Mathematical logic. In this work we want to underline the necessity to apply the methods and elements of the Mathematical logic in order to solve some problems related to negation. The majority of the students do not know that in order to negate a complicate sentence in many cases it is necessary to change the entire structure of the sentence and it is not necessary, in all of the cases, to negate all of the clauses. In this case the methods of Mathematical logic are useful tools to solve complicated Grammar problems. In our opinion the elements and operations of Mathematical logic are necessary to be introduced in the primary school, in order to underline the usefulness of Mathematics in other areas, such as Grammar. It is also necessary to improve the students' logical thinking and their inclination to manipulate the rigorous rules of the Mathematics. In many cases the rigorous rules of Mathematics are not interesting for pupils therefore it is necessary to show the relationship between Mathematics and other sciences.

References

Conditional sentence – Wikipedia, the free encyclopedia, http://en.wikipedia.org/wiki/Conditional_sentence [01.07.2013]

KISS K., É., KIEFER F., & SIPTÁR P. (1998). *Új magyar nyelvtan.* Budapest: Osiris.

KOSZTOLÁNYI J., PINTÉR K., KOVÁCS I., URBÁN J., & VINCZE I. (2006). *Sokszínű matematika 9-12*. Szeged: Mozaik.

PÓLYA Gy. (1988). *A matematikai gondolkodás művészete. II. Kötet: A plauzibilis következtetés.* Budapest: Gondolat.

SZALAY I. (2010*). Holistic Approach to the Teaching of Mathematics. Practice and Theory in Systems of Education*, *5* (1), 49-64.

Truth table – Wikipedia, the free encyclopedia. Retrieved from http://en.wikipedia.org/wiki/Truth_table [01.07.2013]

Appendix

1) Early bird catches the worm.
    - A. The bird is late and it does not catch any worm.
    - B. The bird is late and it catches the worm.
    - C. The bird is early and it does not catch any worm.
    - D. If the bird is late then it does not catch any worm.
    - E. If the bird is late then it catches the worm.
    - F. If the bird is early then it does not catch any worm.

2) It is snowing and Winnie the Pooh is cold.
    - A. It is not snowing and Winnie the Pooh is not cold.
    - B. It is not snowing and Winnie the Pooh is cold.
    - C. It is not snowing or Winnie the Pooh is not cold.
    - D. It is snowing and Winnie the Pooh is not cold.
    - E. It is not snowing or Winnie the Pooh is cold.
    - F. It is snowing or Winnie the Pooh is not cold.

3) If it snows I do not go to cinema.
    - A. It does not snow and I go to cinema.
    - B. If it does not snow then I do not go to cinema.
    - C. It does not snow and I do not go to cinema.
    - D. If it snows then I go to cinema.
    - E. It snows and I go to cinema.
    - F. If it does not snow then I go to cinema.